Intermediate Microeconomic Theory

Monopoly

OUTLINE

- BARRIERS TO ENTRY
- PROFIT MAXIMIZATION PROBLEM (PMP)
- COMMON MISUNDERSTANDINGS OF MONOPOLY
- THE LERNEX INDEX AND INVERSE ELASTICITY PRICING RULE
- MULTIPLANT MONOPOLIST
- WELFARE ANALYSIS UNDER MONOPOLY
- ADVERTISING IN MONOPOLY
- MONOPSONY

BARRIERS TO ENTRY

BARRIERS TO ENTRY

"WHY DO MONOPOLIES EXIST IN THE FIRST PLACE IF THEY ARE BAD FOR SOCIETY?"

- STRUCTURAL BARRIERS: INCUMBENT FIRMS MAY HAVE ADVANTAGES THAT ARE UNATTRACTIVE FOR POTENTIAL ENTRANTS.
  - COST ADVANTAGE (E.G., SUPERIOR TECHNOLOGY)
  - DEMAND ADVANTAGE (E.G., LARGE GROUP OF LOYAL CUSTOMERS)
- LEGAL BARRIERS: INCUMBENTS FIRMS MAY BE LEGALLY PROTECTED.
  - *EXAMPLE*: PATENTS.
- STRATEGIC BARRIERS: INCUMBENT FIRMS CAN TAKE ACTIONS TO DETER ENTRY, BY BUILDING A REPUTATION OF BEING A TOUGH COMPETITOR.
  - *EXAMPLE*: PRICE WARS.

PROFIT MAXIMIZATION PROBLEM (PMP)

PROFIT MAXIMIZATION PROBLEM

- IN A MONOPOLIZED INDUSTRY,
  - A SINGLE FIRM DECIDES THE OUTPUT LEVEL, $q = Q$.
  - A CHANGE IN q AFFECTS MARKET PRICES, AS MEASURED BY THE INVERSE DEMAND FUNCTION $p(q)$, WHICH DECREASES IN q.
  - *EXAMPLE* (LINEAR INVERSE DEMAND):

$$p(q) = a - bq, \text{ WHERE } a, b > 0$$

    - WHEN THE MONOPOLIST SELLS FEW UNITS (LOW VALUES OF q), CONSUMERS ARE WILLING TO PAY A RELATIVELY HIGH PRICE FOR THE SCARCE GOOD.
    - AS THE FIRM OFFERS MORE UNITS (LARGER VALUES OF q), CONSUMERS ARE WILLING TO PAY LESS FOR THE RELATIVELY ABUNDANT GOOD.

PROFIT MAXIMIZATION PROBLEM

- PMP: THE MONOPOLIST CHOOSES ITS OUTPUT q TO MAXIMIZE ITS PROFITS π

$$\max_{q} \; \pi = TR(q) - TC(q) = p(q)q - TC(q).$$

- DIFFERENTIATING WITH RESPECT TO q,

$$p(q) + \frac{\partial p(q)}{\partial q} q - \frac{\partial TC(q)}{\partial q} = 0.$$

- REARRANGING,

$$\underbrace{p(q) + \frac{\partial p(q)}{\partial q} q}_{\text{Marginal revenue, } MR(q)} = \underbrace{\frac{\partial TC(q)}{\partial q}}_{\text{Marginal cost, } MC(q)}.$$

PROFIT MAXIMIZATION PROBLEM

- THEREFORE, TO MAXIMIZE PROFITS, THE MONOPOLIST INCREASES ITS OUTPUT q UNTIL

$$MR(q) = MC(q).$$

  - IF $MR(q) > MC(q)$, THE MONOPOLIST WOULD HAVE INCENTIVES TO INCREASE OUTPUT q BECAUSE ITS REVENUES INCREASES MORE THAN ITS COST.
  - IF $MR(q) < MC(q)$, THE MONOPOLIST WOULD HAVE INCENTIVES TO DECREASE ITS OUTPUT q.

PROFIT MAXIMIZATION PROBLEM

- A CLOSER LOOK AT MARGINAL REVENUE,

$$MR(q) = \underbrace{p(q)}_{\text{Positive effect}} + \underbrace{\frac{\partial p(q)}{\partial q} q}_{\text{Negative effect}}.$$

- WHEN MONOPOLIST INCREASES OUTPUT BY 1 UNIT, THIS ADDITIONAL UNIT PRODUCES 2 EFFECTS ON FIRM'S REVENUE:
  - POSITIVE EFFECT. IF THE FIRM SELLS 1 MORE UNIT, IT EARNS $p(q)$, AND THE FIRM'S REVENUE INCREASES.
  - NEGATIVE EFFECT. WHEN OFFERING 1 MORE UNIT, THE FIRM NEEDS TO DECREASE THE PRICE OF PREVIOUS UNITS SOLD, $\frac{\partial p(q)}{\partial q} < 0$.
- IN SUMMARY, THE TOTAL EFFECT OF INCREASING OUTPUT MUST EXACTLY OFFSET THE ADDITIONAL COSTS OF PRODUCING 1 MORE UNIT, $MR(q) = MC(q)$.

PROFIT MAXIMIZATION PROBLEM

- *EXAMPLE 10.1:* *POSITIVE AND NEGATIVE EFFECTS OF SELLING MORE UNITS.*
  - CONSIDER $p(q) = 10 - 3q$. IF THE FIRM WERE TO MARGINALLY INCREASE ITS OUTPUT,

$$MR(q) = p(q) + \frac{\partial p(q)}{\partial q} q$$
$$= (10 - 3q) + (-3)q$$
$$= 10 - 6q.$$

  - IF THE FIRM SELLS $q = 2$ UNITS,

$$TR(1) = p(2)2 = (10 - 3 \times 2)2 = \$8.$$

PROFIT MAXIMIZATION PROBLEM

- *EXAMPLE 10.1* (CONTINUED):
  - EVALUATING $MR(q)$ AT $q = 2$ UNITS YIELDS

$$MR(2) = (10 - 3 \times 2) + (-3)2 = 4 - 6 = -\$2.$$

  - THE MONOPOLIST'S REVENUE EXPERIENCES:
    - A POSITIVE EFFECT OF $\$4$ BECAUSE IT NOW SELLS 1 MORE UNIT AT PRICE $\$4$.
    - A NEGATIVE EFFECT BECAUSE SELLING 1 MORE UNIT ENTAILS APPLYING A PRICE DISCOUNT OF $\$3$ ON ALL PREVIOUS UNITS.
    - OVERALL, THESE TWO EFFECT GENERATES A TOTAL (NET) DECREASE IN REVENUE OF $\$2$.

