Strategic Merger Approvals Under Incomplete Information

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Abstract

We examine a signaling game where the merging entity privately observes the cost-reduction effect from the merger, but the competition authority does not. The latter, however, observes the firm's submission costs in the merger request, using them to infer its type. We identify pooling equilibria where all firm types, even those with small efficiencies, submit a merger request, which is approved by the regulator. This merger profile cannot be supported under complete information, thus leading to inefficiencies. We investigate under which parameter conditions inefficient mergers are less likely to arise in equilibrium, and which policies hinder them, ultimately improving information transmission from firms to the competition authority.

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1 Introduction

The US Horizontal Merger Guidelines explicitly consider the cost-reduction effects (efficiencies) when evaluating merger requests, mentioning that this efficiency can yield to lower prices passed on to consumers and, ultimately, higher welfare.[1] However, merger efficiencies are relatively difficult to observe by competition authorities (CA) when receiving a merger request and, as the empirical literature shows, even after the merger occurs. For a review of these empirical studies see, for instance, Kolaric and Schiereck (2014), and for a description of the challenges estimating post-merger efficiency effects, see Knittel and Metaxoglou (2008 and 2011), and the solutions in Jaffe and Weyl (2013).[2]

Horizontal merger requests are facing greater scrutiny by regulatory authorities, with more of them being blocked by the US Department of Justice than in previous decades, as they may lead to higher markups and market power.[3] Examples include the merger between insurance brokers Aon and Willis Towers Watson, blocked in July 2020; railway companies Kansas City Southern and Canadian National, blocked in September 2021; and the recent opposition to the merger of publishers Simon and Schuster with Penguin Random House in December 2021, suggesting a potential blocking decision in coming months. This increased opposition comes after empirical studies finding that mergers tend to increase prices of consumer goods, Ashenfelter and Hosken (2010), and airline tickets, Kwoka and Shumilkina (2010), and can decrease labor demand by merged firms, ultimately leading to an increase in unemployment, Gugler and Yurtoglu (2003). Similarly, Loecker et al. (2020) and Eeckhout (2021) show that, from a macroeconomic perspective, the increase in market concentration in recent decades can explain the reduction in labor and capital shares, and a decrease in labor market dynamism, such as job-to-job transitions and transitions from nonemployment to employment status.

In this paper, we study how the CA can use firms' investment decisions during the submission process to infer the cost-reduction effect of the merger. Firms invest large amounts in their merger requests, with three main motives: (1) administrative fees, as in Besanko and Spulber (1993); (2) searching for hard evidence about the merger's efficiency to share with the CA, as in Lagerloff and Heidhues (2005); and (3) investment in public relations campaigns, advertising, and lobbying.[4]

[1] The European Commission follows similar guidelines, see "Guidelines on the assessment of horizontal mergers under the Council Regulation on the control of concentrations between undertakings," specifically section VII, which describes the assessment of merger efficiencies.

[2] For empirical studies estimating cost-reduction effects in the US airline industry, see Kim and Singal (1993), Johnston and Ozment (2013), Gayle and Le (2013), both finding significant cost savings from the merger; Barros et al. (2013), who finds no effects; Yan et al. (2019), who find positive effects in Chinese airlines; and Gudmundsson et al. (2017) who found nil or negative cost saving effects for international airlines. Similarly, Ashenfelter et al. (2015) examines the US beer industry after the MillerCoors merger, showing that costs savings were dominated by adverse competitive effects; and so do Kwoka and Pollitt (2010) for the US electricity industry in 1994-2003 and Bloningen and Pierce (2016) for a large panel of US manufacturing industries during 1997-2007. In contrast, Ganapati (2021) shows that the mergers increased productivity in a large data set of US industries for 1972-2012, ultimately expanding output.

[3] From a macroeconomic perspective, recent studies show that the increase in market power can explain the reduction in labor and capital shares and even a decrease in labor market dynamism, such as job-to-job transitions and transitions from nonemployment to employment status); see Loecker et al. (2020) and Eeckhout (2021).

[4] While the third motive does not produce hard evidence, it is often used by firms seeking to merge, and may

We seek to identify separating equilibria where only firms with substantial efficiencies submit such requests, investing enough resources to signal their type, and the CA responds approving the merger. We also search for pooling equilibria, where all firm types submit requests and the CA responds approving them, leading to inefficient mergers being approved due to the CA's incomplete information. In this setting, we examine different policies to hinder the emergence of inefficient mergers in equilibrium.

Our model considers an industry where a subset of firms evaluates whether to merge into a single entity. In the first stage, firms choose whether to submit a merger request to the CA. In the second stage, the CA responds by approving or blocking the request. In the third stage, if the merger is approved, the firms submitting the request form a single entity, thus coordinating their output decisions; while the outsiders independently choose their output; whereas if the merger is blocked, every firm in the industry independently selects its output level. The merger, if approved, produces a cost-reduction effect (synergy) in the merging firms, which the CA may not observe upon receiving a request.

As a benchmark, we first analyze equilibrium behavior under complete information. We show that only merger requests that are both profitable and welfare improving are submitted to the CA, which responds approving them. In this setting, firms invest no resources in the merger request, as the CA observes the exact cost-reduction effect from the merger; and, importantly, only welfare improving mergers are approved.

Under incomplete information, however, the converse of these results can be sustained: firms may invest large resources in their merger request to convey or conceal their efficiency to the CA and this authority may end up approving mergers that are welfare reducing due to its lack of information. In this context, the CA uses the firms' investment in the submission process as a signal to infer the cost-reducing effect of the merger. For presentation purposes, we first consider that firms, if submitting a merger request, only need to pay administrative fees, denoting this case as "exogenous costs," as firms' investment is given by the fee. In this setting, we identify a separating equilibrium, where only welfare-improving mergers are approved, as under complete information; and a pooling equilibrium where all firm types submit a merger request and the CA responds approving it. As a consequence, mergers that reduce welfare are approved when the CA is uninformed while they would have been blocked under complete information. For compactness, we refer to them as "inefficient mergers." We measure the output distortion arising from this inefficient merger, showing that a more expensive administrative fee can shrink the region of parameter values supporting the pooling equilibrium more significantly than that sustaining the separating equilibrium if the merging firms account for a relatively large share of the industry. In this case, an increase in administrative fees can convey information from the firm to the CA, helping the latter's task of approving or blocking requests under incomplete information. In contrast, when merging firms represent a small fraction of the market, our results suggest that increasing administrative fees hinder information

represent a large monetary outlay, given the number of consulting public relations companies offering services to firms planning to start a merger, such as TV advertising, press realeases, and training for media interviews.

transmission, ultimately hampering the CA's task. This may be the effect of the recently enacted Merger Filing Fee Modernization Act of 2021, which introduces small fee changes for mergers below US$1 billion, but significantly increases fees otherwise.[5]

We then allow for firms to invest any resources in their submission process, beyond the (given) administrative fee, which we refer as "endogenous costs," such as public relations campaigns or consulting companies. In this context, we also identify separating and pooling equilibria, with inefficient mergers being approved in the latter. Relative to exogenous costs, we demonstrate that the additional investment that firms must incur to conceal their type hinders the emergence of pooling equilibria, regardless of whether the merging entity accounts for a large share of the industry, which ultimately helps the CA's task at assessing these requests. Informally, this means that allowing firms to invest unrestricted amounts in their submission process are good news for the CA, particularly when this agency cannot accurately observe the cost-reduction effect arising.

Our paper then explores the effect of different policy tools on information transmission. First, we consider an increase in administrative fees, showing that they facilitate information transmission more significantly under endogenous than exogenous submission costs, suggesting that increasing these fees yields larger welfare gains when firms can invest in their submission process than otherwise. The increase in fees for large merger deals under the Filing Fee Modernization Act of 2021 may, then, be particularly beneficial if firms operate under an endogenous cost setting (allowed to invest in the submission process), but the decrease in fees for small deals under this act may be detrimental, hindering information transmission to the CA. Second, we investigate whether setting investment limits can hinder the pooling equilibria, showing that, instead, these limits do not promote information transmission, thus making inefficient mergers more likely to arise.[6]

Third, we examine whether a "naive CA," which approves requests solely based on its initial priors, would lead to different equilibrium outcomes than a "strategic CA" which updates its beliefs upon observing firms' investment decisions. We show that firms anticipate that a naive CA will approve merger requests regardless of their investment decisions, making these requests less costly, expanding the region of pooling equilibria and, ultimately, making inefficient mergers more likely to arise under naive than strategic CAs. As a consequence, naive CAs which, essentially, ignore the investment effort an entity made upon submitting a merger request, can give rise to more inefficient mergers being approved than strategic CAs which take into account this investment effort. Our result does not imply that, upon receiving merger requests, CAs should ignore hard evidence, but it highlights that they should also consider overall submission costs. Otherwise, CAs only examining hard evidence could inadvertently facilitate the submission, and subsequent approval, of inefficient

[5] As a reference, filing fees in merger deals between US$1 and $2 billion increased from $280,000 to $400,000; those between $2 and $5 billion increased from $280,000 to $800,000; while those above $5 billion increased from $280,000 to $2,250,000. In contrast, deals between $92 and $161.5 million saw a reduction in filing fees from $45,000 to $30,000; and those between $184 to $500 million also decreased their fees from $125,000 to $120,000. For more details on this act, see https://www.congress.gov/bill/117th-congress/house-bill/3843?s=1&r=61.

[6] In particular, if the investment limit is higher than the investment that firms choose in the pooling equilibrium, this limit is not binding. Otherwise, the limit helps firms coordinate into a lower pooling invesment level, actually expanding the set of parameter values that sustain the pooling equilibrium, ultimately hindering information transmission.

mergers.

