EconS301 - Quiz #5

In Tacoma, WA there are two suppliers of distilled water, labeled as firm Aqua and firm Blue. Distilled water is a homogenous good. Let *p* denote the price per gallon, q_A quantity sold by firm Aqua, and q_B the quantity sold by firm Blue. Both firms are located close to a spring, so the only production cost is the cost of bottling. Formally, each firm bears a production cost of $c_A = c_B = \$2$ per one gallon of water.
Tacoma's aggregate inverse demand function for distilled water is given by $p = 6\text{-}Q = 6\text{-}q_A\text{-}q_B$, where $Q = q_A + {}_{qB}$ denotes the aggregate industry supply of distilled water in Tacoma. Solve the following problems:

a) Solve for firm A's best-response function, $q_A = R_A(q_B)$. Also solve for firm B's best-response function, $q_B = R_B(q_A)$. Show your derivations.
b) Solve for the Cournot equilibrium output levels $q^c{}_A$ and $q^c{}_B$. State which firm sells more water (if any) and why.
c) Solve for the aggregate industry supply and the equilibrium price of distilled water in Tacoma.
d) Solve for the profit level made by each firm, and for the aggregate industry profit. Which firm earns a higher profit and why?

Solution:

a) The best-response functions are given by
$q_A(q_B) = (4 - q_B)/2$ and $q_B(q_A) = (4 - q_A)/2$ or $q_A(q_B) = 2 - 1/2\, q_B$ and $q_B(q_A) = 2 - 1/2 q_A$
b) The above best-response functions constitute two linear equations with two variables, q_A and q_B. The unique solution is $q^c{}_A = q^c{}_B = 4/3$ gallons. Both firms produce the same amount since there equally efficient in the sense that the bear identical production costs.
c) $Q = qA + qB = 4/3 + 4/3 = 8/3$ gallons. The equilibrium price is $p = 6 - Q = \$10/3$.
d) Since there are no fixed costs, $\pi_A = (p - c_A)q_A = (10/3 - 2)4/3 = \$16/9$. Similarly, $\pi_B = (p - c_B)q_B = (10/3 - 2)4/3 = \$16/9$. Industry profit is then $\Pi = \pi_A + \pi_B = 16/9 + 16/9 = \$32/9 = 3.56$. Both firms earn the same profit since the bear identical production costs