PROFIT MAXIMIZATION PROBLEM

- *EXAMPLE 10.2: FINDING MARGINAL REVENUE WITH LINEAR DEMAND.*
  - CONSIDER $p(q) = a - bq$. MARGINAL REVENUE IS

$$MR(q) = p(q) + \frac{\partial p(q)}{\partial q} q = (a - bq) + (-b)q = a - 2bq.$$

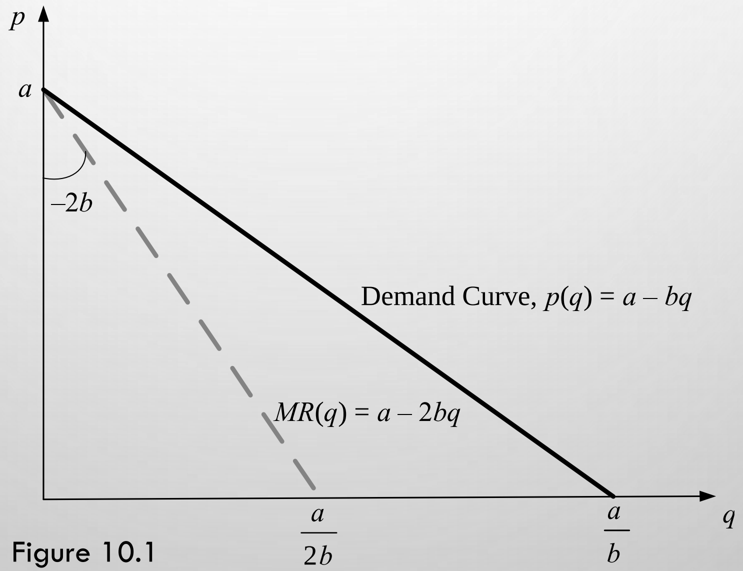

Figure 10.1

PROFIT MAXIMIZATION PROBLEM

- TWO PROPERTIES OF MARGINAL REVENUE CURVE:

1) $MR(q)$ *lies below the demand curve.*
   We need,

$$MR(q) \leq p(q),$$

$$p(q) + \frac{\partial p(q)}{\partial q} q \leq p(q) \rightarrow \frac{\partial p(q)}{\partial q} q \leq 0.$$

2) $MR(q)$ *and the demand curve originate at the same height.*
   At $q = 0$,

$$p(0) = a - b \times 0 = a,$$

$$MR(0) = p(0) + \frac{\partial p(q)}{\partial q} q = p(0) = a.$$

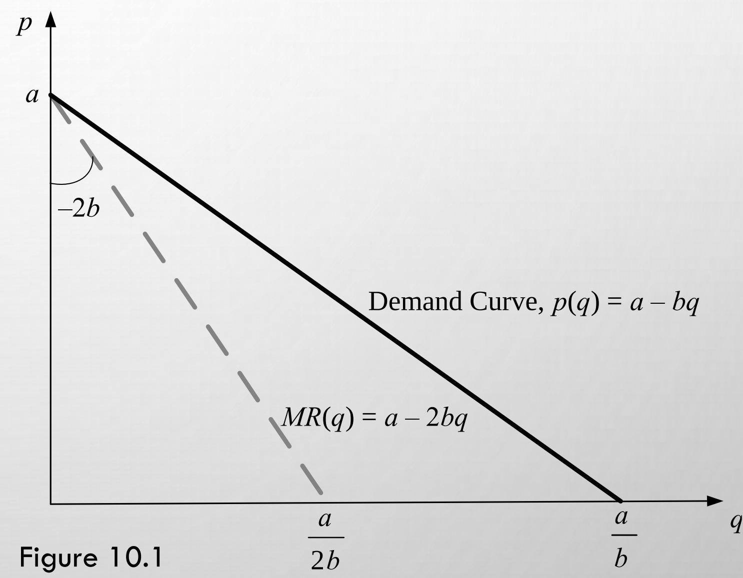

Figure 10.1

PROFIT MAXIMIZATION PROBLEM

- *EXAMPLE 10.3: FINDING MONOPOLY OUTPUT WITH LINEAR DEMAND.*
  - CONSIDER $p(q) = a - bq$, AND $TC(q) = cq$, WHERE $c > 0$.
  - THE MONOPOLIST MAXIMIZES ITS PROFITS BY SOLVING

$$\underset{q}{\text{MAX}}\,\pi = TR(q) - TC(q) = \underbrace{(a - bq)q}_{TR} - \underbrace{cq}_{TC}.$$

  - DIFFERENTIATING WITH RESPECT TO Q YIELDS

$$a - 2bq - c = 0.$$

  - REARRANGING,

$$\underbrace{a - 2bq}_{MR(q)} = \underbrace{c}_{MC(q)}.$$

PROFIT MAXIMIZATION PROBLEM

- *EXAMPLE 10.3* (CONTINUED):
  - REARRANGING,

$$a - c = 2bq.$$

  - SOLVING FOR OUTPUT q,

$$q^M = \frac{a-c}{2b}.$$

  - WE FIND THE MONOPOLY PRICE BY INSERTING THIS OUTPUT INTO THE INVERSE DEMAND FUNCTION

$$p(q^M) = a - bq^M = a - b\overbrace{\left(\frac{a-c}{2b}\right)}^{q^M}$$

$$= \frac{2ab - b(a-c)}{2b} = \frac{a+c}{2}.$$

PROFIT MAXIMIZATION PROBLEM

- *EXAMPLE 10.3* (CONTINUED):

