Answer key Midterm #2 EconS 301

Question #1

The market demand function for Pierogi in Pittsburgh Pennsylvania has a constant elasticity of -3. More precisely the actual daily demand was estimated to be $Q=34560p^{-3}$, where p is the price per pound. Each pound costs c=$8 to produce. Pittsburgh is served by a local monopoly producer. Compute the monopoly's profit-maximizing price and the monopoly's profit level. Show your computations.

Solution

Proposition 3.3!

The monopoly equates marginal revenue to marginal cost, c. Therefore,

$$p^m\left(1+\frac{1}{Elasticity}\right) = p^m\left(1+\frac{1}{-3}\right) = \frac{2p^m}{3} = c = 8$$

Therefore, p^m = $12. Next, Q = $34560p^{-3}$ = 20 units. Hence, π= (p - c)Q =(12 - 8)20 = $80.

Question #2

In Pullman there is only one fortune teller who acts as a monopoly. The inverse demand function for this service is given by $P = 9 - \frac{3Q}{2}$, where P denotes the price charged per visit, and Q the quantity demanded for fortune telling.

a) Suppose the cost function of this fortune teller is given by C(Q) = 3 + Q. That is, the marginal cost is c = $1 (consisting of her value time and other “communication" expenses), and the fixed cost is F = $3 (say, monthly rent on her office space). Compute and draw the fortune teller's marginal cost and average functions, as well as the marginal revenue function.
b) Algebraically compute the fortune teller's profit-maximizing output, price, and profit.
c) Compute the price elasticity at the profit-maximizing output.

Solution

a) The marginal and average cost functions are given by:

$$MC(Q) = \frac{dC(Q)}{dQ} = 1 \ \ and \ \ AC(Q) = \frac{C(Q)}{Q} = \frac{3}{Q} + 1$$

The marginal revenue function is MR(Q) = 9 – 3Q

b) The monopoly equates MR(Q) = 9 - 3Q =1 = MC(Q) to obtain the profit-maximizing output level Q = 8/3=2.67. The monopoly price is then p = 9 – 3Q/2 = $5. Finally, the profit is ∏ = 5x 2.67- 1 x2.67 - 3 = $7.67
c) The direct demand function is Q(p) = 6 – 2/3p. Then, the price elasticity is:

$$\epsilon_p = \frac{dQ}{dp} \times \frac{p}{q} = -\frac{2}{3} \times \frac{5}{2.67} = -1.25$$

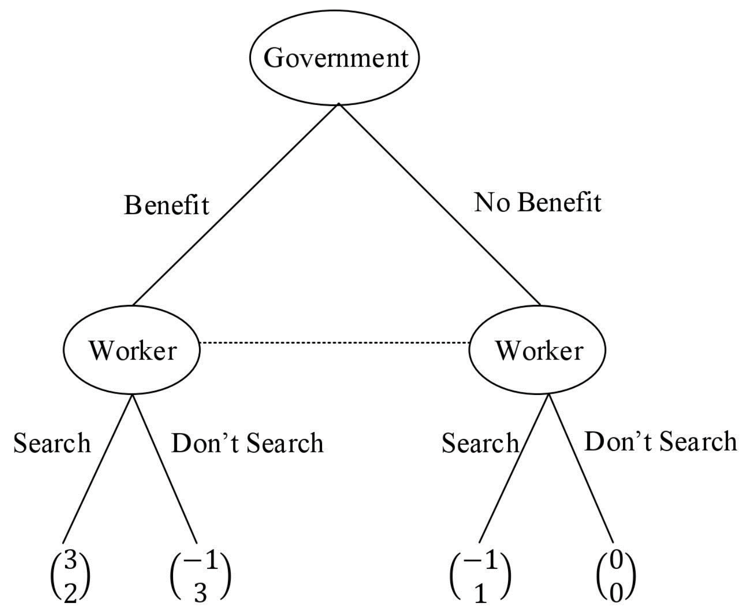
Thus, the demand is inelastic.

Question #3

Consider the following simultaneous-move game between the government (row player), which decides whether to offer unemployment benefits, and an unemployed worker (column player), who chooses whether to search for a job. As you interpret from the payoff matrix below, the unemployed worker only finds it optimal to search for a job when he receives no unemployment benefit; while the government only finds it optimal to help the worker when he searches for a job.

		Worker	
		Search	*Don't Search*
Government	*Benefit*	3, 2	-1, 3
	No benefit	-1, 1	0, 0

a. Represent this game in its extensive form (game tree), where the government acts first and the worker responds without observing whether the government offered unemployment benefits.



b. Does government has strictly dominant strategies? How about the worker?

