EconS 301- Intermediate Microeconomic Theory
Quiz #2 - September 15th.

1. Consider an individual with utility function $u(x,y) = (x+3)y$, and income $I = \$30$. The price of good x is $p_x = \$2$, while that of good y is normalized to $p_y = \$1$ (that is, good y represents the money left for purchasing all other goods but x, which we refer as the "numeraire").

   (a) Find the optimal consumption bundle of this individual. Evaluate her utility function at this optimal bundle.

   - *Step 1.* To solve for our optimal consumption, first we must use the tangency condition, $\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$.
     - For the left-hand side, we calculate the ratio of our marginal utilities, where $MU_x = \frac{\partial u(x,y)}{\partial x} = y$ and $MU_y = \frac{\partial u(x,y)}{\partial y} = x+3$. Therefore,

       $$\frac{MU_x}{MU_y} = \frac{y}{x+3}.$$

     - For the right-hand side, it is simply the ratio of the prices, $\frac{p_x}{p_y} = \frac{2}{1}$.
     - Setting them equal to one another gives

       $$\frac{y}{x+3} = 2,$$

       which we can rearrange to obtain $y = 2x+6$, or $x = \frac{1}{2}y - 3$. Since this contains both x and y, we move on to step 2a.
   - *Step 2a.* Next, we use our budget line, $2x + y = 30$ and substitute $\frac{1}{2}y - 3$ for x, obtaining

     $$\underbrace{2\left(\frac{1}{2}y - 3\right)}_{x} + y = 30.$$

     - Combining terms, we have $2y = 36$, and we can divide both sides of this equation to obtain the amount of good y that she consumes, $y = 18$ units. With a positive value of y, we can move on to step 4.
   - *Step 4.* Last, we return to our tangency condition to determine how much of good x she consumes.
     - Plugging in her value for y gives $x = \frac{1}{2}(18) - 3 = 6$ units.
   - Last, evaluating our utility by plugging in our equilibrium values,

     $$u(6,18) = (6+3)(18) = 162.$$

   (b) Assume now that her income was increased by \$10 (for a total of $I' = \$40$). What is her new optimal consumption bundle? What is the new utility level that she can reach?

   - *Step 1.* To solve for our optimal consumption, first we must use the tangency condition, $\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$.

- For the left-hand side, we calculate the ratio of our marginal utilities, where $MU_x = \frac{\partial u(x,y)}{\partial x} = y$ and $MU_y = \frac{\partial u(x,y)}{\partial y} = x + 3$. Therefore,

$$\frac{MU_x}{MU_y} = \frac{y}{x+3}.$$

- For the right-hand side, it is simply the ratio of the prices, $\frac{p_x}{p_y} = \frac{2}{1}$.
- Setting them equal to one another gives

$$\frac{y}{x+3} = 2,$$

which we can rearrange to obtain $y = 2x + 6$, or $x = \frac{1}{2}y - 3$. Since this contains both x and y, we move on to step 2a.

- *Step 2a.* Next, we use our budget line, $2x + y = 40$ and substitute $\frac{1}{2}y - 3$ for x, obtaining

$$2\underbrace{\left(\frac{1}{2}y - 3\right)}_{x} + y = 40.$$

  - Combining terms, we have $2y = 46$, and we can divide both sides of this equation to obtain the amount of good y that she consumes, $y = 23$ units. With a positive value of y, we can move on to step 4.

- *Step 4.* Last, we return to our tangency condition to determine how much of good x she consumes.
  - Plugging in her value for y gives $x = \frac{1}{2}(23) - 3 = 8.5$ units.
- Last, evaluating our utility by plugging in our equilibrium values,

$$u(8.5, 23) = (8.5 + 3)(23) = 264.5.$$

(c) Assume now that the price of good x decreases in \$1, so its new price is $p'_x = \$1$. What is her new optimal consumption bundle? What is the new utility level that she can reach?

- *Step 1.* To solve for our optimal consumption, first we must use the tangency condition, $\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$.
  - For the left-hand side, we calculate the ratio of our marginal utilities, where $MU_x = \frac{\partial u(x,y)}{\partial x} = y$ and $MU_y = \frac{\partial u(x,y)}{\partial y} = x + 3$. Therefore,

$$\frac{MU_x}{MU_y} = \frac{y}{x+3}.$$

  - For the right-hand side, it is simply the ratio of the prices, $\frac{p_x}{p_y} = \frac{1}{1}$.
  - Setting them equal to one another gives

$$\frac{y}{x+3} = 1,$$

which we can rearrange to obtain $y = x + 3$, or $x = y - 3$. Since this contains both x and y, we move on to step 2a.

- *Step 2a.* Next, we use our budget line, $x + y = 30$ and substitute $y - 3$ for x, obtaining

$$\underbrace{y - 3}_{x} + y = 30.$$

  - Combining terms, we have $2y = 33$, and we can divide both sides of this equation to obtain the amount of good y that she consumes, $y = 16.5$ units. With a positive value of y, we can move on to step 4.

- *Step 4.* Last, we return to our tangency condition to determine how much of good x she consumes.
  - Plugging in her value for y gives $x = 16.5 - 3 = 13.5$ units.

- Last, evaluating our utility by plugging in our equilibrium values,

$$u(13.5, 16.5) = (13.5 + 3)(16.5) = 272.75.$$

d. In which version of parts b–c is the consumer better off? That is, describe whether the consumer prefers the change in income from part (b), or the change in prices from part (c).

- The consumer is better off in part (c), under the price change. This allows her to consume more of both goods, increasing her utility.