Finally, we extend our model along different dimensions, showing that our results are qualitatively unaffected. First, we consider type-dependent submission costs, where the cost of investing differs across firm types. Second, we allow the CA to assess merger requests based on different guidelines, such as whether they increase consumer surplus or total welfare, demonstrating that the pooling equilibrium (and, hence, inefficient mergers) are less likely to arise when the CA uses welfare than consumer surplus as a guideline. Third, we examine how our findings would be affected if firms could dedicate part of their investment to provide verifiable information (hard evidence) to the CA, which would modify this agency's prior before the beginning of the signaling game. And fourth, we allow for the CA to have a continuous, instead of binary, response to a merger request, exerting different effort levels to challenge the request. In this setting, the CA is less likely to approve inefficient mergers in equilibrium, which are good news for the CA. Efficient mergers, however, can now be blocked with a positive probability, which did not happen when the CA's responses were binary.

Related literature. The literature on mergers is extensive and covers topics such as firms' incentives to merge if their market share is sufficiently high, Salant et al. (1983); its welfare effects when the merger reduces the production costs of the merging firms, Williamson (1968), Perry and Porter (1986) and Farrell and Shapiro (1990); mergers when firms compete in prices, Deneckere and Davidson (1985) and Besanko and Spulber (1993); CAs receiving merger proposals over time, and choosing which ones to approve in a dynamic setting, Nocke and Whinston (2010); or firms choosing between alternative merger sizes and the CA responding with a tougher standard on large mergers, Nocke and Whinston (2013). For literature reviews see, for instance, Whinston (2006), Kaplow and Shapiro (2007), and Werden and Froeb (2008).

Our model is, however, more related to the literature examining the role of incomplete information in mergers, either because firms are not informed about some other firm's costs, the merger's efficiency, or because the CA does not observe this efficiency.[7] Specifically, our paper considers a signalling game similar to that in Besanko and Spulber (1993) but allowing for filing fees (merger submission costs) to be endogenous, as opposed to exogenous in their setting. This helps us evaluate how equilibrium results are affected, test if information transmission is improved, and identify if the approval of socially inefficient mergers is ameliorated when firms choose their investment in submission costs while preparing the merger request. In addition, they assume an industry with two firms competing in prices, so a full merger reduces consumer surplus with certainty but may increase or decrease social welfare. Our setting allows, instead, for a merger between a share of all firms competing in quantities, which may increase or decrease consumer surplus depending on the severity of its cost-reducing effect.

Lagerloff and Heidhues (2005) also study merger approval decisions, but assume that firms are

[7]For studies introducing incomplete information on the firm's side, see Saloner (1987) and Loertscher and Marx (2021), analyzing mergers where firms have private information about its production costs. They evaluate how this information asymmetry affects firms' decision to merge and their pre-merger market outcomes (e.g., price wars).

initially uninformed about the cost-reducing effect of the merger, and must invest in producing hard evidence about this cost reduction to present to the CA.[8] As a consequence, firms and CA are symmetrically informed in their setting, thus not allowing for signaling behaviors to occur in equilibrium. In contrast, our model allows firms to have more accurate information about the cost-reducing effect than the CA does, so their investment in expert reports and consulting companies can help the CA update its beliefs about the firm's type. In a pooling equilibrium, where both firm types invest positive amounts in the submission process, the CA is left "in the dark" about the merger's efficiency, which cannot occur in their model.[9] As similar signaling models, our setting considers that submission costs can help firms convey information about their efficiency, even if their investment does not produce hard evidence to present to the CA. (This is analogous to education not being labor-enhancing in standard labor-market signaling games where, nonetheless, investment in education can help workers convey their types to uninformed employers.) Allowing for a share of the firm's investment to produce hard evidence would not qualitatively affect our main results, as that evidence would only shift the CA's priors, making this agency assign a larger probability to the merger's efficiency being high.[10]

The following section describes the model, section 3 identifies equilibrium behavior under a complete information context, which we use later to evaluate merger inefficiencies under incomplete information settings. Section 4 (5) considers that the CA is incompletely informed about the merger efficiency, and the merging entity only pays administrative fees when submitting a merger request (can invest resources beyond the administrative fee, respectively). Section 6 examines different extensions to our model, and section 7 discusses our main results and their policy implications.

2 Model

Consider an industry with $n \geq 2$ firms competing a la Cournot, facing inverse demand function $p(Q) = 1 - Q$, where Q denotes aggregate output, and a marginal cost c, where $1 > c \geq 0$. As in Perry and Porter (1985), we allow for "synergies" among the k merging firms (also known as insiders), where k satisfies $n \geq k \geq 2$. In particular, insiders see their marginal costs decrease from c to $c - x$, where parameter x denotes the cost-reduction effect of the merger (e.g., better management practices, avoid cost duplicities, etc.). In contrast, the marginal cost of the remaining $n - k$ firms (outsiders) remains at c. Therefore, after the merger, $n - k + 1$ firms operate in the industry.

The time structure of the game is the following:

[8] In particular, if the merging entity invests in hard evidence about the efficiency of the merger, it can produce this evidence with probability τ_i, where $i = \{H, L\}$ denotes the merger's efficiency which satisfies $\tau_H > \tau_L$. With probability $1 - \tau_i$, the merging entity does not find hard evidence, meaning that the CA does not receive any information.

[9] This result cannot occur in their extension either, where they allow for firms to fabricate false evidence to submit to the CA, as the CA does not update its believes about the firm's type in that setting.

[10] Other recent articles analyzing welfare effects in mergers include Prat and Valletti (2021), in digital platforms, and Miller et al. (2021), in the US beer industry.

1. In the first stage, the k firms that seek to merge choose whether to submit, as an entity, a merger approval request to the competition authority (CA), at a fixed cost $R \geq 0$. (In subsequent sections, we assume that this cost either coincides with an administrative fee, $R = f$; or allow the merging entity to choose its spending, $R \geq f$.)

2. In the second stage, the CA responds approving or blocking the merger request. If no merger request was submitted, then the CA is not called to move. For simplicity, we consider that the CA evaluates mergers considering whether they increase consumer surplus, but also describe how our results are qualitatively affected if, instead, the CA evaluates mergers considering whether they increase total welfare, which includes consumer and producer surplus.

3. In the third stage, firms observe the CA's decision, and compete a la Cournot.

As a benchmark, we first analyze equilibrium behavior in a complete information context where the CA perfectly observes the cost-reduction effect of the merger (i.e., the realization of parameter x). Afterwards, we examine an incomplete information setting where the CA cannot observe the realization x, only holding a prior probability distribution over x, but may infer its realization by observing whether firms filed a merger approval.

The realization of x is, however, observed by all firms (insiders and outsiders). Otherwise, their decision about whether to join the merger in the first stage would have to made in expectation (if, for instance, some firms in the industry do not observe the realization of x) and, subsequently, their output decision in the third stage would have to be computed in expectation as well. Equilibrium behavior between the complete and incomplete information games would, then, differ along several dimensions, making comparisons more obscure, ultimately hindering our ability to identify how the CA's inability of observe the cost-reduction effect of the merger impacts its approval decisions in equilibrium.

3 Complete Information

Solving the complete information game by backward induction, we find the following output in the third stage. (For compactness, superscript NM denotes "no merger," whereas M means "merger.")

Lemma 1. *In the third stage, if the merger is blocked, the equilibrium output of every firm i is $q_i^{NM} = \frac{1-c}{n+1}$ and equilibrium profits are $\pi_i^{NM} = \left(q_i^{NM}\right)^2$. If the merger of k firms is approved, the equilibrium output of insiders (as a group) is $q_I^M = \frac{1-c+(n-k+1)x}{n-k+2}$ with equilibrium profits $\pi_I^M = \left(q_I^M\right)^2$; while that of each outsider is $q_O^M = \frac{1-c-x}{n-k+2}$ with equilibrium profits $\pi_O^M = \left(q_O^M\right)^2$.*

Outsiders produce a positive output if $x < 1 - c$ or, alternatively, if the cost-reduction effect, as captured by $\theta \equiv \frac{x}{1-c}$, is not excessive, $\theta < 1$, which we assume throughout the paper. Anticipating equilibrium output and profits in the third stage, the CA compares the welfare that arises from approving a merger with that from blocking the merger.

Lemma 2. *In the second stage, the CA approves a k-firm merger if and only if $\theta > \overline{\theta}$ ($\theta > \overline{\theta}_W$) when considering consumer surplus alone (considering consumer and producer surplus, respectively), where $\overline{\theta} \equiv \frac{k-1}{n+1}$ and $\overline{\theta}_W \equiv \frac{(n-k+2)\sqrt{n^2-2k^2+4kn+6k}-(n+1)(n-k+3)}{(n+1)[2(n^2+k^2+3n-2kn-3k)+3]}$.*

As described by Perry and Porter (1985), the CA faces a trade-off when approving the merger: it suffers from a more concentrated industry, but it benefits from a cost-reduction effect on insiders. If this cost-reduction effect is sufficiently strong, i.e., $\theta > \overline{\theta}$, then the merger enhances consumer surplus. A similar argument applies when the CA considers both consumer and producer surplus in its merger evaluation, where the merger is approved if $\theta > \overline{\theta}_W$.[11] The remainder of the paper considers that the CA evaluates mergers using the consumer surplus criterion, as proposed by Pittman (2007), among others; and because cutoff $\overline{\theta}$ is more analytically tractable than cutoff $\overline{\theta}_W$, as shown in Lemma 2. For robustness, section 6.3 discusses how our results would be affected if the CA considers, instead, cutoff $\overline{\theta}_W$ to evaluate merger requests.

We finally analyze firms' decision to request a merger approval in the first stage of the game. In the absence of the CA ($R = 0$), k firms merge if and only if $\pi_I^M \geq k\pi_i^{NM}$, where profits π_I^M and π_i^{NM} were defined in Lemma 1. Solving for θ, yields that firms merge if and only if $\theta > \theta_F$, where $\theta_F \equiv \frac{(n-k+2)\sqrt{k}-(n+1)}{(n-k+1)(n+1)}$. As we next show, when the CA is present, firms file merger approval under more restrictive parameter conditions.

