EconS 301- Intermediate Microeconomic Theory
Homework #2 - Due date: Tuesday September 20th, 2022.

1. Sarah has utility function $u(x, y) = x^{1/2}y^{1/4}$, facing prices $p_x = \$3$ and $p_y = \$2$, and income $I = \$16$. Using the same steps as in example 3.2, find Sarah's optimal consumption of goods x and y.

   - *Step 1.* To solve for Sarah's optimal consumption, first we must use the tangency condition, $\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$.
     - For the left-hand side, we calculate the ratio of our marginal utilities, where $MU_x = \frac{\partial u(x,y)}{\partial x} = \frac{1}{2}x^{-1/2}y^{1/4}$ and $MU_y = \frac{\partial u(x,y)}{\partial y} = \frac{1}{4}x^{1/2}y^{-3/4}$. Therefore,
     $$\frac{MU_x}{MU_y} = \frac{\frac{1}{2}x^{-1/2}y^{1/4}}{\frac{1}{4}x^{1/2}y^{-3/4}} = \frac{2y}{x}.$$
     - For the right-hand side, it is simply the ratio of the prices, $\frac{p_x}{p_y} = \frac{3}{2}$.
     - Setting them equal to one another gives
     $$\frac{2y}{x} = \frac{3}{2},$$
     which we can rearrange to obtain $4y = 3x$. Since this contains both x and y we move on to step 2a.
   - *Step 2a.* Next, we use Sarah's budget line, $3x + 2y = 16$ and substitute $4y$ for $3x$, obtaining
   $$\underbrace{4y}_{3x} + 2y = 16.$$
     - Combining terms, we have $6y = 16$, and we can divide both sides of this equation to obtain the amount of good y that Sarah consumes, $y = 2.67$ units. With a positive value of y, we can move on to step 4.
   - *Step 4.* Last, we return to our tangency condition to determine how much of good x Sarah consumes.
     - Dividing both sides of the tangency condition by 3, we have $x = \frac{4}{3}y$. Plugging in our value for y gives
     $$x = \frac{4}{3}(2.67) = 3.56 \text{ units.}$$

2. Kevin's utility function is $u(x, y) = 3x + 4y$ and faces prices $p_x = \$1$ and $p_y = \$2.5$ and income $I = \$23$. Comparing his $MRS_{x,y}$ and the price ratio, find his optimal consumption of goods x and y.

   - First, we need to calculate Kevin's marginal rate of substitution,
   $$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{3}{4} = 0.75,$$
   and compare it to the ratio of prices,
   $$\frac{p_x}{p_y} = \frac{1}{2.5} = \frac{2}{5} = 0.4.$$

- Since $\frac{3}{4} > \frac{2}{5}$, Kevin receives more benefit by solely consuming good x. Alternatively, the "bang for the buck" he obtains from good x, $\frac{MU_x}{p_x} = \frac{3}{1} = 3$, is larger than that for good y, $\frac{MU_y}{p_y} = \frac{4}{2.5} = 1.6$, inducing him to keep increasing his purchases of good x, while reducing those of good y, until he only consumes the former.
- In this case, Kevin can consume

$$x = \frac{I}{p_x} = \frac{23}{1} = 23 \text{ units}$$

of good x and no units of good y. This budget line and its associated maximum utility are depicted in figure 1.

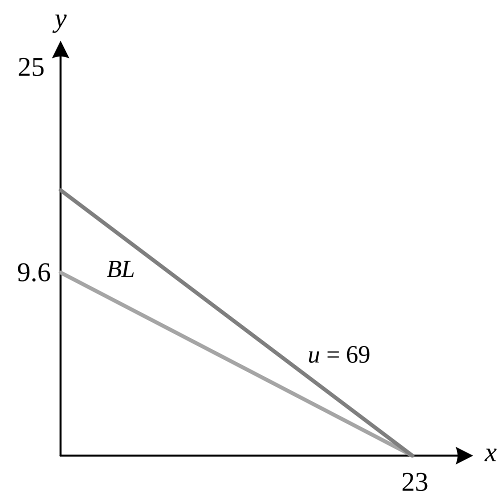

Figure 1. Perfect substitutes corner solution.

