EconS 301- Intermediate Microeconomic Theory
Recitation - Friday October 28th, 2022.

1. Consider the Prisoner's Dilemma game and assume that, when a player confesses while her partner does not, the police do not offer any deal to the confessing player. As a consequence, payoff becomes $(-10, -1)$ or $(-1, -10)$. When both players confess their payoff is $(-5, -5)$ and when both players do not confess their payoff is $(-1, -1)$. Payoffs represent the time they spend in jail. Find the NE of the game, and compare your results against those in example 12.5 (Chapter 12). Interpret.

   - Updating Matrix 12.9a from the textbook with our new values, we obtain Matrix 12.6.

		Player 2	
		Confess	Not confess
Player 1	Confess	$\underline{-5}, \underline{-5}$	$\underline{-1}, -10$
	Not confess	$-10, \underline{-1}$	$\underline{-1}, \underline{-1}$

Matrix 12.6. Prisoner's Dilemma game-II.

   - *Player 1's best responses.* When Player 2 chooses *Confess,* Player 1's best response is also *Confess*, $BR_1(C) = C$ (since $-5 > -10$). When Player 2 chooses *Not confess*, Player 1's best response is to either *Confess* or *Not confess*, $BR_1(NC) = \{C, NC\}$ (since both strategies yield a payoff of -1).
   - *Player 2's best responses.* Similarly, when Player 1 chooses *Confess*, Plater 2's best response is *Confess*, $BR_2(C) = C$ (once again since $-5 > -10$). When Player 1 chooses *Not confess*, Player 2's best response is to either *Confess* or *Not confess*, $BR_2(NC) = \{C, NC\}$ (since both strategies yield a payoff of -1).

   - *Comparison.* Now that no benefit is provided for a confessing player when her partner does not confess, there is less incentive for either player to strictly confess. This leads to a situation where *Confess* only weakly dominates *Not confess*, as either player will always be as well off playing *Confess* over *Not confess*, but they may not be strictly better off. As a result, we have two Nash equilibria in this game where the strategy profiles for each player are underlined:

$$\textit{(Confess, Confess)} \text{ and } \textit{(Not confess, Not confess)}.$$

2. Consider again the Anticoordination game in Matrix 12.12. While we found two psNEs in that game, we can still find one msNE. Repeat the analysis in example 12.9 to find the msNE of the Anticoordination game, and depict the best responses for each player. Show that the best responses cross at three points: (1) at $(p, q) = (0, 1)$ at the corner of the graph, which corresponds to the psNE (*Stay*, *Swerve*); (2) at $(p, q) = (1, 0)$ at the top-left corner of the graph, corresponding to the psNE (*Swerve*, *Stay*); and (3) at an interior point where both p and q are strictly between 0 and 1, illustrating the msNE of the game.

- Reproducing Matrix 12.12a from the textbook, we obtain Matrix 12.11.

		Player 2	
		Swerve	Stay
Player 1	Swerve	$-1, -1$	$-10, 10$
	Stay	$10, -10$	$-20, -20$

Matrix 12.11. Anticoordination game-III.

- *Calculating the msNE.* We assume that Player 1 chooses *Swerve* with probability p and *Stay* with probability $1-p$. Similarly, we assume that Player 2 chooses *Swerve* with probability q and *Stay* with probability $1-q$.
  - Since Player 1 is randomizing, we know that he must be indifferent between choosing *Swerve* and *Stay*, so his expected payoff from *Swerve* must be equal to his expected payoff from *Stay*.

$$\begin{aligned} EU_1(Swerve) &= EU_1(Stay) \\ q(-1) + (1-q)10 &= q(-10) + (1-q)(-20) \\ 10 - 11q &= -20 + 10q. \end{aligned}$$

  Rearranging this expression, we have $21q = 10$. Dividing both sides of this equation by 21 provides the probability that Player 2 chooses *Swerve*, $q = \frac{10}{21} = 0.48$. In words, Player 1 is indifferent between swerving and staying if Player 2 swerves with 48 percent probability.
  - We can perform the exact same analysis for player 2 to find that $p = 0.48$ (since the players are symmetric in their payoffs).
- *Building a graphical representation.* Now that we have our point where both players are indifferent between choosing *Swerve* and *Stay*, we can find for which values of their probabilities that they prefer to act in pure strategies. Player 1 prefers *Swerve* over *Stay* if his expected payoff from *Swerve* is greater than his expected payoff from *Stay*,

$$\begin{aligned} EU_1(Swerve) &> EU_1(Stay) \\ q(-1) + (1-q)10 &> q(-10) + (1-q)(-20) \\ 10 - 11q &> -20 + 10q. \end{aligned}$$

  Once again, this rearranges to $21q < 10$, which we can solve by dividing both sides of this inequality by 21 to obtain $q < 0.48$. Thus, if Player 2 randomizes by choosing *Swerve* less than 48 percent of the time, Player 1 prefers to choose *Swerve* exclusively. Likewise, if Player 2 chooses *Swerve* more than 48 percent of the time, Player 1 prefers to choose *Stay* exclusively. Again, these results are identical for Player 2. We compile these results in figure 12.1 where we can observe three intersections among the best response functions:
  - one at $(p, q) = (1, 0)$ which corresponds with our pure strategy profile (*Swerve*, *Stay*),

– one at $(p, q) = (0, 1)$ which corresponds with our pure strategy profile (*Stay*, *Swerve*), and
– one at $(p, q) = (0.48, 0.48)$ which corresponds with our mixed strategy profile where each player randomly chooses *Swerve* 48 percent of the time.

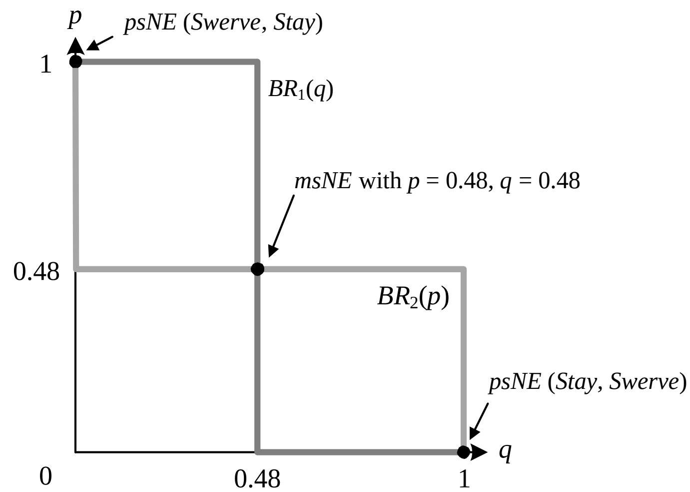

Figure 12.1. Anticoordination game msNE.