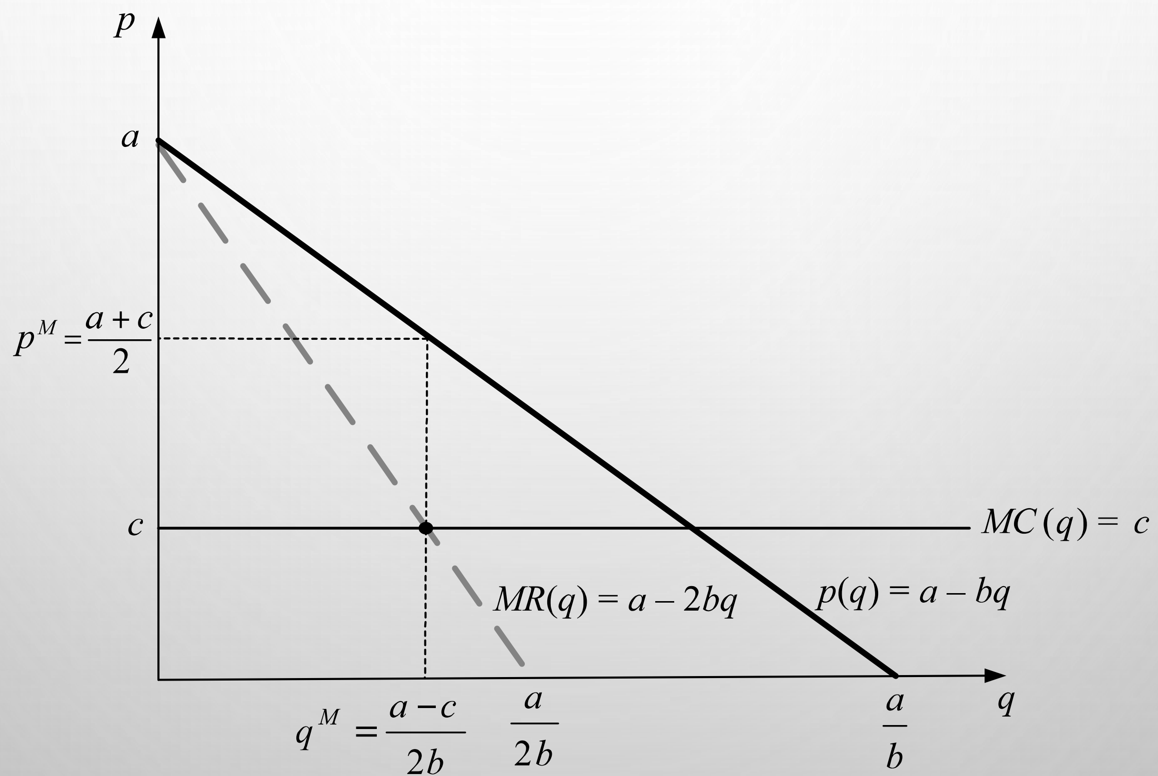

Figure 10.2

PROFIT MAXIMIZATION PROBLEM

- *EXAMPLE 10.3* (CONTINUED):
  - MONOPOLY PROFITS ARE

$$\pi^M = p(q^M)q^M - cq^M$$

$$= \frac{a+c}{2} \cdot \frac{a-c}{2b} - c\,\frac{a-c}{2b}$$

$$= \left(\frac{a+c}{2} - c\right)\frac{a-c}{2b}$$

$$= \frac{(a-c)^2}{4b}.$$

PROFIT MAXIMIZATION PROBLEM

- *EXAMPLE 10.3* (CONTINUED):
  - CONSUMER SURPLUS UNDER THIS MONOPOLY IS

$$CS^M = \underbrace{\frac{1}{2}\left(a - \frac{a+c}{2}\right)}_{\text{height}} \cdot \underbrace{\frac{a-c}{2b}}_{\text{base}} = \frac{1}{2} \cdot \frac{2a-a-c}{2} \cdot \frac{a-c}{2b} = \frac{(a-c)^2}{8b}.$$

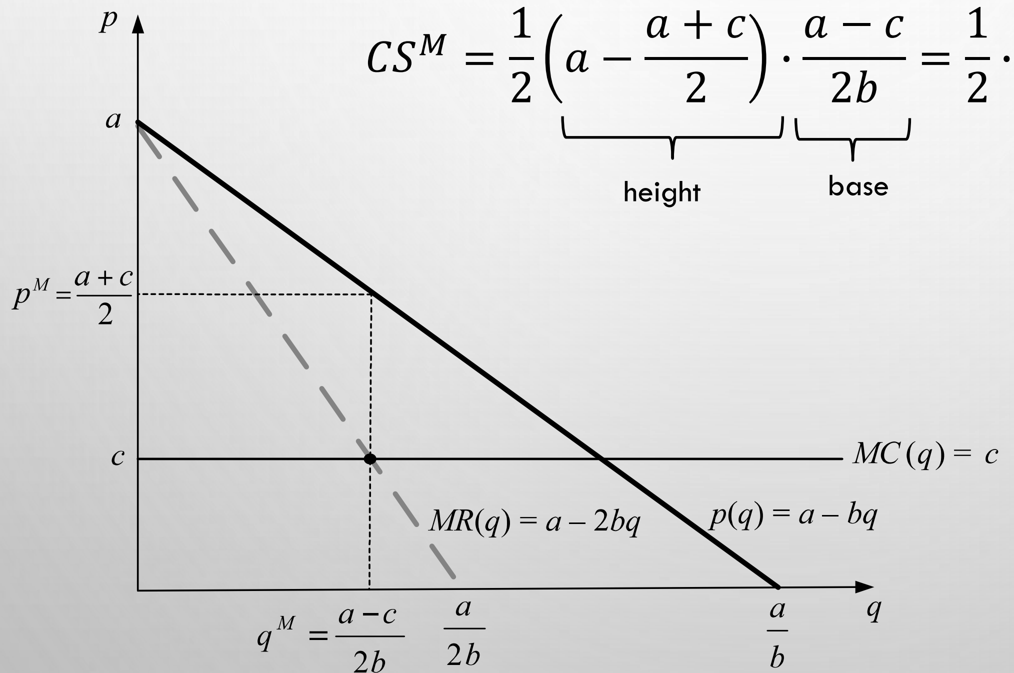

Figure 10.2

PROFIT MAXIMIZATION PROBLEM

- *EXAMPLE 10.3* (CONTINUED):
  - IF INVERSE DEMAND IS $p(q) = 10 - q$ (I.E., $a = 10$ AND $b = 1$), AND $TC(q) = 4q$ (I.E., $c = 4$)
    - $q^M = \frac{a-c}{2b} = \frac{10-4}{2} = 3$ UNITS.
    - $p^M = a - bq^M = \$7.$
    - $\pi^M = \frac{(a-c)^2}{4b} = \frac{(10-4)^2}{4} = \frac{36}{4} = \$9.$
    - $CS^M = \frac{(a-c)^2}{8b} = \frac{(10-4)^2}{8} = \frac{36}{8} = \$4.5.$

COMMON MISUNDERSTANDINGS OF MONOPOLY

COMMON MISUNDERSTANDINGS

1. *MONOPOLIES DO NOT SET INFINITELY HIGH PRICES.*
   - WHILE THE MONOPOLIST IS THE ONLY FIRM IN ITS INDUSTRY, IT FACES A DEMAND CURVE $p(q)$, SUCH AS $p(q) = a - bq$.
   - SETTING HIGHER PRICES MIGHT BE ATTRACTIVE BUT COULD LEAD TO FEWER SALES.
   - THIS TRADE-OFF IMPLIES THE MONOPOLIST DOES NOT SET AN INFINITELY HIGH PRICE BECAUSE IT WOULD IMPLY NO SALES AT ALL.
     - IN EXAMPLE 10.3, ANY PRICE ABOVE $p = \$a$ (E.G., $\$10$ IF $a = 10$) ENTAILS NO SALES.