Proposition 1. *In the first stage, a k-firm merger is submitted for approval if and only if $\theta > \max\left\{\widehat{\theta}, \overline{\theta}\right\}$, where $\widehat{\theta} \equiv \frac{n-k+2}{(1-c)(n-k+1)}\sqrt{k\left(\frac{1-c}{n+1}\right)^2 + R} - \frac{1}{n-k+1}$. Cutoffs satisfy $\overline{\theta} > \widehat{\theta} > \theta_F$ when $R \leq \widehat{R}$, but $\widehat{\theta} > \overline{\theta} > \theta_F$ otherwise, where $\widehat{R} \equiv \left(\frac{1-c}{n+1}\right)^2\left[\left(\frac{(n+1)+(n-k+1)(k-1)}{(n-k+2)}\right)^2 - k\right]$. Cutoff $\widehat{\theta}$ is increasing in R and c.*

First, note that cutoff $\widehat{\theta}$ coincides with θ_F when administrative fees are negligible, $R = 0$, but otherwise $\widehat{\theta}$ increases in R, satisfying $\widehat{\theta} > \theta_F$ for all $R > 0$. Intuitively, the merging entity finds the merger less attractive when the CA is present (and must incur a submission cost f) than when the CA is absent.[12]

Second, figure 1 plots cutoffs $\widehat{\theta}$, $\overline{\theta}$, and $\widehat{R}$, thus illustrating that the ranking of cutoffs $\widehat{\theta}$ and $\overline{\theta}$ satisfies $\widehat{\theta} < \overline{\theta}$ for all $R < \widehat{R}$ (to the left of $\widehat{R}$ in the figure), but $\widehat{\theta} \geq \overline{\theta}$ otherwise. The figure helps summarize that, in equilibrium, four regions arise where: (1) the merger is both unprofitable and welfare reducing (when R is relatively high and θ is low); (2) the merger is still unprofitable but welfare improving (when R and θ are relatively high); (3) the merger is profitable but welfare reducing (when R and θ are low); and (4) the merger is profitable and welfare improving (when R is low and θ is high). Firms anticipate these regions, choosing to only file merger approvals in

[11] The ranking of cutoffs $\overline{\theta}$ and $\overline{\theta}_W$ is a non-linear function of k and n but, intuititvely, when $\overline{\theta} > \overline{\theta}_W$, condition $\theta > \overline{\theta}$ is sufficient for $\theta > \overline{\theta}_W$ to also hold, implying that if a merger improves consumer surplus then it must also be welfare improving. The opposite occurs when cutoffs satisfy $\overline{\theta} \leq \overline{\theta}_W$, namely, if a merger is welfare enhancing, it must also increase consumer surplus.

[12] Cutoff $\widehat{\theta}$ is increasing in R and c, but its comparative statics with respect to k and n yield nonlinear equations.

the fourth region, where their submission costs are low and the cost-reduction effect of the merger is sufficiently strong, $\theta > \max\left\{\widehat{\theta}, \overline{\theta}\right\}$, as they anticipate that the request will be approved by the CA. Figure 1 also represents the special case where $R = 0$ along the vertical axis.

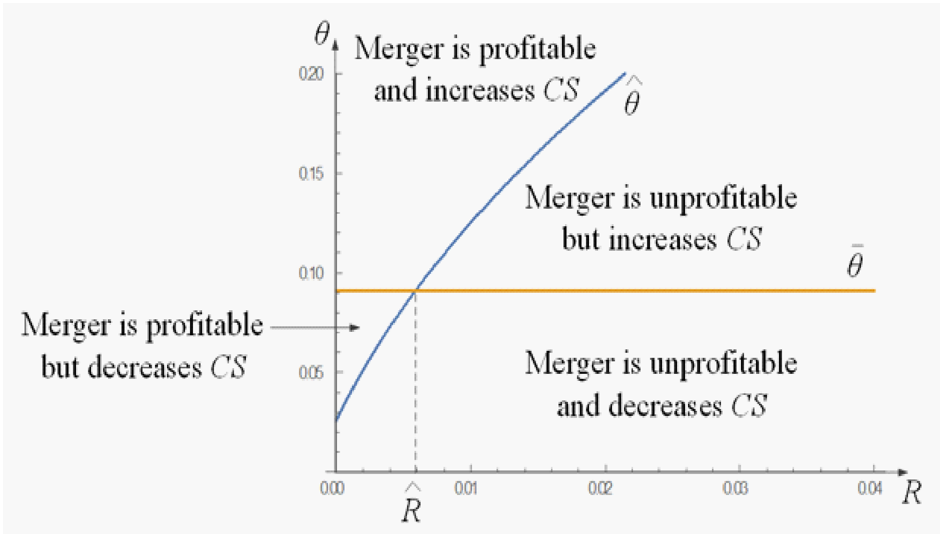

Figure 1. Equilibrium mergers and welfare.

In summary, the subgame perfect equilibrium (SPE) of the game prescribes that k firms file a merger approval if and only if $\theta > \max\{\widehat{\theta}, \overline{\theta}\}$; in region (4). The CA responds approving this request in the subsequent period and, then, insiders produce q_I^M units while outsiders produce q_O^M, as reported in Lemma 1. If $\theta > \max\{\widehat{\theta}, \overline{\theta}\}$ does not hold (in regions 1-3), no firm files a merger approval and every firm i responds producing the Cournot output q_i^{NM} in the last stage.

For simplicity, the remainder of the paper assumes that submission costs are relatively low, $R < \widehat{R}$, implying that, if a merger is welfare improving it must also be profitable. The converse is, however, not necessarily true, meaning that a merger may be profitable but may not be welfare improving.

4 Incomplete Information - Exogenous costs

Consider now that the CA does not observe the realization of the cost-savings effect, x, which implies that the CA does not observe $\theta \equiv \frac{x}{1-c}$ either. The CA, however, holds a prior probability of θ, where $\theta = \theta_H$ with probability p, and $\theta = \theta_L$ otherwise. Importantly, θ_H and θ_L satisfy $\theta_H > \overline{\theta} > \theta_L$, entailing that the CA would approve mergers if it was perfectly informed that $\theta = \theta_H$ but block them otherwise.[13]

[13] Assuming, instead, that $\overline{\theta}$ satisfies $\overline{\theta} > \theta_H$ ($\theta_L > \overline{\theta}$) would make incomplete information uninteresting, as the CA would have blocked (approved, respectively) mergers from all firm types under complete information.

In addition, let

$$\begin{aligned} E[\theta] &\equiv p\frac{x_H}{1-c} + (1-p)\frac{x_L}{1-c} \\ &= p\theta_H + (1-p)\theta_L \end{aligned}$$

denote the expected cost-reduction effect of the merger. We consider that $E[\theta] > \overline{\theta}$, which entails that $p > \frac{\overline{\theta}-\theta_L}{\theta_H-\theta_L} \equiv \widehat{p}$. Intuitively, the high type is relatively likely or, alternatively, its cost-reduction advantage as captured by $\theta_H - \theta_L$ is sufficiently strong.

As in previous sections, a merger is profitable for type-i firm if and only if $\theta_i > \widehat{\theta}$. As defined in Proposition 1, cutoff $\widehat{\theta}$ is a function of submission cost, which in this section coincides with administrative fees, $R = f$. (Section 5 allows for firms to strategically choose the resources they invest in their submission, instead of assuming that this cost is exogenous.)

Therefore, under complete information, the high-type (low-type) entity submits (does not submit) a merger request since it anticipates that the CA will respond approving (blocking, respectively) it in the next stage. As we study next, this separating strategy profile may not arise when the CA is imperfectly informed about the firm's type.

In this incomplete information context, consider the following signaling game:

1. In the first stage, the merging entity privately observes its type, θ_H or θ_L, and decides whether to submit a merger request to the CA. In this section, this merger request is interpreted as an administrative form which cannot include additional technical reports describing the potential efficiency gains from the merger. Section 5 allows firms to invest in these reports from experts and consulting companies, thus making the modeling of the merger request more similar to real-life settings.

2. In the second stage, the CA does not observe the merging entity's private type, but observes whether it submitted a merger approval request (M) or not (NM). Upon observing M, the CA updates its beliefs about the merging entity's type being high, $\mu(\theta_H|M)$, or low, $\mu(\theta_L|M)$, applying Bayes' rule whenever possible. Given this set of beliefs, the CA responds to the merger request approving it or blocking it.[14]

3. In the third stage, firms observe the CA's decision, and compete a la Cournot.

Unmerged firms can observe the cost-reduction effect of the merger, theta, in the last stage of the game. (Recall that, when the merger does not ensue, no cost-reduction effects arise.) This assumption is, then, similar to that in standard limit pricing models, where a potential entrant observes the incumbent's production costs after joining the industry.[15] Intuitively, the uninformed player has enough time to observe this information. Alternatively, the Cournot profits in the last

[14] When the CA observes, instead, that the merging entity did not submit a request, the CA is not called to move, implying that its belief updating upon observing NM is inconsequential.

[15] See Milgrom and Roberts (1982) and extensions by Harrington (1987), Bagwell and Ramey (1991), Schultz (1999), and Ridley (2008).

stage of the game describe the average per-period payoff in a sufficiently long game that begins after the potential entrant joins the industry and, given its length, all firms can eventually infer each other's production costs with precision. A similar argument applies to merged and unmerged firms in our model.

4.1 Separating equilibria

We first study separating Perfect Bayesian Equilibria (PBEs) where the θ_H-type merging entity submits a merger request, while the θ_L-type does not. This strategy profile allows the CA to concentrate its beliefs to $\mu(\theta_H|M) = 1$ and $\mu(\theta_L|M) = 0$. In other words, the merger approval request conveys the merging entity's type (i.e., the cost-reduction effect arising from the merger) to the uninformed CA, which can respond approving the request, as only requests originating from the θ_H-type are submitted. The following proposition identifies under which conditions this separating PBE can be sustained.

Proposition 2. *A separating PBE where only the θ_H-type merging entity submits a merger request can be supported if and only if $\theta_H > \widehat{\theta} \geq \theta_L$.*

When $\theta_H > \widehat{\theta} \geq \theta_L$, only the θ_H-type entity submits a merger request, which also holds under complete information (Proposition 1), implying that no inefficiencies emerge due to the CA's inability to observe the cost-reduction effect of the merger.