There are no strictly dominant strategies for both government and worker. Specifically, when the worker chooses (not) to search for a job, the government will be better off (not) offering unemployment benefits. Whereas, when the government chooses (not) to offer unemployment benefits, the worker will be better off (not) searching for a job.

c. Find which strategy profile (or profiles) survive the application of IDSDS.

In this context, every strategy survives IDSDS, which contribute to a strategy profile of

{(Benefit,Search),(Benefit,Don't Search),(No Benefit,Search),(No Benefit,Don't Search)}

Question #4

The demand function for concert tickets to be played by the Ann Arbor symphony orchestra varies between nonstudents (N) and students (S). Formally, the two demand functions of the two consumer groups are given by qN = 7,290(pN)-3 and qS = 40,960(pS)-4

Assume that the orchestra's total cost function is TC(Q) = 6Q, where Q = qN +qS is to total number of tickets sold. Compute the concert ticket prices set by this monopoly orchestra, and the resulting ticket sales, assuming that the orchestra can price discriminate between the two consumer groups, say by requiring students to submit their student ID cards. [**Hint**: MR=P(1+1/ε), where MR denotes marginal revenue and ε is the demand elasticity]

In the market for nonstudents,

$$MR_N = p_N = \left[1 + \frac{1}{-3}\right] = c = 6 \Longrightarrow p_N = \$9.$$

$$MR_S = p_S = \left[1 + \frac{1}{-4}\right] = c = 6 \Longrightarrow p_N = \$8.$$

To find the amount of tickets sold to each group, solve

$$q_N = 7290 \cdot 9^{-3} = 10 \quad \text{and} \quad q_S = 40960 \cdot 8^{-4} = 10 \quad \text{hence} \quad Q = q_N + q_S = 20.$$

5. Consider a firm with production function $q = \sqrt{z}$, using one input (e.g., labor) to produce units of output q. The price of every unit of input is $w > 0$, and the price of every unit of output is $p > 0$.

   (a) Set up the firm's profit-maximization problem (PMP), and solve for its unconditional factor demand $z(w, p)$.

   - The firm chooses the units of input z to solve

   $$\max_{z \geq 0} p\sqrt{z} - wz$$

   where the first term indicates total revenue, whereas the second reflects total costs. Taking first-order condition with respect to z, we obtain

   $$p\frac{1}{2}z^{-1/2} - w \leq 0.$$

   In the case of interior solutions, we can solve for z to find the unconditional factor demand

   $$z(w, p) = \frac{p^2}{4w^2}.$$

   (b) What is the output level that arises from using the amount of inputs $z(w, p)$? Label this output level $q(w)$.

   - Inserting $z(w, p)$ into the firm's production function $\sqrt{z}$, we obtain

   $$q(w) = \frac{p}{2w}$$

   (c) Set up the firm's cost-minimization problem (CMP), and solve for its conditional factor demand $z(w, q)$ for any output level q. (For now, we write the constraint of the CMP to be $f(z) \geq q$, where the output level q that the firm seeks to reach does not necessarily coincide with that found in part (b), $q(w)$.)

   - The firm chooses the units of input z to solve

   $$\min_{z \geq 0} w \cdot z$$

   $$\text{subject to} \quad \sqrt{z} \geq q$$

   Setting up the Lagrangian, we obtain

   $$L = w \cdot z - \lambda \left(\sqrt{z} - q \right).$$

Taking first-order condition with respect to z, we find that

$$w - \frac{\lambda}{2\sqrt{z}} = 0,$$

and solving for z, we find

$$z = \frac{\lambda^2}{4w^2}.$$

Now, note that the constraint must be binding in equilibrium, so that $\sqrt{z} = q$. Otherwise, the firm could still reduce its total costs and satisfy the output constraint (reaching output target q). Using the binding constraint $\sqrt{z} = q$ into the above result, we obtain that

$$\lambda = 2qw$$

Last, we solve for z, to find the conditional factor demand

$$z(w, q) = q^2$$

(d) Evaluate the conditional factor demand $z(w, q)$ at output level $q = q(w)$, to obtain $z(w, q(w))$. Show that it coincides with the unconditional factor demand $z(w, p)$ found in part (a), that is,

$$z(w, q(w)) = z(w, p).$$

- We find that

$$z(w, q(w)) = \left(\frac{p}{2w}\right)^2 = \frac{p^2}{4w^2} = z(w, p)$$

which coincides with the unconditional factor demand $z(w, p)$ found in part (a).