COMMON MISUNDERSTANDINGS

2. *THE MONOPOLIST DOES NOT HAVE A SUPPLY CURVE.*

- A COMMON MISUNDERSTANDING IS TO CONSIDER THAT q^M, WHERE $MR(q) = MC(q)$, CONSTITUTES THE MONOPOLIST'S SUPPLY CURVE.
- IN PERFECTLY COMPETITIVE MARKETS, THE FIRM OBSERVES THE GIVEN MARKET PRICE OFFERS THE OUTPUT THAT SATISFIES $p = MC(q)$, OBTAINING THE SUPPLY FUNCTION $q(p)$.
- IN A MONOPOLY, THE MONOPOLIST DETERMINES OUTPUT AND PRICE SIMULTANEOUSLY.
  - IN EXAMPLE 10.3, WHEN THE MONOPOLIST CHOOSES $q^M = 3$ UNITS, IT SIMULTANEOUSLY DETERMINES $p^M = 10 - 3 = \$7$, NOT ALLOWING THE FIRM TO CHOOSE DIFFERENT OUTPUT LEVELS FOR A GIVEN MARKET PRICE OF $p^M = \$7$.

COMMON MISUNDERSTANDINGS

3. *THE MONOPOLIST PRODUCES IN THE ELASTIC PORTION OF THE DEMAND CURVE.*
   - GOODS WITH FEW (OR NO) CLOSE SUBSTITUTES TEND TO HAVE A RELATIVELY INELASTIC DEMAND CURVE.
   - MONOPOLIES OFTEN PRODUCE GOODS WITH NO CLOSE SUBSTITUTES. HOWEVER, IT IS DOES NOT MEAN THAT IT PRODUCES IN THE INELASTIC PORTION OF THE DEMAND CURVE.

COMMON MISUNDERSTANDINGS

3. *THE MONOPOLIST PRODUCES IN THE ELASTIC PORTION OF THE DEMAND CURVE.*

- CONSIDER THE FORMULA OF PRICE ELASTICITY OF DEMAND

$$\varepsilon_{q,p} = \frac{\%\Delta q}{\%\Delta p},$$

  - IF THE MONOPOLIST PRODUCES IN THE INELASTIC PORTION OF THE DEMAND CURVE, $|\varepsilon_{q,p}| < 1$, AN INCREASE IN PRICE BY 1% REDUCES SALES BY LESS THAN 1%. IT WOULD INCREASE ITS PRICE, AS SALES WOULD NOT BE GREATLY AFFECTED BUT IT WOULD NOT BE PROFIT MAXIMIZING.
  - IF IT PRODUCES IN THE ELASTIC SEGMENT, $|\varepsilon_{q,p}| > 1$, AN INCREASE IN PRICE BY 1% REDUCES SALES BY MORE THAN 1%. THE FIRM DOES NOT HAVE INCENTIVES TO ADJUST ITS PRICE.

COMMON MISUNDERSTANDINGS

- *EXAMPLE 10.5: PRICE ELASTICITY OF OUTPUT q^M UNDER LINEAR DEMAND.*
    - CONSIDER THE MONOPOLIST IN EXAMPLE 10.3, FACING $p(q) = 10 - q$.
    - WE FOUND $q^M = 3$ UNITS, AND $p^M = \$7$.
    - WE FIND PRICE ELASTICITY AS

$$\varepsilon_{q,p} = \frac{\%\Delta q}{\%\Delta p} = \frac{\Delta q}{\Delta p} \cdot \frac{p}{q}.$$

    - IF THE CHANGE IN PRICE IS SMALL, $\varepsilon_{q,p} = \frac{\partial q(p)}{\partial p} \cdot \frac{p}{q}$.

COMMON MISUNDERSTANDINGS

- *EXAMPLE 10.5* (CONTINUED):
  - FROM THE INVERSE DEMAND FUNCTION, WE OBTAIN THE DIRECT DEMAND FUNCTION, $q(p) = 10 - p$. THEN,

$$\varepsilon_{q,p} = \frac{\partial q(p)}{\partial p} \cdot \frac{p^M}{q^M} = -1\frac{7}{3} \cong -2.33.$$

  - IF THE MONOPOLIST INCREASES PRICES BY 1%, ITS SALES DECREASE BY 2.33%.
  - THEREFORE, $\left|\varepsilon_{q,p}\right| = 2.33 > 1$ → THE MONOPOLIST SETS A PRICE p^M LYING IN THE ELASTIC PORTION OF THE DEMAND CURVE.

THE LERNEX INDEX AND INVERSE ELASTICITY PRICING RULE

THE LERNEX INDEX

- WE CAN REWRITE THE PROFIT-MAXIMIZING CONDITION FOR THE MONOPOLIST, $MR(q) = MC(q)$, TO SHOW A RELATIONSHIP BETWEEN MARGIN, $p - MC(q)$, AND PRICE ELASTICITY, $\varepsilon_{q,p}$,

$$p(q) + \frac{\partial p(q)}{\partial q} q = MC(q).$$

- MARGINAL REVENUE CAN BE REARRANGED AS

$$MR(q) = p\left(1 + \frac{\partial p(q)}{\partial q} \cdot \frac{q}{p}\right),$$

$$MR(q) = p\left(1 + \frac{1}{\frac{\partial q(p)}{\partial p} \cdot \frac{p}{q}}\right) = p\left(1 + \frac{1}{\varepsilon_{q,p}}\right).$$

THE LERNEX INDEX

- SUBSTITUTING THIS EXPRESSION OF $MR(q)$ INTO $MR(q) = MC(q)$,

$$p\left(1 + \frac{1}{\varepsilon_{q,p}}\right) = MC(q).$$

- REARRANGING, $p + p\frac{1}{\varepsilon_{q,p}} = MC(q)$, OR $p - MC(q) = -p\frac{1}{\varepsilon_{q,p}}$.
- DIVIDING BOTH SIDES BY p YIELDS,

$$\frac{p - MC(q)}{p} = -\frac{1}{\varepsilon_{q,p}}.$$

- WHICH IS KNOWN AS THE "LERNEX INDEX":
  - A MONOPOLIST'S ABILITY TO SET A PRICE ABOVE MARGINAL COST IS INVERSELY RELATED TO THE PRICE ELASTICITY OF DEMAND.