3. John's utility function is $u(x, y) = 5 \min\{2x, 3y\}$ and he faces prices $p_x = \$1$ and $p_y = \$2$ and income $I = \$100$. Find his optimal consumption of goods x and y.

- Since we cannot define John's marginal rate of substitution, we must use the fact that John prefers to consume goods x and y in fixed proportions to maximize his utility (i.e., the two arguments inside the min operator must coincide, $2x = 3y$) or, after dividing both sides by 2,

$$x = \frac{3}{2}y.$$

This is the kink of John's indifference curves, as depicted in the following figure. This budget line and its associated maximum utility are depicted in figure 2.

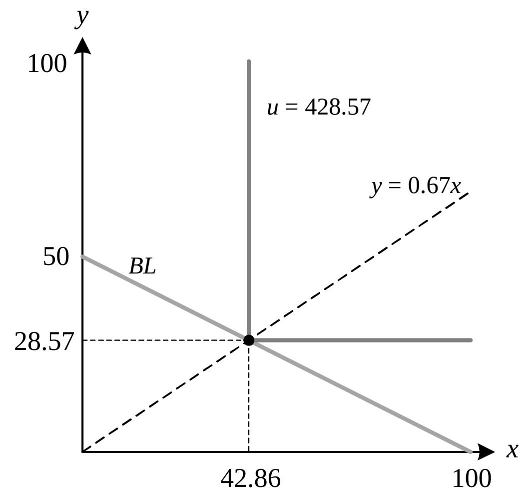

Figure 2. Perfect complements optimal consumption.

- Using the condition we found above, $x = \frac{3}{2}y$, and John's budget line, $x+2y = 100$, we can substitute $\frac{3}{2}y$ for x, giving the equation

$$\underbrace{\frac{3}{2}y}_{x} + 2y = 100.$$

  - Combining terms on the left-hand side of the equation gives us $\frac{7}{2}y = 100$. Dividing both sides of this expression by $\frac{7}{2}$ provides our equilibrium consumption level of good y, $y = 28.57$ units.

- Returning to our condition for John's consumption bundle, $x = \frac{3}{2}y$, we can plug in our value of $y = 28.57$ to find our equilibrium consumption level of good x,

$$x = \frac{3}{2}\underbrace{(28.57)}_{y} = 42.86 \text{ units.}$$

4. Luke has a weekly income of $I = \$40$ that he allocates between purchasing goods x and y. When the price of good x is \$4 and the price of good y is \$4, Luke purchases 3 units of good x and 7 units of good y in equilibrium. Suppose now that the price of good x falls to \$2.

   (a) Find the equation of his original and new budget lines, and represent it graphically.

   - Setting up Luke's original budget line, we have

$$4x + 4y = 40,$$

     which, solving for y, we obtain,

$$y = 10 - x.$$

From here, we find the vertical intercept of Luke's budget line by setting $x = 0$, which yields $y = 10$ units. Next, we find the horizontal intercept of Luke's budget line by setting $y = 0$ and solving for x,

$$0 = 10 - x$$

which yields $x = 10$ units.

- Setting up Luke's new budget line, we have

$$2x + 4y = 40,$$

which, solving for y, we obtain,

$$y = 10 - \frac{1}{2}x.$$

From here, we find the vertical intercept of Luke's budget line by setting $x = 0$, which yields $y = 10$ units. Next, we find the horizontal intercept of Luke's budget line by setting $y = 0$ and solving for x,

$$0 = 10 - \frac{1}{2}x$$

which yields $x = 20$ units. Intuitively, this implies that Luke's budget line rotates outward on the x-axis as a result of the price decrease. The resulting budget lines are plotted in figure 3.

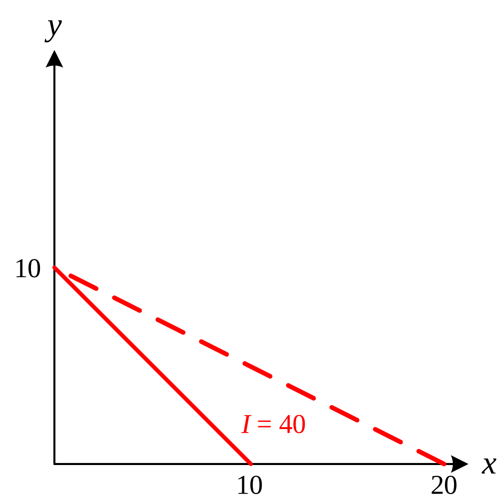

Figure 3. Budget lines.