4.2 Pooling equilibria

We next examine pooling PBEs where both firm types submit a merger approval request. Unlike in the separating PBE, the CA cannot update its beliefs upon observing M, which coincide with its priors, i.e., $\mu(\theta_H|M) = p$ and $\mu(\theta_L|M) = 1-p$. Therefore, the expected cost-reduction effect from the merger remains at $E[\theta]$ and, since $E[\theta] > \overline{\theta}$ by assumption, the CA responds to the request by approving the merger. In the opposite pooling strategy profile, where none of the firm types submits a merger request, the CA is only called to move off-the-equilibrium path, thus sustaining beliefs $\mu \equiv \mu(\theta_H|M)$ and $1-\mu \equiv \mu(\theta_L|M)$ upon observing a merger request.

The next proposition investigates under which conditions these two pooling PBEs can be supported where, for compactness, we only present those strategy profiles surviving the Cho and Kreps' (1987) Intuitive Criterion.[16]

Proposition 3. *A pooling PBE where every merging entity $i = \{H, L\}$ submits (does not submit) a merger request can be supported if and only if $\theta_i \geq \widehat{\theta}$ ($\theta_i < \widehat{\theta}$) for every type i, and beliefs satisfy $\mu(\theta_H|M) = p$ ($\mu(\theta_H|M) = \mu$, respectively).*

[16] Other refinement criteria, such as the Banks and Sobel's (1987) Divinity Criterion would yield the same results (i.e., it has the same bite) because our setting only has two types (high and low) and two possible messages (request or not request). Section 5.5 elaborates on refinement criteria.

For completeness, Appendix 2 investigates under which conditions semiseparating equilibria can be sustained, where firms and the CA randomize their decisions. We show, however, that it can only be sustained under relatively restrictive parameter conditions, i.e., θ_L must coincide with cutoff $\widehat{\theta}$.

4.3 Socially inefficient mergers

Figure 2 summarizes our results in Propositions 2 and 3, comparing them against those under complete information (Proposition 1). This comparison helps us identify the presence of inefficiencies, i.e., mergers that would have not been approved under complete information. The horizontal (vertical) axis plots θ_L (θ_H) which lies below $\overline{\theta}$ since $\theta_L < \overline{\theta}$ (above $\overline{\theta}$ since $\theta_H > \overline{\theta}$) by definition. In addition, we focus on the region above the 45-degree line since $\theta_H > \theta_L$ by assumption. Starting at the northwest of the figure, where θ_H is significantly larger than θ_L, only the high-type submits a merger under incomplete information, as predicted in the separating equilibrium (SE) of Proposition 2. As described after that proposition, this equilibrium behavior coincides with that under complete information, thus giving rise to no inefficiencies. A similar argument applies in the southwest region of the figure, where both θ_H and θ_L are relatively low, driving both firms to not submit a merger request (supported as one of the pooling PBEs); a finding that coincides with that under complete information.

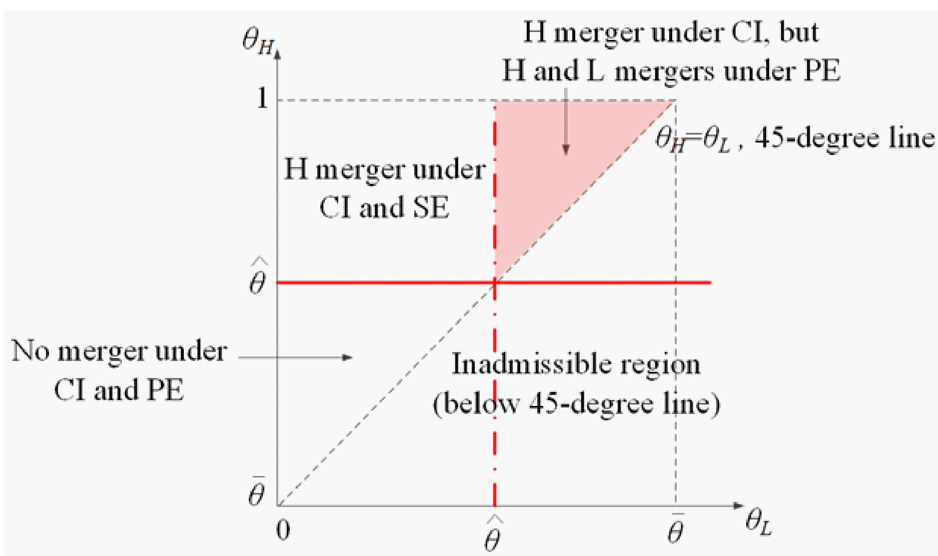

Figure 2. Equilibrium behavior under incomplete information and inefficiencies.

However, in the northeast region of the figure, where both θ_H and θ_L are relatively high (shaded area), both firm types submit a merger request, as predicted in the first pooling PBE in Proposition 3, which are approved in equilibrium by the CA. Under complete information, in contrast, only the high-type would have submitted a merger request, implying that the CA approves an inefficient merger (that of the low-type entity) under incomplete information. Intuitively, this occurs because both firm types experience a relatively large cost-reduction effect from the merger (as captured by high values of θ_H and θ_L), which still satisfies $\theta_L < \overline{\theta}$, implying that the low-type merger is welfare decreasing. Importantly, this region expands when cutoff $\widehat{\theta}$ decreases, which occurs when this firm's submission cost, R_L, decreases.[17] Corollary 1 summarizes the regions in which inefficient mergers can be sustained in equilibrium.

Corollary 1. *Inefficient mergers arise in equilibrium if $\theta_i > \widehat{\theta}$ for every firm type i, where a low-type merger is approved (blocked) under incomplete (complete) information.*

4.3.1 Measuring efficiency losses

The output distortion from approving the merger between low-type firms under incomplete information is given by

$$Q^{NM} - Q^M(\theta_L) = \frac{(1-c)\left[k-1-(n+1)\theta_L\right]}{(n+1)(n-k+2)},$$

where the CA considers the aggregate output that the low-type firm would produce under complete information, where it observes the firm's type and the merger does not ensue, $Q^{NM} = nq_i^{NM}$, which is unaffected by θ_L; and the aggregate output pooling equilibrium, where the merger is approved, $Q^M(\theta_L) = q_I^M + (n-k)q_O^M$. As the low-type firm's cost-reduction effect increases (higher θ_L), the output distortion, as captured by $Q^{NM} - Q^M(\theta_L)$, decreases. Intuitively, the high and low-type firms become more symmetric, reducing the output distortion of incorrectly approving a merger under incomplete information.

The output distortion is, then, weighted by the size of the shaded triangle in figure 2 to measure if the pooling PBE can be sustained under a large or small set of parameter values, yielding the "merger inefficiency" with exogenous costs ($MI_{exog.}$), as follows

$$MI_{exog.} = \frac{\overbrace{\left(1-\widehat{\theta}\right)\left(\overline{\theta}-\widehat{\theta}\right)}^{\text{Shaded area in figure 2}}}{2} \times \overbrace{\left[Q^{NM} - Q^M(\theta_L)\right]}^{\text{Output distortion}}.$$

The administrative fee f is not included in $MI_{exog.}$. While the low-type firm would have not submitted a merger request under complete information, this fee is just a transfer from the firm to the government, thus being welfare neutral.[18] We next analyze how this inefficiency is affected by more expensive administrative fees.

Corollary 2. *A marginal increase in $\widehat{\theta}$ produces:*

[17] Technically, the pooling PBE where both firms submit a merger request is sustained when both $\theta_H \geq \widehat{\theta}$ and $\theta_L \geq \widehat{\theta}$, implying that the region supporting this equilibrium expands when both cutoff $\widehat{\theta}$ decreases. However, since $\theta_H \geq \theta_L$ by assumption, this region expands as long as cutoff $\widehat{\theta}$ decreases.

[18] If a share $\gamma \in [0,1]$ of the administrative fee is lost, not transferred between agents, the expression of $MI_{exog.}$ would include term γf, thus augmenting the inefficiencies that arise from the merger. Our qualitative results, however, would be unaffected.

1. *A shrink in the region of parameters supporting the pooling PBE (Proposition 3), thus reducing $MI_{exog.}$, which holds under all parameter values.*

2. *A shrink in the region of parameters sustaining the separating PBE (Proposition 2) if and only if $\widehat{\theta} > 1/2$.*

3. *The shrinking effect in point #1 (#2) dominates if $\overline{\theta}$ satisfies $\overline{\theta} > \max\{6\widehat{\theta} - 3, \widehat{\theta}\}$ ($6\widehat{\theta} - 3 > \overline{\theta} > \widehat{\theta}$, respectively).*

Therefore, an increase in f produces an increase in cutoff $\widehat{\theta}$ which shrinks area F, ultimately reducing $MI_{exog.}$. Intuitively, a higher fee hinders the emergence of the pooling PBE, thus making inefficient mergers less likely to occur. This increase in f, however, expands the northwest area of figure 2, where efficient mergers arise, if $\widehat{\theta} < 1/2$. Otherwise, a higher f hinders both the emergence of efficient and inefficient mergers in equilibrium.[19]

The last point of Corollary 2 identifies that the pooling PBE shrinks more significantly than the separating PBE, thus facilitating information transmission, if $\overline{\theta}$ is sufficiently high. Intuitively, recall that $\overline{\theta} \equiv \frac{k-1}{n+1}$, implying that high values of $\overline{\theta}$ indicate that the merging firms account for a larger share of the industry. In this case, increasing administrative fees (and, thus, cutoff $\widehat{\theta}$) can help improve the CA's information when deciding whether to approve the merger request. In contrast, for intermediate values of $\overline{\theta}$ (mergers between a small share of firms), the separating PBE shrinks more substantially, implying that administrative fees should actually be lowered to improve the CA's information.

