

# EconS 301- Intermediate Microeconomic Theory

## Quiz #3 - October 11th.

1. Consider a firm with production function  $q = 5L^{1/3}K^{2/3}$ . Find the firm's MRTS. Then, assuming that the firm seeks to produce  $q = 220$  units of output, find and depict the isoquant.

- The *MRTS* is

$$MRTS = \frac{MP_L}{MP_K} = \frac{\frac{5}{3}L^{-2/3}K^{2/3}}{\frac{10}{3}L^{1/3}K^{-1/3}} = \frac{K^{\frac{2}{3} - (\frac{1}{3})}}{2L^{\frac{1}{3} - (\frac{2}{3})}} = \frac{K}{2L}.$$

- To find the isoquant, we need to solve  $200 = 5L^{1/3}K^{2/3}$  for  $K$ . Dividing each side by 5 gives us  $40 = L^{1/3}K^{2/3}$ . Cubing both sides yields  $40^3 = LK^2$ . Next, divide by  $L$  to get  $K^2 = \frac{40^3}{L}$ , and square-root each side to get the isoquant

$$K = \frac{40^{3/2}}{L^{1/2}}.$$

This isoquant will approach both axes, but never cross them; however, it will approach the  $K$ -axis more quickly than the  $L$ -axis.

2. Consider a firm with the production function  $q = 5L^{1/3}K^{2/3}$ . Find if this production function exhibits increasing, decreasing, or constant returns to scale. What if the firm's production function changes to  $q = 7L + 8K$ ?

- $q = 5L^{1/3}K^{2/3}$ : If we increase all inputs by  $\lambda$ , we get

$$5(\lambda L)^{1/3}(\lambda K)^{2/3} = 5\lambda^{1/3}L^{1/3}\lambda^{2/3}K^{2/3} = \lambda^{1/3+2/3}5L^{1/3}K^{2/3} = \lambda \underbrace{(5L^{1/3}K^{2/3})}_q = \lambda q.$$

The resulting increase in output is the same as the increase in the input, so this production function exhibits constant returns to scale.

- $q = 7L + 8K$ : If we increase all inputs by  $\lambda$ , we get

$$7\lambda L + 8\lambda K = \lambda \underbrace{(7L + 8K)}_q = \lambda q.$$