(b) Suppose that Luke's new equilibrium bundle is 5 units of good x and 5 units of good y. Does this new bundle violate WARP? Explain why or why not.

- Using Luke's original prices, this bundle had a total cost of $4(5)+4(5) = \$40$. This implies that Luke's new bundle was affordable under his original prices. In addition, Luke's original bundle is also affordable under the new prices, as it now only costs $2(3)+4(7)=\$34$ to purchase it under the new prices. This implies that Luke's new bundle violates WARP as both of his bundles were affordable under the original prices, while the original bundle is still affordable under the new prices.

(c) Suppose now that Luke's new equilibrium bundle contained 4 units of good y. How many units of good x must be consumed such that our equilibrium allocation does not violate WARP?

- In order for this new bundle to not violate WARP, it must be unaffordable under the original prices, which would mean that the premise of WARP does not hold and thus cannot be violated. To set this up, we must have that,

$$4x+4(4)>40.$$

Solving this expression for x, we have $x>6$.

- Second, this bundle must be affordable under the new prices. To set this up, we must have that

$$2x+4(4)\leq 40$$

Solving this expression for x, we have $x\leq 12$. Thus, any bundle satisfying $6<x\leq 12$. would not violate WARP in this context.

5. Eric wishes to reach a utility level of $U=50$ and has a quasilinear utility function of the type $u(x,y)=4x+y^{1/2}$. The price of good x is \$2 while the price of good y is \$2.

(a) Find Eric's tangency condition following step 1 of the expenditure minimization procedure.

- *Step 1.* We begin the expenditure minimization problem by finding Eric's tangency condition, $\frac{MU_x}{MU_y}=\frac{p_x}{p_y}$.
  - Calculating Eric's marginal utilities, we obtain $MU_x=\frac{\partial u(x,y)}{\partial x}=4$ and $MU_y=\frac{\partial u(x,y)}{\partial y}=\frac{1}{2}y^{-1/2}$. Combining these into our marginal rate of substitution gives

$$MRS_{x,y}=\frac{MU_x}{MU_y}=\frac{4}{\frac{1}{2}y^{-1/2}}=8y^{1/2}.$$

  - Next, we obtain our ratio of prices, $\frac{p_x}{p_y}=\frac{2}{2}$.
  - Setting them equal to one another, we have our tangency condition,

$$8y^{1/2}=1.$$

(b) Find Eric's equilibrium quantities for goods x and y.

- Since Eric's tangency condition contains only y we move on to step 2b.
- *Step 2b.* Rearranging this expression, we have $y^{1/2}=\frac{1}{8}$. Squaring both sides gives our optimal purchase of good y,

$$y^E=\frac{1}{64}=0.015 \text{ units.}$$

– Now we substitute this value, $y^E = 0.016$, into our utility function, $4x + y^{1/2} = 50$ to obtain

$$4x + (0.25)^{1/2} = 50,$$

or $4x + 0.125 = 50$, which further simplifies to $4x = 49.875$. We can now solve for x, to obtain the optimal purchase of good x,

$$x^E = 12.47 \text{ units.}$$

(c) How much income does Eric require to reach his target utility level?

- To find Eric's income requirement, we multiply his equilibrium quantities by their respective prices and add them together, obtaining

$$I = 2(12.47) + 2(0.016) = \$24.97$$

6. Brandon's utility function is $u(x, y) = x^{1/3}y^{2/3}$, his income is $I = \$150$, and the price of good y is $p_y = \$1$. The price of good x decreases from $p_x = \$3$ to $p'_x = \$1$. Using the steps in example 4.8 (Chapter 4, pages 91-92), find the substitution and income effects.

