Quiz #1 EconS 301 - September 1st, 2022

1. Consider the following utility functions

$$u(x,y) = 3x + 2y \text{ and } v(x,y) = 5x^{1/2}y^{1/4}$$

   (a) Find the Marginal Rate of Substitution $MRS_{x,y}$ for $u(x,y)$ and $v(x,y)$. Does the consumer regard goods x and y as perfect substitutes or complements? Interpret your results.

   (b) Illustrate the indifference curves that represent each utility function, that is $u(x,y)$ and $v(x,y)$, considering utility levels $u = 10$ and $u = 20$.

Solution

- The Marginal Rate of Substitution is

$$MRS_{x,y} = \frac{MU_x}{MU_x} = \frac{3}{2}$$

$$MRS_{x,y} = \frac{MU_x}{MU_x} = \frac{5\frac{1}{2}x^{-1/2}y^{1/4}}{5\frac{1}{4}x^{1/2}y^{-3/4}} = 2\frac{y}{x}$$

- In the case of $u(x,y)$ the consumer regards these two goods as perfect substitutes. However, when the utility is represented by $v(x,y)$, Cobb-Douglas utility function, the consumer regards goods as neither perfectly substitutable nor complementary.

2. - Since we know that utility function $u(x,y) = 3x + 2y$ is a straight line and that desired utility level is $u = 10$, we can find her utility curve by just finding two bundles on this utility curve and connecting the dots.
     - *Indifference curve when reaching utility level* $u = 10$. If we set $x = 0$, we obtain $2y = 10$, and thus $y = 5$ units, which gives us our first bundle. If we then set $y = 0$, we find $3x = 10$, and thus $x = 3.33$ units, which gives us our second bundle.
     - *Indifference curve when reaching utility level* $u = 20$. We now perform the same steps but when utility level is $u = 20$. If we set $x = 0$, we obtain $2y = 20$, and thus $y = 10$ units, which gives us our first bundle. If we set $y = 0$, we have $3x = 20$, and thus $x = 6.66$ units, which gives us our second bundle.
     - Connecting the dots gives us our indifference curves, as detailed in figure 1.

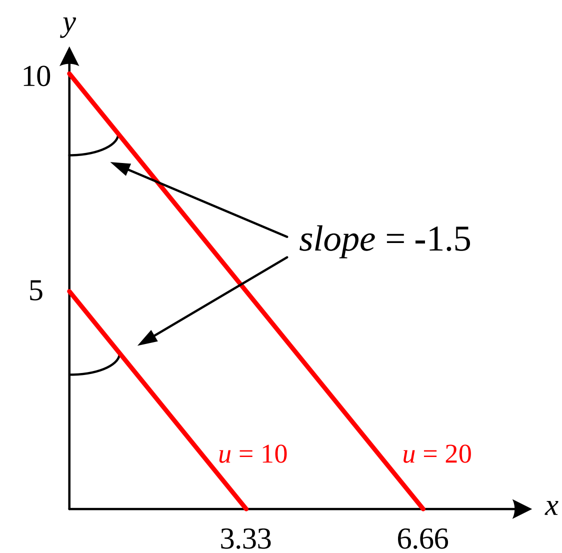

Figure 1. indifference curves for $u(x,y)$.

– First, we divide both sides of $u(x,y) = 5x^{1/2}y^{1/4}$ by $5x^{1/2}$ to obtain $y^{1/4} = \frac{u}{5x^{1/2}}$.

– Next we raise both sides of this equation to the fourth power to obtain our equation for Maria's indifference curves,

$$y = \frac{u^4}{625x^2}.$$

– Plugging in our different values for u gives us two different indifference curves,

$$\begin{aligned} y &= \frac{10,000}{625x^2} \text{ for } u = 10, \text{ and} \\ y &= \frac{160,000}{625x^2} \text{ for } u = 20. \end{aligned}$$

– We can plot these indifference curves by examining a few bundles that fall on these indifference curves:

  * For $u = 10$, if we set $x = 2$, we have that $y = 4$ units. Likewise, when $x = 4$, good y becomes $y = 1$ units.
  * For $u = 20$, if we set $x = 4$, good y is $y = 16$ units. When $x = 8$, good y becomes $y = 4$ units.
  * These bundles allow us to find a few relevant points along our indifference curves, which are depicted in figure 2.

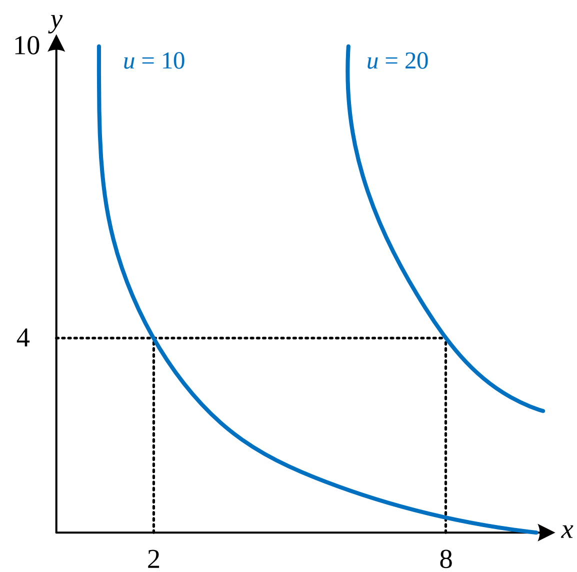

Figure 2. Indifference curves for $v(x, y)$.