5 Incomplete information - Endogenous costs

Previous sections assume that submission costs were exogenous. In several contexts, however, firms can often choose how many resources to spend on preparing their merger request before submitting it to the CA, such as hiring consulting companies and experts. This cost could be interpreted more generally as also including lobbying efforts to politicians who could influence the CA's opinion about the merger request, or ad campaigns on different media outlets describing the merger as beneficial for consumers.

In this section, we allow every entity i to invest $R_i = f + r_i$ dollars on their submission to the CA, where $f \geq 0$ denotes the administrative fee and r_i represents any additional investment associated with the merger request. This investment is observable by the CA, which could be explained by this agency inspecting the number of expert reports included in the merger request and/or the consulting companies involved.[20]

[19] As a reference, condition $\widehat{\theta} < 1/2$ holds for sufficiently low values of f; a condition that becomes more restrictive when the number of outsiders, $n - k$, increases. Intuitively, when fewer firms merge, the cost-reduction effect of the merger must become more significant for the merger to be profitable, thus requiring lower administrative fees, f, for condition $\widehat{\theta} < 1/2$ to hold.

[20] Alternatively, R_i could take a functional form $R_i(e_i, \theta_i)$, where e_i denotes the hours of effort that θ_i-type entity

Therefore, $R_i = 0$ means that the merging entity did not submit a merger request (not even incurring the administrative fee of submitting a form requesting the approval to the CA), while $R_i \geq f$ indicates that it did.[21] Technically, the presence of exogenous submission costs, then, changes the structure of the signaling game, from one in which the merging entity sends binary messages (submit or not a merger request) to one where it can send a continuum of messages (investment $R_i \geq 0$). The CA has only two available responses (approve or block) under both exogenous and endogenous submission costs, but updates its beliefs by observing a potentially larger set of messages under the latter. (For robustness, we consider other assumptions behind the submission process in the Extensions section, showing that our results are qualitative unaffected.)

5.1 Separating equilibria

We first examine under which conditions a separating PBE can be sustained where entities invest different amounts in preparing their submissions, R_H and R_L, where $R_H \neq R_L \geq 0$. As in Proposition 2, this strategy profile helps the CA infer the entity's type upon observing its investment level.

Proposition 4. *If $\pi_I^{M,L} - k\pi_i^{NM} \geq f$, a separating PBE can be sustained if $\theta_H > \widehat{\theta}(R_H^{SE}) > \theta_L$ where the merging entity invests $R_H^{SE} = \pi_I^{M,L} - k\pi_i^{NM}$ when its type is high and $R_L^{SE} = 0$ when its type is low, and cutoff $\widehat{\theta}(R_H^{SE})$ satisfies $\widehat{\theta}(R_H^{SE}) = \widehat{\theta}$.*

Intuitively, the low-type entity does not submit a merger request (investing zero) as it anticipates that any investment R such that $f < R_H^{SE}$ leads the CA to believe that R must originate from the low-type entity, responding blocking the merger. In addition, investing R_H^{SE}, while inducing a merger approval, is unprofitable for the low-type entity, as it anticipates a relatively minor cost-reducing effect due from the merger (low θ_L). In contrast, the high-type entity submits a merger request, investing $R_H^{SE} = \pi_I^{M,L} - k\pi_i^{NM}$, which conveys its type to the CA, responding with the approval of the merger.[22] While other separating PBEs can be sustained where $R_H \geq \pi_I^{M,L} - k\pi_i^{NM}$ and $R_L^{SE} = 0$, the separating equilibrium in Proposition 4 is the least-costly separating PBE, also known as the "Riley outcome" after Riley (1979), as it allows the high-type to invest the least to distinguish itself from the low-type entity. Indeed, investing more than R_H^{SE} would render the merger unprofitable for the low-type entity. (Note that if administrative fees are relatively high, $\pi_I^{M,L} - k\pi_i^{NM} < f$, the high-type firm does not submit a merger request, implying that this separating PBE cannot be sustained.)

exerts in the submission, yielding a total cost of R_i, satisfying standard properties ($R_i(\cdot)$ is increasing and convex in effort, and its cross-partial derivative satisfies $R_{e_i,\theta_i} < 0$). The CA, however, is more likely to observe total investment levels in hiring experts and/or lobbying politicians, as captured by R_i, than the specific effort levels exerted by the merging entity. For this reason, we focus on the signaling role of R_i.

[21] If we allowed for investment levels such that $f > R_i > 0$, they would indicate that the merging entity did not incur the administrative fee, and thus the merger request is not submitted, which are strictly dominated by $R_i = 0$. As a result, the merging entity either does not submit the request, $R_i = 0$, or submits it, $R_i \geq f$, as specified above.

[22] Investment R_H^{SE} lies below cutoff $\widehat{f}$ (the upper bound on R defined in Proposition 1) for all $\theta_L < \overline{\theta}$, which holds by definition.

Relative to the separating equilibrium under exogenous submission costs, note that cutoffs satisfy $\widehat{\theta}(R_H^{SE}) \geq \widehat{\theta}$ because $R_H^{SE} \geq f$. Intuitively, the high-type entity exerts a more significant separating effort, by investing more in the submission process, than when firms only pay administrative fees. As a consequence, the high-type (low-type) has less (more) incentives to behave as prescribed by this separating strategy profile, yielding an ambiguous effect on the region of parameter values sustaining this equilibrium. However, as we show next, the increase in submission costs hinders the emergence of pooling equilibria where inefficient mergers can be sustained.

5.2 Pooling equilibria

Before examining how the equilibrium results differ from those under complete information or with exogenous submission costs, we identify under which conditions a pooling PBE can be sustained.

Proposition 5. *A pooling PBE can be supported if $\theta_i > \widehat{\theta}(R^{PE})$ for every $i = \{H, L\}$, where both types of the merging entity choose a common investment R^{PE}, where $f \leq R^{PE} \leq \pi_I^{M,L} - k\pi_i^{NM}$, and CA responds approving the merger request, given beliefs $\mu\left(\theta_H | R^{PE}\right) = p$ and $\mu\left(\theta_H | R'\right) = 0$ for all off-the-equilibrium investment levels $R' \neq R^{PE}$. However, if $\theta_i \leq \widehat{\theta}(0)$ for every i, a pooling PBE can be sustained where no firm type submits a merger request, $R^{PE} = 0$.*

Therefore, when both types benefit from a sufficiently high cost-reducing effect from the merger, $\theta_H > \theta_L > \widehat{\theta}(R^{PE})$, both types submit a merger request, making relatively low investments (R^{PE} ranges from f to $\pi_I^{M,L} - k\pi_i^{NM}$). This common investment does not help the CA infer the type of firm it faces, and thus responds approving the merger since $p > \widehat{p}$. In addition, cutoff $\widehat{\theta}(R^{PE})$ decreases as R^{PE} decreases, thus expanding the parameters sustaining this pooling PBE as, intuitively, firms pool into a less expensive investment, thus implying that firms can more easily invest in R^{PE}, even when merger synergies are less significant. Finally, when R^{PE} decreases to $R^{PE} = f$, then cutoff $\widehat{\theta}(f)$ separates the pooling equilibrium where both firm types invest, if $\theta_i > \widehat{\theta}(f)$, and that in which they do not, if $\theta_i \leq \widehat{\theta}(f)$.

Figure 3a summarizes the equilibrium results in propositions 4 and 5. Cutoff $\widehat{\theta}(R_H^{SE})$ in the separating PBE applies to both firm types, thus crossing each other at the 45-degree line. Similarly, cutoff $\widehat{\theta}(R^{PE})$ applies to both firm types in the pooling PBE, thus crossing each other at the 45-

degree line as well. In addition, $\widehat{\theta}(R^{PE})$ lies below $\widehat{\theta}(R_H^{SE})$ since R^{PE} satisfies $f \leq R^{PE} \leq R_H^{SE}$.

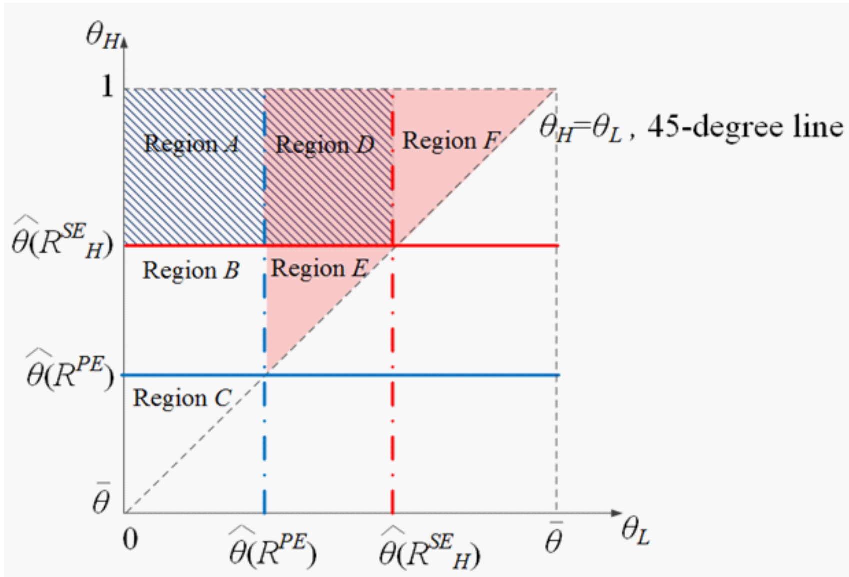

Figure 3a. Separating and pooling PBEs with endogenous submission costs, $R^{PE} < R_H^{SE}$.

From Proposition 4, a separating PBE can be sustained in regions A and D; while from Proposition 5, a pooling PBE can be supported in regions D, E, and F.[23] Intuitively, when firm types are relatively asymmetric (region A) only a separating equilibrium arises; when they are relatively symmetric (region F), only a pooling equilibrium emerges; if their synergies are intermediate and high (region D) both equilibria coexist; and if they are intermediate and low (region E) only a pooling PBE can be supported.