- Before applying the steps in example 4.8, it would be useful to calculate Brandon's marginal rate of substitution,

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{\frac{1}{3}x^{-2/3}y^{2/3}}{\frac{2}{3}x^{1/3}y^{-1/3}} = \frac{y^{\frac{2}{3}+\frac{1}{3}}}{2x^{\frac{1}{3}+\frac{2}{3}}} = \frac{y}{2x}.$$

- *Finding initial bundle A.* With Brandon's initial prices, his tangency condition, $\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$ is $\frac{y}{2x} = \frac{3}{1}$. This expression rearranges to $y = 6x$. Inserting this expression into the budget line gives

$$3x + \underbrace{6x}_{y} = 150.$$

From here, we can divide both sides of this expression by 9 to obtain

$$x = \frac{150}{9} = 16.67 \text{ units.}$$

From Brandon's tangency condition, we can solve for Brandon's consumption of good y,

$$y = 6x = 6(16.67) = 100 \text{ units.}$$

This leads to Brandon's initial bundle $A = (16.67, 100)$. This initial bundle is depicted in figure 4a.

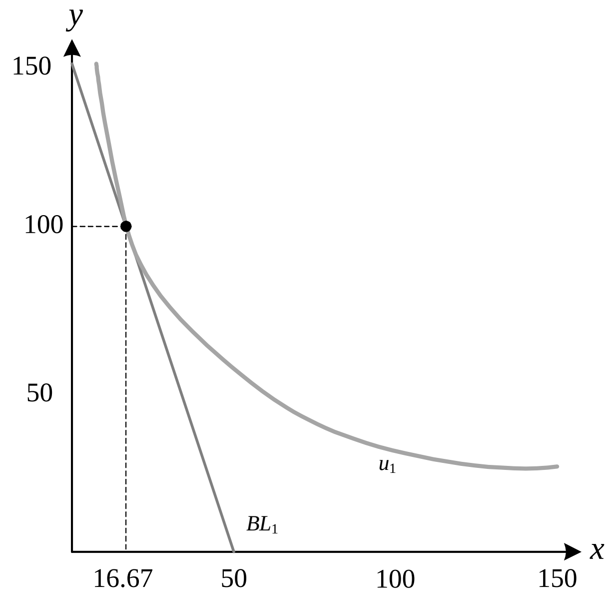

Figure 4a. Brandon's initial bundle.

- *Finding final bundle C.* Now with Brandon's final prices, his tangency condition, $\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$ is $\frac{y}{2x} = \frac{1}{1}$. Once again, we can rearrange this expression to obtain $y = 2x$. Inserting this expression into the new budget lines gives

$$x + \underbrace{2x}_{y} = 150.$$

As before, we can divide both sides of this expression by 3 to obtain $x = \frac{150}{3} = 50$ units. Using Brandon's tangency condition, we can solve for Brandon's consumption of good y, $y = 2x = 2(50) = 100$. This gives us Brandon's final bundle $C = (50, 100)$. The final bundle, along with the initial bundle for comparison, is depicted in figure 4b.

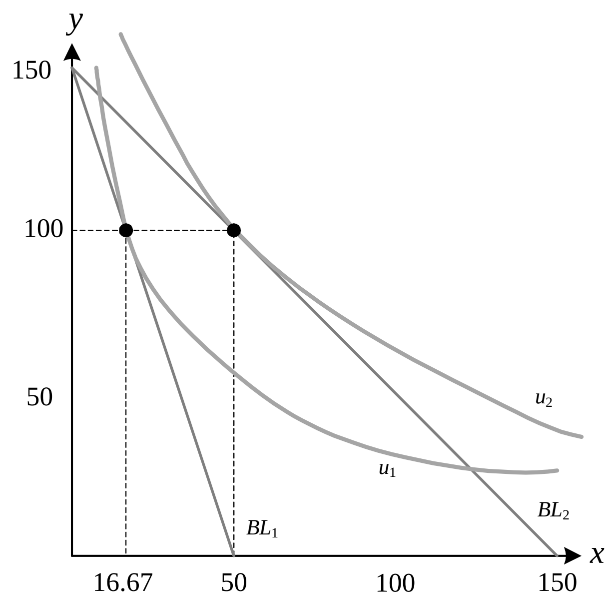

Figure 4b. Brandon's initial and final bundles.