THE LERNEX INDEX

- THE "LERNEX INDEX" IS ALSO KNOWN AS THE "MARKUP INDEX" BECAUSE IT MEASURES THE PRICE MARKUP OVER MARGINAL COST.
  - AS DEMAND BECOMES RELATIVELY ELASTIC, (I.E., A MORE NEGATIVE NUMBER) THE PRICE MARKUP DECREASES.
    - *EXAMPLE*: IF $\varepsilon_{q,p} = -4$,

$$-\frac{1}{\varepsilon_{q,p}} = -\frac{1}{-4} = 0.25,$$

PRICE MARKUP OVER MARGINAL COST DECREASES TO 25%.

THE LERNEX INDEX

- AS DEMAND BECOMES RELATIVELY INELASTIC, THE PRICE MARKUP INCREASES.
  - *EXAMPLE*: IF $\varepsilon_{q,p} = -0.5$,

$$-\frac{1}{\varepsilon_{q,p}} = -\frac{1}{-0.5} = 2,$$

PRICE MARKUP OF 200%.

THE LERNEX INDEX

- *EXAMPLE 10.6: LERNEX INDEX WITH A LINEAR DEMAND.*
  - CONSIDER MARKET INVERSE DEMAND FUNCTION IS $p(q) = 10 - q$.
  - SOLVING FOR q, WE OBTAIN DIRECT DEMAND $q(p) = 10 - p$.
  - WHICH YIELDS AND ELASTICITY OF

$$\varepsilon_{q,p} = \frac{\partial q(p)}{\partial p} \cdot \frac{p}{q}$$

$$= -1 \frac{p}{10 - p}.$$

THE LERNEX INDEX

- *EXAMPLE 10.6* (CONTINUED):
  - ASSUMING $MC(q) = 4$, THE LERNEX INDEX BECOMES

$$\frac{p - MC(q)}{p} = -\frac{1}{\varepsilon_{q,p}},$$

$$\frac{p-4}{p} = -\left(\frac{1}{-1\frac{p}{10-p}}\right).$$

THE LERNEX INDEX

- *EXAMPLE 10.6* (CONTINUED):
  - REARRANGING TERMS,

$$\frac{p-4}{p} = \frac{10-p}{p}.$$

  - WHICH SIMPLIFIES TO

$$p - 4 = 10 - p.$$

  - AND SOLVING FOR PRICE, $p = \$7$.

THE LERNEX INDEX

- *EXAMPLE 10.7*: *LERNEX INDEX WITH CONSTANT ELASTICITY DEMAND.*
  - CONSIDER MONOPOLIST FACING DEMAND CURVE
$$q(p) = 5p^{-\varepsilon}.$$
  - ASSUMING $MC(q) = \$4$, THE LERNEX INDEX BECOMES
$$\frac{p-4}{p} = -\frac{1}{\varepsilon_{q,p}}.$$

THE LERNEX INDEX

- *EXAMPLE 10.7* (CONTINUED):
  - IF DEMAND CURVE IS $q(p) = 5p^{-2}$ (I.E., $\varepsilon = -2$),

$$\frac{p-4}{p} = -\frac{1}{-2},$$

WHICH SIMPLIFIES TO $2p - 8 = p$, OR $p = \$8$.

  - IF DEMAND FUNCTION CHANGES TO $q(p) = 5p^{-5}$,

$$p = \frac{20}{4} = \$5.$$

AS DEMAND BECOMES MORE ELASTIC, PRICE DECREASES.

INVERSE ELASTICITY PRICING RULE (IEPR)

- USING THE LERNEX INDEX, AND SOLVING FOR PRICE

$$\frac{p - MC(q)}{p} = -\frac{1}{\varepsilon_{q,p}},$$

$$p = \frac{MC(q)}{1 + \frac{1}{\varepsilon_{q,p}}}.$$

WHICH IS KNOWN AS THE "INVERSE ELASTICITY PRICE RULE" (IEPR)

- *EXAMPLE*: IF $MC(q) = \$4$ AND $\varepsilon_{q,p} = -2$,

$$p = \frac{4}{1 + \frac{1}{-2}} = \frac{4}{\frac{1}{2}} = \$8.$$

MULTIPLANT MONOPOLY

MULTIPLANT MONOPOLY

- CONSIDER A MONOPOLY PRODUCING IN TWO PLANTS (FACTORIES),
  - q_1 IS THE OUTPUT PRODUCED IN PLANT 1,
  - q_2 IS THE OUTPUT PRODUCED IN PLANT 2,
  - $Q = q_1 + q_2$ REPRESENTS TOTAL OUTPUT ACROSS PLANTS.
- THE MONOPOLIST MAXIMIZES THE JOINT PROFITS FROM BOTH PLANTS

$$\max_{q_1, q_2} \pi = \pi_1 + \pi_2 = \underbrace{TR_1(q_1, q_2) - TC_1(q_1)}_{\pi_1} + \underbrace{TR_2(q_1, q_2) - TC_2(q_2)}_{\pi_2}$$

$$= [p(q_1, q_2) \times q_1 - TC_1(q_1)] + [p(q_1, q_2) \times q_2 - TC_2(q_2)]$$

$$= p(q_1, q_2) \times (q_1 + q_2) - TC_1(q_1) - TC_2(q_2)$$

MULTIPLANT MONOPOLY

- DIFFERENTIATING WITH RESPECT TO q_1, YIELDS

$$\underbrace{p(q_1, q_2) + \frac{\partial p(q_1, q_2)}{\partial q_1}}_{MR_1} = \underbrace{\frac{\partial TC_1(q_1)}{\partial q_1}}_{MC_1},$$

- AND DIFFERENTIATING WITH RESPECT TO q_2,

$$\underbrace{p(q_1, q_2) + \frac{\partial p(q_1, q_2)}{\partial q_2}}_{MR_2} = \underbrace{\frac{\partial TC_2(q_2)}{\partial q_2}}_{MC_2},$$

- IN THE SPECIAL CASE THAT $\frac{\partial p(q_1,q_2)}{\partial q_1} = \frac{\partial p(q_1,q_2)}{\partial q_2}$, $MR_1 = MR_2 = MR$.
- THE MULTIPLANT MONOPOLY MAXIMIZES ITS JOINT PROFITS AT

$$MR = MC_1 = MC_2.$$

MULTIPLANT MONOPOLY

- WHEN $\frac{\partial p(q_1,q_2)}{\partial q_1} = \frac{\partial p(q_1,q_2)}{\partial q_2}$,

$$MR_1 = MR_2 = MR.$$

  - THIS OCCURS WHEN PRICES ARE AFFECTED TO THE SAME EXTENT WHEN EITHER PLANT INCREASES ITS PRODUCTIONS, IF $p(q_1, q_2) = 300 - q_1 - q_2$.
  - THE MULTIPLANT MONOPOLY ONLY NEEDS TO EQUATE MARGINAL COSTS ACROSS PLANTS.