Since R^{PE} satisfies $f \leq R^{PE} \leq R_H^{SE}$, regions B, D, and E arise only when $R^{PE} < R_H^{SE}$, that is, when the pooling investment level is relatively low, as in that context $\widehat{\theta}(R^{PE}) < \widehat{\theta}(R_H^{SE})$. To understand this result, note that if R^{PE} increases, cutoff $\widehat{\theta}(R^{PE})$ increases as well, approaching $\widehat{\theta}(R_H^{SE})$. If R^{PE} increases to its upper bound, $R^{PE} = R_H^{SE}$, cutoffs $\widehat{\theta}(R^{PE})$ and $\widehat{\theta}(R_H^{SE})$ coincide

[23] In region C, a pooling PBE where no firm type invests can be supported (see Proposition 5). Finally, in region B, neither separating or pooling equilibria can be sustained.

(as depicted in figure 3b), and only regions A, C, and F can be supported.

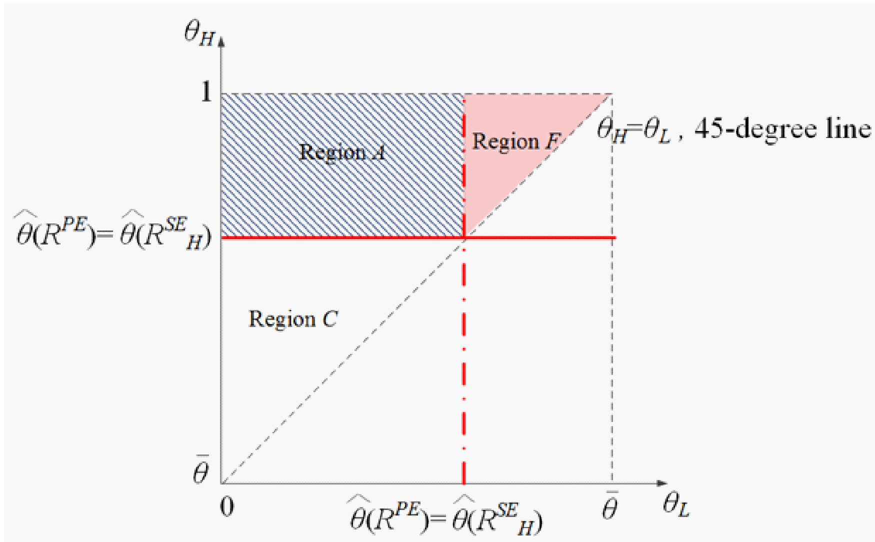

Figure 3b. Separating and pooling PBEs with endogenous submission costs, $R^{PE} = R_H^{SE}$.

Another result from Proposition 5 is that the profit difference $\pi_I^{M,L} - k\pi_i^{NM}$ increases in k, thus decreasing the number of outsiders, $n - k$. If the filing fee f is constant in the profit gain from the merger, as captured by $\pi_I^{M,L} - k\pi_i^{NM}$, the range of R^{PE} values supporting the pooling PBE, $f \leq R^{PE} \leq \pi_I^{M,L} - k\pi_i^{NM}$, expands, thus facilitating the emergence of socially inefficient mergers. To prevent this result, regulators often set filing fees that increase in the profit gain from the merger, or the deal size as a proxy; as in the US, where filing fees became more rapidly increasing in the merger deal after the Filing Fee Modernization Act of 2021.

5.3 Socially inefficient mergers

Complete information. Relative to complete information, where the CA only approves high-type mergers, socially inefficient mergers arise in the pooling PBE of Proposition 5. In particular, if $\theta_i > \widehat{\theta}(R^{PE})$ for every type i (as depicted in regions D, E, and F in figure 3a), mergers of both firm types are approved, while the low-type merger request would have been blocked under complete information.

Exogenous submission costs. A natural question is whether allowing firms to strategically invest in their merger reports (endogenous submission costs) facilitates inefficient mergers, relative to a setting where firms invest a given amount on their the merger requests (exogenous submission costs). Under exogenous costs, recall that both firm types merge if $\theta_i > \widehat{\theta} \equiv \widehat{\theta}(f)$; only the high-type does if $\theta_H > \widehat{\theta}$; and no firm types do if $\theta_i \leq \widehat{\theta}$ for every i. Therefore, equilibrium results under exogenous costs depend on cutoff $\widehat{\theta}$ alone. The next corollary identifies under which conditions endogenous submission costs emphasize or ameliorate the inefficient mergers identified in the pooling equilibrium of Proposition 3.

Corollary 3. *With endogenous submission costs, inefficient mergers are less likely to arise than under exogenous submission costs.*

Intuitively, since firms invest more under endogenous submission costs, $R^{PE} \geq f$, cutoffs satisfy $\widehat{\theta}(R^{PE}) > \widehat{\theta}$, entailing that inefficient mergers are hindered by the firm's ability to invest beyond minimal administrative fees. These are good news for the CA: while investment R^{PE} in the pooling equilibrium conceals the firm's type and, thus, becomes an uninformative signal, it helps this agency reduce the likelihood of an inefficient merger occurring in equilibrium.

5.3.1 Measuring efficiency losses

Approving the merger yields the same output distortion as in section 4.3.2, $Q^{NM} - Q^{M}(\theta_L)$. However, the area of triangle F in figure 3 shrinks relative to that in figure 2, thus entailing a different merger inefficiency with endogenous costs of

$$MI_{endog.} = \frac{\overbrace{\left[1 - \widehat{\theta}(R^{PE})\right]\left[\overline{\theta} - \widehat{\theta}(R^{PE})\right]}^{\text{Area } F \text{ with endog. costs}}}{2} \times \overbrace{\left[Q^{NM} - Q^{M}(\theta_L)\right]}^{\text{Output distortion}}.$$

Therefore, the merger inefficiency decreases by

$$\nabla MI = MI_{endog.} - MI_{exog.} = \nabla F \times \left[Q^{NM} - Q^{M}(\theta_L)\right]$$

where ∇F denotes the shrink in area F. Then, allowing for endogenous submission costs ameliorates inefficiencies.

Corollary 4. *The decrease in merger inefficiencies, ∇MI, is decreasing in the administrative fee f.*

Hence, a more expensive administrative fee, f, produces a larger decrease in merger inefficiencies under endogenous than under exogenous submission costs, that is, $\left|\frac{\partial MI_{endog.}}{\partial f}\right| > \left|\frac{\partial MI_{exog.}}{\partial f}\right|$. In other words, the informational benefits from more expensive fees (hindering the emergence of the pooling PBE) are especially relevant when firms can invest in the submission process, but are minor otherwise.

5.4 Investment limits

Our above results help to inform policy proposals limiting firms' ability to invest in their merger requests. In our setting, these policies would introduce a maximum investment, $\overline{R}$, that firms cannot exceed. This maximum investment could be implemented, for instance, by requiring firms to submit itemized bills to all reports and consulting companies hired while preparing the merger request documentation.

From our previous findings, if $\overline{R} < R^{PE}$, the pooling PBE in Proposition 5 would arise under larger parameter conditions, i.e., for all $\theta_i > \widehat{\theta}(\overline{R})$ instead of for all $\theta_i > \widehat{\theta}(R^{PE})$, where $\widehat{\theta}(\overline{R}) < \widehat{\theta}(R^{PE})$. Intuitively, the investment limit helps firms coordinate into a lower investment level, while not facilitating information transmission from privately informed firms to the CA. However, if the investment limit $\overline{R}$ is relatively low, firms' cost savings could offset the inefficiency that arises from approving the merger of low-type firms. If, instead, $\overline{R}$ satisfies $\overline{R} \geq R^{PE}$, the investment limit becomes not binding for firms, as they would still behave as under the pooling PBE (investing R^{PE} without exceeding the limit). In this context, the investment limit does not facilitate information transmission either.

5.5 Naive CA

A natural question is how the CA's merger approvals differ from those in a setting where, still under incomplete information, the CA does not update its beliefs about the firm's type upon observing the firm's submission costs (i.e., a "naive" CA).[24] In other words, are inefficient mergers affected by the CA's ability to use firm's signals to more accurate infer their types? In that setting, the CA's beliefs would coincide with its priors, $\mu(\theta_H|R) = p$ and $\mu(\theta_L|R) = 1-p$ for all $R \geq 0$, yielding the following equilibrium results.

Proposition 6. *With a naive CA, the following strategy profiles can be sustained in equilibrium:*

1. *A pooling PBE where both firm types submit a merger request, investing $R_H = R_L = 0$, if $\theta_i \geq \widehat{\theta}(0)$ for all $i = \{H, L\}$.*
2. *A separating PBE where only the high-type firm submits a merger request, investing $R_H = 0$, if $\theta_H \geq \widehat{\theta}(0) > \theta_L$.*
3. *A pooling PBE where no firm type submits a merger request if $\theta_i < \widehat{\theta}(0)$ for all $i = \{H, L\}$.*

Therefore, inefficient mergers arise only in the pooling PBE of Proposition 6.1, as in this case the CA approves a merger between low-type firms. Relative to figure 3a, cutoff $\widehat{\theta}(0)$ lies below $\widehat{\theta}(R^{PE})$, entailing that the region of parameters where inefficient mergers arise (region F) is larger when firms interact with a naive than a strategic CA. Intuitively, when facing a naive CA, firms anticipate that a merger request will be approved for all $R \geq 0$, making the merger request more attractive for both firm types, which could have it approved investing zero or a negligible amount, ultimately facilitating the emergence of inefficient mergers. In contrast, a strategic CA which updates its

[24] We do not examine a naive CA under exogenous submission costs, as naivette often refers to a player who ignores some signal or, alternatively, poorly constructing its beliefs. In that context, the only signal is whether the firm submitted a merger request, while under endogenous costs the signal is the submission cost that the firm incurred.

beliefs about the firm's type upon observing its submission costs induces the high-type firm to invest more significantly to separate itself from the low-type firm, thus hindering the emergence of socially inefficient mergers in equilibrium.

5.6 Equilibrium refinements

The pooling PBE of Proposition 3 where both firm types submit a merger request survives equilibrium refinement criteria, such as Cho and Kreps' (1987) Intuitive Criterion or Banks and Sobel's (1987) Divinity Criterion.