- *Finding decomposition bundle B.* To break down Brandon's change from his initial bundle to his final bundle, we need to satisfy both conditions as explained in example 4.8.
  - First, the decomposition bundle must reach the utility level Brandon received under his initial bundle, which we can calculate by inserting bundle A into Brandon's utility function,

$$(16.67)^{1/3}(100)^{2/3} = 55.04.$$

    Thus, Brandon's decomposition bundle B must satisfy $x^{1/3}y^{2/3} = 55.04$.
  - Second, the decomposition bundle must be tangent to Brandon's indifference curve. This happens where the slope of Brandon's indifference curve is equal to the slope of his final budget line, which implies that we use the tangency condition from Brandon's final bundle, $y = 2x$.

  Substituting Brandon's tangency condition into the first condition gives

$$x^{1/3}\underbrace{(2x)}_{y}{}^{2/3} = 55.04.$$

  We can distribute the exponent through the parentheses to obtain

$$x^{1/3}2^{2/3}x^{2/3} = 55.04.$$

  The left-hand side of this expression simplifies to $1.59x = 55.04$. Dividing both sides by 1.59 provides Brandon's decomposition bundle value of good x,

$$x = \frac{55.04}{1.59} = 34.62 \text{ units.}$$

As before, we use Brandon's tangency condition to find his decomposition value for good y,

$$y = 2x = 2(34.62) = 69.24 \text{ units.}$$

This gives us Brandon's decomposition bundle $B = (34.62, 69.24)$. The decomposition bundle is added to our previous figure in figure 4c.

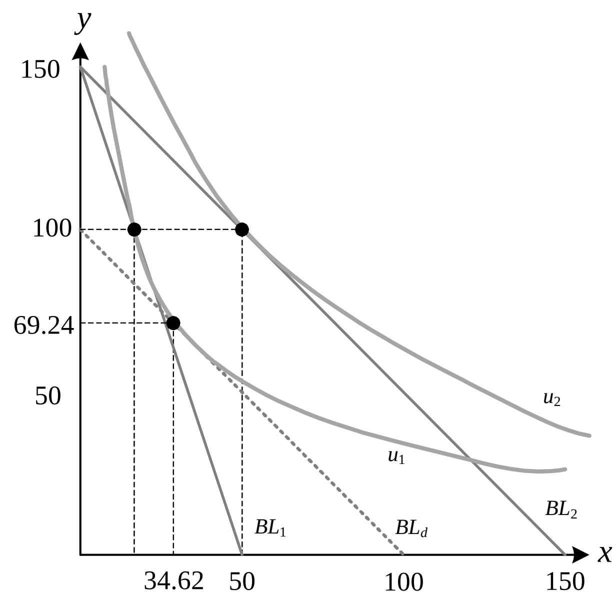

Figure 4c. Brandon's decomposition bundle.

- Last, we can calculate Brandon's substitution and income effects by comparing the values of his different bundles:

$$\begin{aligned} \text{Substitution Effect:} \quad & x_B - x_A = 34.62 - 16.67 = 17.95, \text{ and} \\ \text{Income Effect:} \quad & x_C - x_B = 50 - 34.62 = 15.38. \end{aligned}$$

which we depict in figure 4d.

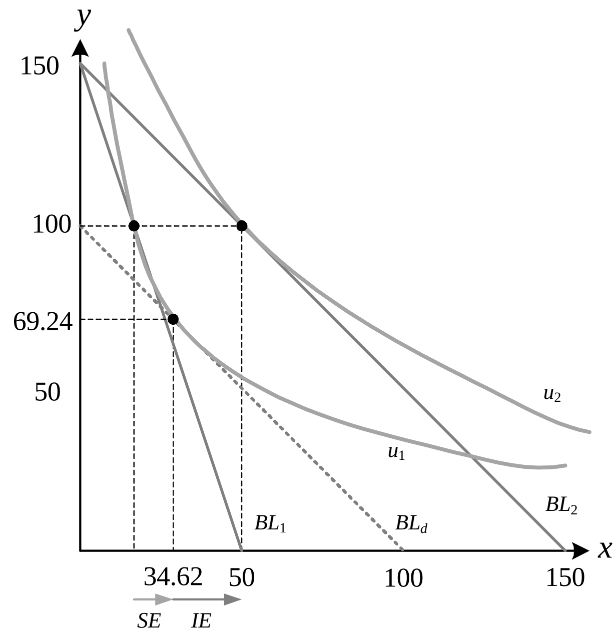

Figure 4d. Brandon's substitution and income effects.