EconS 301- Intermediate Microeconomic Theory
Homework #2 - Due date: Tuesday December 6th, 2022.

1. Black Smoke eatery is the only restaurant in small town. They face inverse demand $p = 25 - 0.05q$ and have costs $TC(q) = 3 + 4q$. Unfortunately, the eatery produces a lot of unsightly black smoke at the same rate as output (so, pollution is equal to q).

   (a) Find the unregulated equilibrium.

   - The monopolist's profit maximization problem is

     $$\max_q \ \pi = (25 - 0.05q)q - (3 + 4q)$$

     Differentiating with respect to q

     $$\frac{\partial \pi}{\partial q} = 25 - 0.1q - 4 = 0$$

     rearranging, $0.1q = 21$, and solving for q, we find

     $$q^U = 210 \text{ units}$$

     which is the unregulated equilibrium output. The equilibrium price is

     $$p^U = 25 - 0.05(210) = \$14.5$$

   (b) Assume that the external cost of Black Smoke's pollution is $EC = 2q$. Find the social optimum.

   - The regulator solves

     $$\max_q \ \underbrace{(25 - 0.05q)q - (3 + 4q)}_{\text{Profits}} \underbrace{-2q}_{\text{EC}}$$

     Differentiating with respect to q,

     $$25 - 0.1q - 4 - 2 = 0$$

     rearranging, $0.1q = 19$, solving for q, we find

     $$q^{SO} = 190 \text{ units}$$

     the socially optimal output.

   (c) If the regulator is to seek the socially optimal output, what pollution quota would she set?

   - In this case, the regulator sets a quota equal to the socially optimal output of 190 units.

   (d) If the regulator is to seek the socially optimal output, what emission fee would she set?

- If the social planner uses an emission fee, it solves the two-stage game as described in Example 17.4.
- *Second stage.* If the regulator sets an fee t on every unit of output, the monopolist's profit-maximization problem becomes

$$\max_q \ \pi = (25 - 0.05q)q - (3 + 4q) - tq$$

  Differentiating with respect to output q, we obtain

$$25 - 0.1q - 4 - t = 0$$

  rearranging, $0.1q = 21 - t$, and solving for q, we find the monopolist's output when it faces an emission fee

$$q(t) = 210 - 10t$$

- *First stage.* The regulator anticipates the monopolist's output from the fee and sets the fee so that this output is equal to the socially optimal output, that is $q(t) = q^{SO}$:

$$210 - 10t = 190$$

  rearranging, $10t = 20$, and solving for t, we find the optimal fee $t = \$2$.

2. Can the following situations be effectively addressed under the Coase theorem? Discuss why or why not.

   (a) Air pollution

   - Here, the biggest barrier to addressing air pollution under the Coase theorem is that large negotiation costs are present. Because the dispute is between polluting firms and citizens exposed to it, many agents are in play, which makes it hard for the citizens to coordinate and negotiate. This causes a breakdown in the Coase theorem.

   (b) A homeowner playing loud music (negatively affecting his neighbors) within a homeowners' association (HOA)

   - This situation can likely be settled under the Coase theorem. Property rights are well defined, and the right to play loud music is likely covered under an enforceable contract set by the HOA or local community ordinances. Each of the neighbors is well aware of the benefits and costs at play, and negotiation costs are likely small.

   (c) Light pollution in a town with a powerful telescope (that needs surrounding darkness to be effective)

   - This situation is a little tricky. Although there are many agents, and negotiation costs may be large, these telescopes are usually in low-population areas, minimizing the amount of people (and houses) nearby, which keeps negotiation costs low. Property rights may not be well defined here, though, as citizens and the telescope operator may both claim a right to the light or dark.

(d) Use of an irrigation ditch between two ranches

- Here, the Coase theorem is very much in play if the water rights are well defined and there is observable (measurable) use of water from the irrigation ditch. With few agents, negotiation costs are low.