Intuitive Criterion. In our context, the Intuitive Criterion seeks to identify firm types which can benefit by deviating towards no merger if the CA responded in the most positive way for the firm (approving the merger), while other firm types cannot benefit from such a deviation. In our setting, however, the CA is not called to move when the firm deviates to no merger, entailing that such a deviation yields, with certainty, $k\pi_I^{NM}$ to both firm types. Therefore, the CA does not hold off-the-equilibrium beliefs (upon observing no merger requests), which the Intuitive Criterion cannot help to further restrict.

A similar argument applies in the pooling PBE of Proposition 5 (with endogenous submission costs), where both firm types can improve their equilibrium payoffs when deviating from investing R^{PE} to $0 \leq R' < R^{PE}$. Indeed, if the CA still approves the merger (as it did in equilibrium), every firm type would save $R^{PE} - R' > 0$ in submission costs. Therefore, we cannot find a firm type for which a deviation to R' is equilibrium dominated, as opposed to the result in Spence's (1973) labor market signaling game. In that setting, a set of pooling PBEs can be supported where the firm manager cannot update his beliefs about the worker and pays him a salary equal to the average productivity. When applying the Intuitive Criterion, we can find deviations to lower education levels which improve the low-productivity worker if he is recognized as a high-productivity worker by the firm, but that deviation is not utility improving for the high-productivity worker even if he was paid the high-productivity salary. Therefore, deviations in Spence's model entail a change in the firm's off-the-equilibrium beliefs and, thus, in its wage response.[25] In contrast, in the pooling PBE of Proposition 5, the CA does not change its response, approving the merger in equilibrium and off-the-equilibrium (most positive response for the firm). As a consequence, the merging entity only considers its cost savings from lowering its investment from R^{PE} to R', leading both firm types to have incentives to deviate.

Divinity Criterion. Banks and Sobel's (1987) Divinity Criterion, in our context, seeks to identify how likely it is that each firm type deviates from its equilibrium message (submitting a merger request in Proposition 3). In particular, it compares the best response sets by the CA that weakly improve the firm's equilibrium payoff, and eliminates all firm types but that with the largest best response set. However, in settings where the responder only has two possible

[25] When a worker considers whether to deviate to a lower education level in the first step of the Intuitive Criterion, he compares his savings in education acquisition against his potential salary increase, leading only the low-productivity worker to have incentives to deviate.

responses (approve or block the merger), the Divinity Criterion yields the same results as the Intuitive Criterion, which applies both to the case of exogenous and endogenous submission costs.

6 Extensions

6.1 Low priors

If, instead, priors satisfy $p \leq \widehat{p}$, the following corollary shows our equilibrium results under incomplete information.

Corollary 5. *If $p \leq \widehat{p}$, the separating PBE in Proposition 4 can still be supported under the same parameter conditions. However, the pooling PBE of Proposition 5 where both (no) firm types submit a merger request cannot be sustained under any parameter values (can be sustained for all parameter values).*

Therefore, the pooling PBE where both firm types submit a merger request cannot be sustained, as firms anticipate their requests will be blocked by the CA because $p \leq \widehat{p}$. This case represents, for instance, a CA that believes that the industry will generate negligible cost-saving benefits from the merger. The pooling PBE where no firm type submits a merger request can now be supported under all parameter conditions.

In terms of figure 2, the comparison of equilibrium behavior across information contexts would be unaffected to the left of cutoff $\widehat{\theta}_L$, thus still finding no inefficiencies due to incomplete information. However, the shaded region where a socially inefficient merger of low-type firms was approved when $p > \widehat{p}$, now becomes a region where no mergers occur. In other words, not even the high-type entity submits a merger request, since it anticipates it being declined by the CA. In summary, when the inefficiency of the merger, as captured by $\overline{\theta} - \theta_L$ is sufficiently severe, we may experience inefficient merger approvals stemming from the CA's inability to accurately observe the cost-reduction effect of the merger. Specifically, mergers of firms benefiting from low (high) cost-reduction effects are approved (blocked) under incomplete information when $p > \widehat{p}$ ($p \leq \widehat{p}$), which would have been blocked (approved) under a complete information setting.

6.2 Allowing for type-dependent submission costs

Previous sections considered, for simplicity, type-independent submission costs. Formally, let $R(f, r)$ describe the total cost that a firm incurs when submitting its merger request to the CA, where it pays f in administrative fees and r in additional investments, where only r is a choice variable for the firm. Then, for a given r, total cost $R(f, r)$ coincides across firm types. Appendix 1 shows that our results are qualitatively unaffected if, instead, we allow for type-dependent submission costs, considering $R(f, r|\theta_i) = f + \frac{r^2}{\theta^\alpha}$, where parameter $\alpha \in [0, 1]$ denotes how sensitive total costs are to the firm's type. When $\alpha = 0$, total costs collapse to $R(f, r|\theta_i) = f + r^2$, thus being unaffected by the firm's type (type-independent costs); whereas when $\alpha = 1$, total costs become

$R(f,r|\theta_i) = f + \frac{r^2}{\theta}$. Expression $R(f,r|\theta_i)$ satisfies the single-crossing property since, in a setting with two types, $\frac{\partial R(f,r|\theta_H)}{\partial r} < \frac{\partial R(f,r|\theta_L)}{\partial r}$ holds for all $\alpha > 0$.[26]

Our argument about equilibrium refinements in the previous section still holds when firms face type-dependent submission costs. To understand this point, consider the pooling PBE in Proposition 5, but allowing for type-dependent submission costs, where both firm types invest r^{PE} and the CA responds approving the merger in equilibrium. If, instead, the high-type firm deviates to $r' < r^{PE}$, it can improve its equilibrium payoff if the CA still approves the merger because the firm can reduce its investment by $r^{PE} - r' > 0$. The same argument applies to the low-type firm, which also improves its equilibrium payoff by deviating to r', implying that the CA cannot further update its off-the-equilibrium beliefs and, as a consequence, the pooling equilibrium survives the Intuitive and Divinity Criteria.[27]

6.3 Allowing for other merger evaluation guidelines

We considered that the CA uses consumer surplus to evaluate merger requests, approving a merger under complete information if the firm's cost-reduction effect, θ, satisfies $\theta > \overline{\theta}$. If, instead, the CA considers the sum of consumer and producer surplus (welfare), it would approve a merger if $\theta > \overline{\theta}_W$, as identified in Lemma 2, where cutoff $\overline{\theta}_W$ satisfies $\overline{\theta}_W < \overline{\theta}$. In this context, the condition for the CA's decision to be relevant ($\theta_H > \overline{\theta} > \theta_L$, as defined in section 4) becomes $\theta_H > \overline{\theta}_W > \theta_L$. Our equilibrium results would hold in this setting too, since Propositions 4 and 5 identify parameter conditions for each firm type to behave as prescribed, but do not directly depend on the CA's payoff.[28]

However, the region of parameter values supporting separating and pooling equilibria would be affected. To understand this point, note that in figure 3a, the high-type firm's efficiency satisfies $\theta_H \in [1, \overline{\theta}]$ in the vertical axis while that of the low-firm's satisfies $\theta_L \in [\overline{\theta}, 0]$. When the CA evaluates mergers according to welfare, these admissible values become $\theta_H \in [1, \overline{\theta}_W]$ and $\theta_L \in [\overline{\theta}_W, 0]$, both of them suffering the same reduction, since $\overline{\theta}_W < \overline{\theta}$, graphically shrinking the quadrant of efficiency pairs in figure 3a. Importantly, this quadrant shrinks from below, thus not affecting the region of parameter values supporting the separating equilibrium, and from the right side, hindering the emergence of the pooling equilibrium. Intuitively, the merger between low-type firms that arises in the pooling equilibrium, while still inefficient, produces smaller inefficiencies when the CA uses a less stringent merger evaluation standard (welfare improvement instead of consumer

[26] As in previous sections, we assume that investment, r, is observable by the CA, which could be explained by this agency inspecting the number of expert reports included in the merger request and/or the consulting companies involved.

[27] As a remark, note that the high-type firm's cost savings from deviating to r' are $\frac{(r^{PE})^2 - (r')^2}{\theta_H^\alpha}$, while those of the low-type firm are higher, $\frac{(r^{PE})^2 - (r')^2}{\theta_L^\alpha}$, since $\theta_L < \theta_H$ for all $\alpha > 0$. (When $\alpha = 0$, cost savings coincide across firm types.) However, the Cho and Kreps' Intuitive Criterion and the Banks and Sobel's Divinity Criterion are silent about which sender improves its equilibrium payoff the most by deviating to r', implying that this comparison in cost savings is inconsequential for the application of standard equilibrium refinements.

[28] The CA approves mergers based on its updated beliefs about facing a high-type firm, approving them based on the initial condition $\theta_H > \overline{\theta}_W > \theta_L$.

surplus gain).

6.4 Allowing for hard evidence

Previous sections consider that a higher investment in the firm's submission, R_i, does not provide verifiable information (hard evidence) that the CA can use to modify its prior probability that the firm's type is high, p. Instead, we assume that the investment does not bring hard evidence, yet it can help the CA update its beliefs about the firm's type. If the firm's investment provides hard evidence, however, we could model this information by considering that $R_i = f + r_i + h_i$, where h_i is the investment in hard evidence, f denotes the administrative fee, and r_i captures all investments that the firm carries out during the submission process and that are unrelated with gathering hard evidence, such as advertising campaigns.