6. Some small towns may only have one restaurant, making them a monopoly in that town. Consider Rosie's Diner in a small mountain town. Her inverse demand is $p(q) = 20 - 0.4q$, where q represents meals per week, and her costs are $C(q) = 5q$.

(a) Find Rosie's profit-maximizing price, quantity, and profits.

- *Monopoly output.* We want to set Rosie's marginal revenue equal to marginal cost. Her marginal revenue is

$$MR(q) = p(q) - \frac{\partial p(q)}{\partial q}q = \underbrace{20 - 0.4q}_{p(q)} - 0.4(q) = 20 - 0.8q.$$

Her marginal cost is

$$MC = \frac{\partial C(q)}{\partial q} = 5$$

Setting $MR(q) = MC(q)$, we have that

$$20 - 0.8q = 5$$

solving for q, we find her profit-maximizing quantity

$$q^M = 18.75 \text{ meals.}$$

- *Monopoly price.* Plugging $q^M = 18.75$ into her inverse demand, we obtain the profit-maximizing price

$$p^M = 20 - 0.4(18.75) = \$12.5.$$

- *Monopoly profit.* Rosie's monopoly profit is

$$\begin{aligned}\pi^M &= p^M q^M - C(q^M) \\ &= 12.5(18.75) - 5(18.75) \\ &= \$140.63.\end{aligned}$$

(b) The road into the town has become considerably harder to traverse since a recent mudslide and Rosie's suppliers have increased their delivery price. This has increased her costs to $C(q) = 8q + 10$. How do her equilibrium prices, quantity, and profits change?

- *Monopoly output.* With her increased costs, her marginal costs change to

$$MC = \frac{\partial C(q)}{\partial q} = 8$$

Setting $MR(q) = MC(q)$, we have

$$20 - 0.8q = 8$$

or, simplifying, $12 = 0.8q$. Solving for q, we find that Rosie now sells

$$q^M = 15 \text{ meals,}$$

3.75 meals less than before the cost increase.

- *Monopoly price.* Her price increases to

$$p^M = 20 - 0.4(15) = \$14.$$

- *Monopoly profit.* Her profit now decreases to

$$\begin{aligned}\pi^M &= p^M q^M - C(q^M) \\ &= 14(15) - 8(15) - 10 \\ &= \$80.\end{aligned}$$

  Intuitively, while her price went up, the reduced sales produce a decrease in her overall profits.

(c) After the mudslide, there have been less visitors hiking the trails around town, which has decreased demand to $p(q) = 15 - 4q$. Does Rosie stay in business?

- *Monopoly output.* With the new demand, her marginal revenue changes to

$$MR(q) = p(q) - \frac{\partial p(q)}{\partial q} q = \underbrace{15 - 0.4q}_{p(q)} - 0.4(q) = 15 - 0.8q.$$

  Setting $MR(q) = MC(q)$, we have that

$$15 - 0.8q = 8$$

  or, $0.8q = 7$. Solving for q, we obtain

$$q^M = 8.75 \text{ meals}$$

  , almost half of those in part (b), where we found $q^M = 15$ meals.

- *Monopoly price.* She will now charge a price

$$p^M = 15 - 0.4(8.75) = \$11.5,$$

  which is also lower than that in part (b), where we found $p^M = \$14$.

- *Monopoly profit.* Her monopoly profit is

$$\begin{aligned}\pi^M &= p^M q^M - C(q^M) \\ &= 11.5(8.75) - 8(8.75) - 10 \\ &= \$20.63.\end{aligned}$$

which is lower than in parts (a) and (b), but still positive implying that she stays in business.

7. DEFY store provides entertainment to the Moscow community. The demand for stretchy trampoline is $p(q) = 10 - \frac{q}{10}$, and this company's costs are $C(q) = 1 + \frac{q}{2}$.

   (a) Does DEFY exhibit the properties to be a "natural monopoly"?

   - As discussed in section 10.2, to be a natural monopoly, Duchess energy must have decreasing average costs. Duchess Energy's average costs are

     $$AC(q) = \frac{C(q)}{q} = \frac{1 + 0.5q}{q} = \frac{1}{q} + 0.5.$$

     Since q only shows up in the denominator of average costs, it is the case that average costs are decreasing in q, and thus Duchess Energy is a natural monopoly.

   (b) Find the unregulated monopolist's profit-maximizing price, output, and profit.