MULTIPLANT MONOPOLY

- WHEN $\frac{\partial p(q_1,q_2)}{\partial q_1} \neq \frac{\partial p(q_1,q_2)}{\partial q_2}$,

$$MR_1 \neq MR_2.$$

  - THIS MAY OCCUR IF $p(q_1, q_2) = 300 - q_1 - 0.5q_2$.
  - THE MULTIPLANT MONOPOLY MAXIMIZES JOINT PROFITS WHEN $MR_1 = MC_1$ AND $MR_2 = MC_2$.

MULTIPLANT MONOPOLY

- *EXAMPLE 10.8: MULTIPLANT MONOPOLY.*
  - CONSIDER $p(Q) = 100 - Q = 100 - q_1 - q_2$
  - ASSUME THE E MONOPOLIST OPERATES 2 PLANTS
    - PLANT 1 (US) WITH $TC_1(q_1) = 5 + 12q_1 + 6(q_1)^2$
    - PLANT 2 (CHILE) WITH $TC_2(q_2) = 5 + 18q_2 + 3(q_2)^2$
  - THE MONOPOLIST CHOOSES q_1 AND q_2 TO MAXIMIZE JOINT PROFITS FROM BOTH PLANTS

$$\max_{q_1 \geq 0, q_2 \geq 0} \pi = \pi_1 + \pi_2 = \underbrace{(100 - q_1 - q_2)q_1 - TC_1(q_1)}_{\pi_1} + \underbrace{(100 - q_1 - q_2)q_2 - TC_2(q_2)}_{\pi_2}$$

MULTIPLANT MONOPOLY

- *EXAMPLE 10.8* (CONTINUED):
  - DIFFERENTIATING WITH RESPECT TO q_1,

$$100 - 2q_1 - q_2 - 12 - 12q_1 - q_2 = 0,$$

$$88 - 14q_1 - 2q_2,$$

$$q_1 = \frac{44 - q_2}{7}.$$

  - SIMILARLY, DIFFERENTIATING TOTAL PROFITS WITH RESPECT TO q_2,

$$100 - q_1 - 2q_2 - 18 - 6q_2 - q_1 = 0,$$

$$82 - 2q_1 - 8q_2,$$

$$q_2 = \frac{41 - q_2}{4}.$$

MULTIPLANT MONOPOLY

- *EXAMPLE 10.8* (CONTINUED):
  - INSERTING THE RESULT FOR q_2 INTO q_1, WE OBTAIN

$$q_1 = \frac{44 - q_2}{7} = \frac{44 - \left(\frac{41 - q_1}{4}\right)}{7},$$

    WHICH SIMPLIFIES TO $7q_1 = \frac{135 - q_1}{4}$, YIELDING AN OPTIMAL PRODUCTION IN THE US PLANT OF $q_1 = 5$ UNITS.
  - THE OPTIMAL PRODUCTION IN THE CHILEAN PLANT IS $q_2 = \frac{41-5}{4} = 9$ UNITS.
  - AGGREGATE OUTPUT IS $Q = q_1 + q_2 = 5 + 9 = 14$ UNITS.
  - IN SUMMARY, THE MONOPOLY PRODUCES A SHARE OF $\frac{q_1}{Q} = \frac{5}{9} \cong 0.56$ IN THE US PLANT, AND $\frac{q_2}{Q} = \frac{4}{9} \cong 0.44$ IN THE CHILEAN PLANT.

MULTIPLANT MONOPOLY

- THE ANALYSIS ABOUT HOW THE MULTIPLANT MONOPOLIST DETERMINES Q, AND HOW IT DISTRIBUTES SUCH PRODUCTION AMONG ITS PLANTS, q_1 AND q_2, IS ANALOGOUS TO A "CARTEL" PROBLEM.

- A CARTEL IS A GROUP OF FIRMS (EQUIVALENT TO A MONOPOLIST WITH DIFFERENT PLANTS) COORDINATING THEIR PRODUCTION DECISIONS TO INCREASE THEIR JOINT PROFITS.
  - *EXAMPLE*: ORGANIZATION OF THE PETROLEUM-EXPORTING COUNTRIES (OPEC).
    - SOME COUNTRIES HAVE A LOWER MC (I.E., LOWER COST OF EXTRACTING AN ADDITIONAL BARREL OF OIL), SUCH AS SAUDI ARABIA.
    - OTHER COUNTRIES HAVE HIGHER MC, SUCH AS ANGOLA O VENEZUELA.
    - THEY COORDINATE THEIR TOTAL PRODUCTION AND DISTRIBUTE IT AMONG THE CARTEL PARTICIPANTS.

WELFARE ANALYSIS UNDER MONOPOLY

WELFARE ANALYSIS

- OUTPUT IS LOWER UNDER MONOPOLY THAN UNDER PERFECTLY COMPETITIVE INDUSTRIES, ENTAILING A HIGHER PRICE.
- CONSUMER SURPLUS IS MUCH SMALLER THAN UNDER PERFECT COMPETITION BECAUSE CUSTOMERS PAY MORE PER UNIT AND BUY FEWER UNITS.
- IN CONTRAST, PROFITS ARE LARGER.
- HOWEVER, THE FIRM'S PROFIT GAIN DOES NOT COMPENSATE FOR THE LOSS IN CONSUMER SURPLUS, YIELDING A NET LOSS IN SOCIAL WELFARE.