3. Two neighbors in a rural community were fed up with the town's landfill policies and decided to purchase land together to use as their own landfill. However, the two neighbors did not anticipate the consequences of their purchase and quickly found their new landfill to smell. Each neighbor has 10 bags of trash. Dumping on their own land is cheap, but they have to endure an increased bad smell, however dumping at the town's landfill incurs a cost of \$3 per bag. Neighbor 1 lives downwind of the new landfill and endures the brunt of the smell. Her utility is

$$u_1(b_1, b_2) = -3(10 - b_1) - (b_1 + b_2)^2 - (b_1 + b_2)$$

while the upwind neighbor's utility is

$$u_2(b_1, b_2) = -3(10 - b_2) - (b_1 + b_2)^2$$

where b_i is the number of bags each neighbor dumps at their own landfill. Note that the utilities are negative as both actions are actually

(a) How much will each neighbor dump at their new landfill?

- Neighbor 1 maximizes their utility by solving

$$\max_{b_1} \; u_1(b_1, b_2) = -3(10 - b_1) - (b_1 + b_2)^2 - (b_1 + b_2)$$

Differentiating with respect to b_1, we find

$$3 - 2(b_1 + b_2) - 1 = 0$$

Solving for b_1, we have neighbor 1's best response function

$$b_1(b_2) = 1 - b_2$$

- Neighbor 2's utility-maximization problem is

$$\max_{b_2} u_2(b_1, b_2) = -3(10 - b_2) - (b_1 + b_2)^2$$

Differentiating with respect to b_2 yields

$$3 - 2(b_1 + b_2) = 0$$

Solving for b_2 yields neighbor 2's best response function

$$b_2(b_1) = \frac{3}{2} - b_1$$

Plugging neighbor 1's best response function into neighbor 2's yields

$$b_2 = \frac{3}{2} - \underbrace{1 + b_2}_{b_1(b_2)}$$

we obtain a continuum of solutions. Hence, if we consider that $b_1 = b_2$

$$b_2^* = \frac{3}{2} - b_2 = \frac{3}{4}.$$

Plugging this into neighbor 1's best response, we obtain the equilibrium amount of garbage she dumps at their own landfill

$$b_1^* = 1 - \frac{3}{4} = \frac{1}{4}.$$

A total of $b_1 + b_2 = \frac{1}{4} + \frac{3}{4} = 1$ bag of trash will be dumped in their new landfill.

(b) If the neighbors were to coordinate, how much would they dump at their new landfill?

- If the neighbors coordinate to maximize their joint utility, the solve

$$\max_{b_1, b_2} \; u_1(b_1, b_2) + u_2(b_1, b_2)$$

which, after plugging in for the appropriate utility functions

$$\max_{b_1, b_2} \; \underbrace{-3(10 - b_1) - (b_1 + b_2)^2 - (b_1 + b_2)}_{u_1} \underbrace{-3(10 - b_2) - (b_1 + b_2)^2}_{u_2}$$

This simplifies to

$$\max_{b_1, b_2} -3(20 - b_1 - b_2) - 2(b_1 + b_2)^2 - (b_1 + b_2)$$

Differentiating with respect to b_1 and b_2, respectively, yields

$$3 - 4(b_1 + b_2) - b_1 = 0$$
$$3 - 4(b_1 + b_2) - b_2 = 0$$

Rearranging each of these, we get the following system

$$3 - 4b_2 - 5b_1 = 0$$
$$3 - 4b_1 - 5b_2 = 0$$

Solving the first equation for b_1 yields $b_1 = \frac{3}{5} - \frac{4}{5}b_2$. We can plug this into the second equation to find that

$$3 - 4\underbrace{\left(\frac{3}{5} - \frac{4}{5}b_2\right)}_{b_1} -5b_2 = 0$$

Solving for b_2 yields neighbor 2's jointly-optimal amount of trash to dump in their own landfill

$$b_2 = \frac{3}{8}$$

Plugging this into neighbor 1's equation above yields her equilibrium bags of trash at their own landfill, $b_1 = \frac{3}{5} - \frac{4}{5}\frac{3}{8} = \frac{24-12}{40} = \frac{3}{5}$ bags.

- We find that each neighbor now changes the amount of bags they throw in their landfill, and the total number of bags is

$$\frac{3}{5} + \frac{3}{8} = \frac{24+15}{40} = \frac{39}{40}.$$