EconS 301- Intermediate Microeconomic Theory
Recitation - Friday December 2nd, 2022.

1. Firms within an industry can experience "spillover effects" of investing in new technologies. Many competing industries have firms concentrated in one region or city, where the workers at competing firms may interact with each other leading to the exchange of ideas and this positive externality. Consider two firms that face inverse demand $p = 10 - q_1 - q_2$. Each firm faces total costs

$$TC = (4 - x_i - 0.25x_j)q_i + 0.5x_i^2,$$

where original marginal cost of production ($4) can be reduced by investing in x_i and also decreases by a portion of their rival's investment at a rate of $0.25x_j$. The cost of investing in the cost-reducing technology is $0.5x_i^2$. Assume that, in the first stage, every firm i invests x_i dollars in R&D. In the second stage, every firm observes the R&D investment (x_i, x_j), and thus the cost function of each firm, and firms compete in quantities (à la Cournot).

   (a) If the firms act competitively, how much will each firm produce and how much of the cost-reducing technology will they invest in? (*Hint:* Since this is a sequential-move game where firms are perfectly informed, you need to find the subgame perfect equilibrium (SPE) of the game, operating by backward induction. You should then start analyzing firms' output choices in the second-stage of the game, for any pair of x_i and x_j; and then, anticipating the profits firms make in the second stage, find their equilibrium investment in R&D in the first stage.)

   (b) Discuss how the knowledge spillover impacts investment in the new technology. In other words, how does firm 2's investment x_2 impact firm 1's decisions on output and investment compared to if they were to act cooperatively?

2. A coal company produces electricity with total cost $TC = 4q$ and emissions $e = 2q$. The coal company, being large, is a monopoly in its region and faces demand $p = 20 - q$.

   (a) If the external cost of emissions is $EC = 2e$, what emission fee would induce the social optimum?rearranging, $16 - t = 12$. Solving for t, we find the socially optimal fee $t = \$4$.

   (b) Now assume that the company can invest in a new technology that reduces their emissions to $e = 2(q - \alpha)$ at a cost of $2\alpha^2$. Intuitively, if $\alpha = 0$, the firm does not invest in the technology and its emissions are the same as in part (a), while a larger investment in the technology increases α which decreases emissions and the fee paid by the firm. Assume the fee you found in part (a), how much will the company invest in α?