WELFARE ANALYSIS

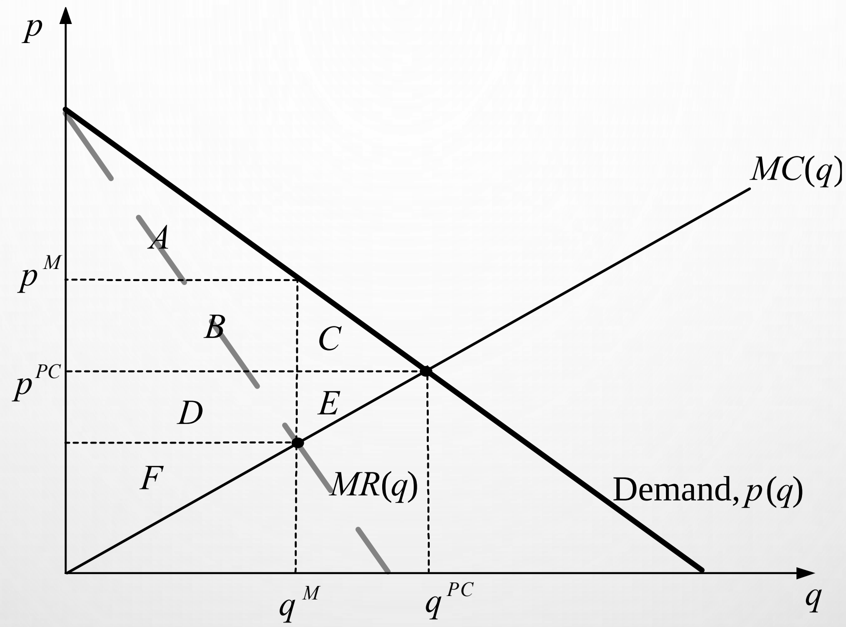

Figure 10.3

Table 10.1

	Perfect Competition	Monopoly	Difference
Consumer Surplus	$A+B+C$	A	$-B-C$
Profits	$D+E+F$	$D+F+B$	$B-E$
Welfare	$A+B+C+D+E+F$	$A+D+F+B$	$-C-E$

"Deadweight loss"

WELFARE ANALYSIS

- *EXAMPLE 10.9: FINDING THE DEADWEIGHT LOSS OF A MONOPOLY.*
  - CONSIDER $p(q) = 10 - q$ AND $MC(q) = 4$.

Monopoly	Perfect Competition
$q^M = 3$ units	$q^{PC} = 6$ units
$p^M = \$7$	$p^{PC} = \$4$
$CS^M = \frac{1}{2}(10-7)3 = \$4.50$	$CS^{PC} = \frac{1}{2}(10-4)6 = \$18$
$\pi^M = (7 \times 3) - (4 \times 3) = \$9$	$\pi^{PC} = (4 \times 6) - (4 \times 6) = \$0$
$W^M = CS^M + \pi^M$ $= 4.5 + 9 = \$13.50$	$W^{PC} = CS^{PC} + \pi^{PC}$ $= 18 + 0 = \$18$

$$W^{PC} - W^M = 18 - 13.50 = \$4.50.$$

WELFARE ANALYSIS

- *EXAMPLE 10.9* (CONTINUED):
  - DEADWEIGHT LOSS UNDER THIS MONOPOLY IS

$$DWL = \frac{1}{2}\underbrace{[p^M - MC(q^M)]}_{\text{height}}\underbrace{(q^{PC} - q^M)}_{\text{base}} = \frac{1}{2}(\$7 - \$4)(6 - 3) = \frac{9}{2} = \$4.50$$

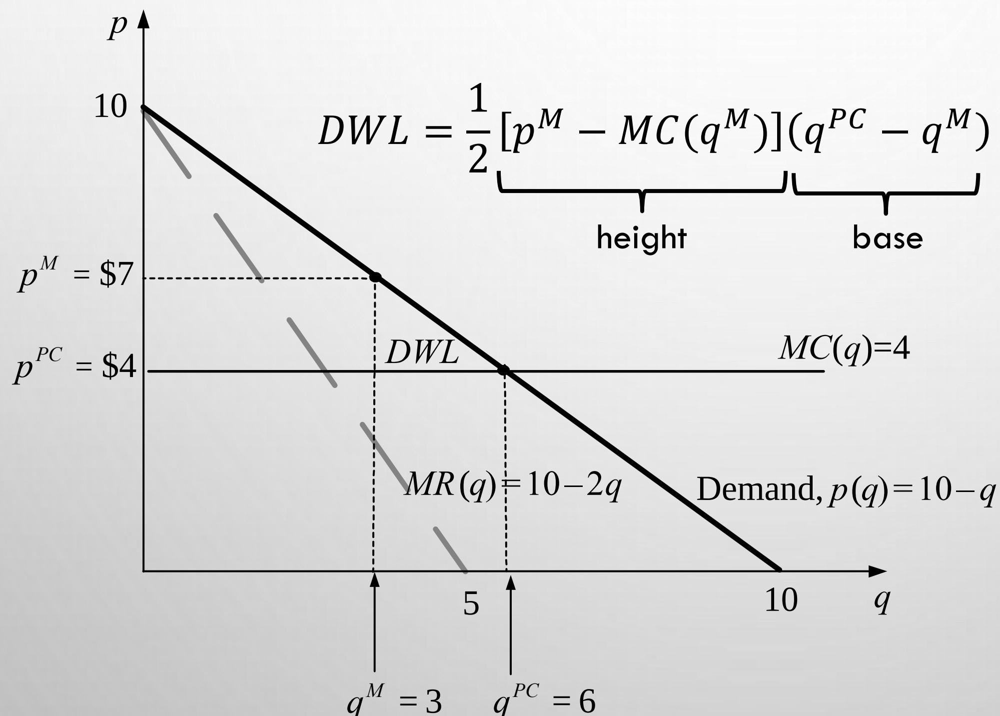

Figure 10.4

ADVERTISING IN MONOPOLY

ADVERTISING IN MONOPOLY

- WHEN INVESTING IN ADVERTISING, THE MONOPOLIST FACES A TRADE-OFF: ADVERTISING INCREASES DEMAND BUT IT IS COSTLY.
- TO FIND THE PROFIT-MAXIMIZING AMOUNT OF ADVERTISING, A,

$$\underset{A}{\text{MAX}}\, \pi = TR - TC - A.$$

- WE CAN REWRITE THIS PROBLEM AS

$$\underset{A}{\text{MAX}}\, \pi = (p \times q) - TC(q) - A$$

$$= [p \times q(p, A)] - TC[q(p, A)] - A.$$

- WHERE $q = q(p, A)$ REPRESENTS THE DEMAND FUNCTION (SALES) WHICH IS DECREASING p, AND INCREASING IN A.