   - The monopolist maximizes

     $$\max_{q \geq 0} (10 - 0.1q)q - (1 + 0.5q).$$

     Differentiating with respect to q, we obtain

     $$10 - 0.2q - 0.5 = 0,$$

     rearranging, we have $9.5 = 0.2q$. Solving for q, we find that the unregulated electric company produces

     $$q^U = 47.5 \text{ units.}$$

     Inserting this output in the inverse demand function, we obtain that the company charges a price of

     $$p^U = 10 - 0.1(47.5) = 10 - 4.75 = \$5.25.$$

     As a result, Duchess Energy earns a profit of

     $$\pi^U = 5.25(47.5) - [1 + 0.5(47.5)] = 249.38 - 24.75 = \$224.63.$$

   (c) The Moscow city government passes a law that requires utility and other electricity providers to practice MC pricing (i.e., $p(q^R) = MC(q^R)$). What is the regulated monopolist's output, price, and profit?

- In this case, the monopolist sets its price such that

$$\underbrace{10 - 0.1q}_{p(q)} = \underbrace{0.5}_{MC(q)},$$

rearranging, we first have $0.1q = 9.5$, and solving for q, we find the regulated monopolist produces an output of

$$q^R = 95 \text{ units.}$$

Inserting this output in the inverse demand function, we confirm that the regulated company charges a price equal to marginal cost, \$0.5,

$$p^R = 10 - 0.1(95) = 10 - 9.5 = \$0.5,$$

implying that the profits of the regulated monopolist are

$$\pi^R = 0.5(95) - [1 + 0.5(95)] = 47.5 - 48.5 = -\$1.$$

When the regulator enforces marginal cost pricing, the firm cannot recover its fixed costs.

(d) What is the lump-sum subsidy that the regulator must provide the electric utility company to practice MC pricing without operating at a loss?

- When the utility company practices marginal cost pricing, its profit is $-\$1$, as shown in part (c). Therefore, if the regulator offers a lump-sum subsidy of $S = \$1$, the utility company would break even.

(e) Compute the consumer surplus from the pricing strategies in parts (a) and (b).

- *Unregulated monopoly.* Under uniform pricing in the unregulated monopoly, consumer surplus is calculated as $CS = \frac{1}{2}(a - p^U)q^U$, where $a = 10$ is the vertical intercept of the inverse demand, yielding

$$CS^U = \frac{1}{2}(10 - \underbrace{5.25}_{p^U})\underbrace{47.5}_{q^U} = \frac{1}{2}(4.75)(47.5) = \$112.81.$$

- *Marginal cost pricing.* Under marginal cost pricing monopoly, consumer surplus is calculated as $CS = \frac{1}{2}(a - p^R)q^R$, where $a = 10$ is the vertical intercept of the inverse demand, obtaining

$$CS^R = \frac{1}{2}(10 - \underbrace{0.5}_{p^R})\underbrace{95}_{q^R} = \frac{1}{2}(9.5)(95) = \$451.25.$$

(f) Discuss the pros and cons of MC pricing in natural monopolies.

- If we add total welfare under each pricing scheme, we find that the sum of profit and consumer surplus in the unregulated monopoly is

$$W^U = \pi^U + CS^U = \$224.63 + \$112.81 = \$337.44,$$

while under the marginal cost pricing welfare becomes

$$W^R = \underbrace{(\pi^R + S)}_{\text{Profit after subsidy}} + CS^R - T = (-\$1 + \$1) + \$451.25 - \$1 = \$450.25.$$

The first term represents the firm's profits after receiving the lump-sum subsidy, the second term reflects consumer surplus in this setting, and the last term $T = \$1$ denotes the taxes that the regulator needs to collect to provide the lump-sum subsidy to the monopolist. Overall, welfare is \$450.25, thus being larger than when the firm is left unregulated.

Intuitively, the increase in consumer surplus (from $CS^U = \$112.81$ to $CS^R = \$451.25$) offsets the loss in profits ($\pi^U = \$224.63$ to $\pi^R + S = \$0$) and the cost of the subsidy in terms of tax collection ($T = \$1$). Therefore, an advantage of marginal cost pricing is that it can help increase total welfare.

A major disadvantage is the decrease in profit (to a negative value before subsidies) for the monopolist. Because of this, many regulated natural monopolies are induced to artificially inflate their marginal cost in order to gain some profit. If the monopolist is successful, prices would increase resulting in a decrease in consumer surplus and an increase in deadweight loss. Another important disadvantage is that regulators rarely have accurate information about the monopolist's cost function and set pricing decisions based on expected, rather than true, marginal costs, and lump-sum subsidy decisions based on expected fixed costs.