ADVERTISING IN MONOPOLY

- DIFFERENTIATING WITH RESPECT TO THE AMOUNT OF ADVERTISING A,

$$p\frac{\partial q(p,A)}{\partial A} - \frac{\partial TC}{\partial q} \cdot \frac{\partial q(p,A)}{\partial A} - 1 = 0.$$

- REARRANGING,

$$(p - MC) \cdot \frac{\partial q(p,A)}{\partial A} = 1.$$

ADVERTISING IN MONOPOLY

- LET US DEFINE THE ADVERTISING ELASTICITY OF DEMAND, $\varepsilon_{q,A}$, AS

$$\varepsilon_{q,A} = \frac{\%\ increase\ in\ q}{\%\ increase\ in\ A} = \frac{\frac{\Delta q}{q}}{\frac{\Delta A}{A}} = \frac{\Delta q}{\Delta A} \cdot \frac{A}{q}.$$

- IN THE CASE OF A SMALL CHANGE IN A, THE ELASTICITY $\varepsilon_{q,A}$ CAN BE WRITTEN AS $\varepsilon_{q,A} = \frac{\partial q(p,A)}{\partial A} \cdot \frac{A}{q}$.
- REARRANGING, WE FIND $\varepsilon_{q,A} \cdot \frac{q}{A} = \frac{\partial q(p,A)}{\partial A}$.

ADVERTISING IN MONOPOLY

- THEREFORE, WE CAN REWRITE THE PROFIT-MAXIMIZING CONDITION AS

$$(p - MC)\underbrace{\varepsilon_{q,A} \cdot \frac{q}{A}}_{\frac{\partial q(p,A)}{\partial A}} = 1.$$

- DIVIDING BOTH SIDES BY $\varepsilon_{q,A}$ AND REARRANGING,

$$p - MC = \frac{1}{\varepsilon_{q,A}} \cdot \frac{A}{q}.$$

- DIVIDING BOTH SIDES BY p, WE FIND

$$\frac{p - MC}{p} = \frac{1}{\varepsilon_{q,A}} \cdot \frac{A}{pq}.$$

ADVERTISING IN MONOPOLY

- FROM THE IERP, WE KNOW

$$\frac{p-MC}{p} = -\frac{1}{\varepsilon_{q,p}}.$$

- HENCE,

$$-\frac{1}{\varepsilon_{q,p}} = \frac{1}{\varepsilon_{q,A}} \cdot \frac{A}{pq}.$$

$$-\frac{\varepsilon_{q,A}}{\varepsilon_{q,p}} = \frac{A}{pq}.$$

  - THE RIGHT SIDE REPRESENTS THE ADVERTISING-TO-SALES RATIO.
  - FOR TWO MARKETS WITH THE SAME $\varepsilon_{q,p}$, THE ADVERTISING-TO-SALES RATIO MUST BE LARGER IN THE MARKET WHERE DEMAND IS MORE SENSITIVE TO ADVERTISING (HIGHER $\varepsilon_{q,A}$).

ADVERTISING IN MONOPOLY

- *EXAMPLE 10.11: MONOPOLIST'S OPTIMAL ADVERTISING RATIO.*
  - CONSIDER A MONOPOLIST WITH PRICE ELASTICITY OF DEMAND OF $\varepsilon_{q,p} = -1.5$ AND ADVERTISING ELASTICITY $\varepsilon_{q,A} = 0.1$.
  - THE ADVERTISING-TO-SALES RATIO SHOULD BE

$$\frac{A}{pq} = -\frac{\varepsilon_{q,A}}{\varepsilon_{q,p}}$$

$$= -\frac{0.1}{-1.5} = 0.067.$$

  - ADVERTISING SHOULD ACCOUNT FOR 6.7% OF THIS MONOPOLIST'S TOTAL REVENUE.

MONOPSONY

MONOPSONY

- MONOPSONY: ONLY ONE BUYER IN THE MARKET AND SEVERAL SELLERS.
  - *EXAMPLES*: SMALL LABOR MARKETS, SUCH AS A MINE OR WALMART SUPERSTORE IN A SMALL TOWN.
- THE BUYER (EMPLOYER) WILL BE ABLE TO PAY LESS FOR EACH HOUR OF LABOR (LOWER WAGES) THAN IF IT HAD TO COMPETE AGAINST OTHER EMPLOYERS, AS IN A PERFECTLY COMPETITIVE MARKET.

MONOPSONY

- CONSIDER A FIRM (E.G., A COAL MINE) WITH PRODUCTION FUNCTION $q = f(L)$, WHICH:
  - INCREASES WITH THE NUMBER OF WORKERS HIRED, $f'(L) > 0$,
  - BUT AT A DECREASING RATE, $f''(L) < 0$.
- THE PROFITS OF THE COAL MINE IS GIVEN BY

$$\pi = TR - TC = pq - w(L)L.$$

  - THE FIRM EXTRACTS q UNITS OF COAL, EACH SOLD AT PRICE p, YIELDING $TR = pq$.
  - THE FIRM HIRES L WORKERS, PAYING EACH OF THEM A WAGE OF $w(L)$.
    - $w'(L) > 0$, AS THE FIRM HIRES MORE WORKERS, LABOR BECOMES SCARCE, AND A MORE GENEROUS WAGE MUST BE OFFERED TO ATTRACT NEW WORKERS.

MONOPSONY

- THE MONOPSONIST'S PMP IS

$$\max_{L \geq 0} \ \pi = pq - w(L)L = pf(L) - w(L)L.$$

  - INTUITIVELY, THIS PROBLEM SAYS "CHOOSE THE NUMBER OF WORKERS YOU PLAN TO HIRE, L, SO AS TO MAXIMIZE YOUR PROFITS."

- DIFFERENTIATING WITH RESPECT TO L,

$$pf'(L) - [w(L) + w'(L)L] = 0.$$

- REARRANGING,

$$\underbrace{pf'(L)}_{MRP_L} = \underbrace{w(L) + w'(L)L}_{ME_L}.$$

MONOPSONY

$$\underbrace{pf'(L)}_{MRP_L} = \underbrace{w(L) + w'(L)L}_{ME_L}.$$

- MRP_L ("MARGINAL REVENUE PRODUCT" OF LABOR):
  - AFTER HIRING 1 MORE WORKER (INCREASE IN L), THE FIRM PRODUCES $f'(L)$ MORE UNITS OF OUTPUT (E.G., COAL), SOLD AT A PRICE p.
- ME_L ("MARGINAL EXPENDITURE" ON LABOR). AFTER HIRING 1 MORE WORKER, THE FIRM EXPERIENCES AN INCREASE IN COST:
  - THIS EXTRA WORKER MUST BE PAID $w(L)$.
  - THE ADDITIONAL WORKER IS ONLY ATTRACTED TO THE JOB IF THE FIRM OFFERS HER A HIGHER SALARY BECAUSE LABOR BECOMES SCARCER. SUCH A WAGE INCREASE, $w'(L)$, MUST BE PASSED ON TO ALL EXISTING WORKER, ENTAILING A COST INCREASE OF $w'(L)L$.