At the beginning of the game, the CA's priors would then be a function of the hard evidence provided by the firm, $p(h_i)$, where $p' > 0$ and $p'' < 0$, intuitively implying that, upon reading the reports, the CA ends up with a prior probability $p(h_i)$ that the entity's type is high. This is equivalent to adding a new stage before the game starts, where the firm chooses h_i, which produces prior $p(h_i)$ given function $p(\cdot)$. As a consequence, the presence of hard evidence would only change the CA's prior to $p(h_i)$, ensuing from that point of the game as we analyzed in previous sections.[29]

6.5 Allowing for continuous responses by the CA

In previous sections, the CA's available responses are binary (approve or block the merger). In richer settings, however, the CA could respond challenging the merger request, exerting different effort levels, yielding a profit of

$$\alpha \left(k\pi_i^{NM} \right) + (1-\alpha)\pi_I^M,$$

where $\alpha \in [0, 1]$ represents the challenge effort that the CA chooses. When $\alpha = 1$ the merger is challenged so effectively that it is blocked, entailing a profit of $k\pi_i^{NM}$ for the merging entity; whereas when $\alpha = 0$, the CA does not challenge the merger at all, which becomes approved, and the merging entity earns π_I^M. For all other values of α, the merging entity earns a linear combination of these two profits, reflecting that it must adjust its production process following the CA's recommendations, or spend additional resources on appeals, reducing its profits below π_I^M. As a consequence, the CA's strategy becomes a continuum, $\alpha \in [0, 1]$, rather than discrete (approve or block). More intense challenging efforts are costly for the CA, however, as it needs to hire a larger team of lawyers and experts, at a total cost $\frac{1}{2}\lambda\alpha^2$, where $\lambda > 0$ represents a higher cost of challenging the merger request.

In this setting, the SE of Proposition 4 is not affected. Indeed, upon observing a submission

[29] We consider here that a given investment in hard evidence produces, for simplicity, the same change in the CA's prior probability for all firm types, that is, $p(h_H) = p(h_L)$ for all $h_H = h_L$. In a more general setting, however, one could argue that the same investment in hard evidence is more effective at increasing the CA's prior when it is carried out by the high-type than the low-type firm, i.e., $p_H(h_H) > p_L(h_L)$ for all $h_H = h_L$. Since priors are a parameter at the beginning of the game, considering this alternative would still produce similar results.

cost that originates from the high-type firm, the CA finds approving the merger welfare improving ($\alpha = 0$) since $\theta_H > \overline{\theta} > \theta_L$, which is welfare superior to any other challenge Similarly, upon observing a submission cost stemming from the low-type firm, the CA responds rejecting the merger ($\alpha = 1$).

However, the PE of Proposition 5 prescribes different equilibrium behavior for the CA, as the following proposition examines.

Proposition 7. *When the CA can respond with $\alpha \in [0, 1]$, the equilibrium in Proposition 5 can be sustained, but upon observing R^{PE} the CA responds with*

$$\alpha^* = \frac{1-c}{\lambda}\left[\frac{1+E[\theta](n-k+1)}{n-k+2} - \left(\frac{k}{n+1}\right)\right]$$

which satisfies $\alpha^* > 0$ under all parameter values, but $\alpha^* < 1$ holds if $E[\theta]$ satisfies

$$E[\theta] < \frac{1}{n-k+1}\left[\left(\frac{\lambda(n+1)+k(1-c)}{(1-c)(n+1)}\right)(n-k+2) - 1\right].$$

In addition, α^* is unambiguously decreasing in c and λ, decreasing in k if and only if $E[\theta] > \frac{2k(n+2)}{n+1} - 3$, and increasing in n if and only if $E[\theta] > 1 - k\left(\frac{n-k+2}{n+1}\right)^2$.

Intuitively, the merging entity submits a merger request under the same conditions as in Proposition 5. The CA, however, does not respond approving it (which would entail a zero challenging effort, $\alpha^* = 0$) but exerts some challenging effort, $\alpha^* > 0$. This challenging effort, α^*, is decreasing in its cost, λ, and increasing in the market share of the merging entity, $n - k$, as an inefficient merger would generate a larger welfare loss in this case.[30]

Therefore, in expectation, some merger requests from low-type firms are rejected, which did not occur when the CA's responses were binary, thus ameliorating the merger inefficiencies due to incomplete information. However, this challenging effort now gives rise to new problems, as merger requests stemming from high-type firms can be incorrectly rejected with positive probability, which did not happen under binary responses. (Merger requests from low-type firms were incorrectly approved when the CA's responses were binary, so the CA's decisions when facing this type of firm unambiguously improve.)

7 Discussion

Information inefficiencies under exogenous costs. Relative to complete information, the presence of incomplete information produces an unambiguous inefficiency. First, under exogenous submission costs (e.g., given administrative fees), inefficient mergers of a low-type firm are approved. Nonetheless, this type of mergers only occur when both firm types are relatively efficient, thus reducing the output inefficiency that arises from approving the merger. In addition, the low-type firm incurs

[30] The proof of Proposition 7 confirms that the pooling PBE survives both the Intuitive and Divinity Criteria.

submission costs to file the merger request, R, which did not incur under complete information. As this cost does not improve the CA's information about the firm's type, it can be interpreted as a social loss, further augmenting the inefficiencies that emerge due to incomplete information.

Information inefficiencies under endogenous costs. When firms can strategically choose their investment during the preparation of the merger request (hiring experts, consulting companies, and even lobbying), in a pooling equilibrium both firm types invest R^{PE} which, in most settings, may exceed the administrative fee under exogenous costs, R. In this context, we showed that the output inefficiency can still be sustained, but only arises under more restrictive parameter conditions. Intuitively, allowing firms to invest large amount of resources above their administrative fees gives rise to two effects: on one hand, this investment is a social loss, as it does not help the CA infer the firm's type, and such a loss is larger than under exogenous costs; but, on the other hand, this substantial investment hinders the emergence of socially inefficient mergers, as the pooling PBE is less likely to arise. If the first form of inefficiency dominates, allowing firms to invest in the submission process emphasizes inefficiencies, relatively to a setting of exogenous costs. Otherwise, letting firms invest could actually reduce the inefficiencies stemming from approving a socially inefficient merger. This case happens when firm types invest a common, but relatively small, amount in the submission process and, in contrast, the efficiency differential between the high- and low-type firm is significant. In such a setting, the CA seeks to hinder the emergence of pooling equilibria, as output inefficiencies from approving a low-type merger would be substantial. Informally, letting firms invest vast resources in the technical reports they submit in their merger request is socially undesirable, but mergers between firms that produce negligible cost-reduction effects is even more welfare reducing.

Don't limit investments. Our results also suggest that limiting firm's investments while preparing its merger request, while reducing the social loss described above (first form of inefficiency), may actually facilitate the emergence of socially inefficient mergers in equilibrium (second form of inefficiency). As a consequence, regulatory authorities considering setting investment limits should balance both inefficiencies before implementing these policies. Specifically, limits may be welfare improving when the second inefficiency is minor, such as when the efficiency differential between firm types is small. Intuitively, while setting a relatively high investment limit facilitates socially inefficient mergers in equilibrium, the output inefficiency these mergers generate is small in this context. When the efficiency differential is high, however, our findings indicate that investment limits can be particularly welfare reducing.

Declarations of interest: none.

8 Appendix

8.1 Appendix 1 - Allowing for type-dependent submission costs

We first examine under which conditions a separating PBE can be sustained where each firm type invests different amounts in their submissions, r_H and r_L, where $r_H \neq r_L \geq 0$.

Proposition A1. *A separating PBE can be sustained if* $\theta_H > \widehat{\theta}(R_H^*) > \theta_L$ *where the merging entity invests* $r_H^* = \left[\theta_H^\alpha \left(\pi_I^{M,L} - k\pi_i^{NM} - f\right)\right]^{1/2}$ *when its type is high and* $r_L^* = 0$ *when its type is low, where total cost* R_H^* *is defined as* $R_H^* \equiv R_H(r_H^*) = \pi_I^{M,L} - k\pi_i^{NM}$.

Proof of Proposition A1. *Updated beliefs.* In this separating strategy profile, the CA updates its beliefs according to Bayes' rule, obtaining $\mu(\theta_H|r_H) = 1$ and $\mu(\theta_H|r_L) = 0$. For simplicity, we consider that, upon observing an off-the-equilibrium investment $r \neq r_H \neq r_L$, the CA's off-the-equilibrium beliefs satisfy $\mu(\theta_H|r) = 0$.

Receiver's response. Given the above beliefs, the CA is convinced of facing a high-type firm upon observing r_H, and responds approving a merger since $\theta_H > \overline{\theta}$ by definition. In contrast, upon observing r_L, the CA is convinced of facing a low-type firm, thus blocking the merger since $\theta_L < \overline{\theta}$ by assumption. A similar argument applies upon observing any other investment $r \neq r_H$, leading to a merger blocking decision.

Sender's messages. Anticipating these responses, the θ_H-type entity invests r_H, as prescribed in this separating strategy profile, if $\pi_I^{M,H} - R_H \geq k\pi_i^{NM}$, where $R_H \equiv R(r_H|\theta_H) = f + \frac{r_H^2}{\theta_H^\alpha}$, and the right side of the inequality assumes that the high-type deviates to zero (no merger request). This inequality simplifies to $\pi_I^{M,H} - k\pi_i^{NM} \geq R_H$ or, solving for θ_H, as shown in Proposition 2, this inequality yields $\theta_H > \widehat{\theta}_H$, where recall that cutoff $\widehat{\theta}_H$ is evaluated at R_H, i.e., $\widehat{\theta}_H = \widehat{\theta}(R_H) = \widehat{\theta}\left(f + \frac{r_H^2}{\theta_H^\alpha}\right)$. Alternatively, the high-type firm could deviate to R_L, but doing so would yield even a lower payoff on the right-side of the above inequality, that is, $\pi_I^{M,H} - R_H \geq k\pi_i^{NM} - R_L$, implying that $\theta_H > \widehat{\theta}_H$ is a sufficient condition for $\pi_I^{M,H} - R_H \geq k\pi_i^{NM} - R_L$.

In contrast, the θ_L-type entity chooses R_L, instead of deviating to R_H, if and only if $k\pi_i^{NM} - R_L \geq \pi_I^{M,L} - R_H$, as the CA denies the merger upon observing r_L but approves it upon observing r_H. Among all the values of r_L that lead to a merger decline, the most profitable is, of course, $r_L = 0$ (minimizing submission costs), so the above inequality becomes $k\pi_i^{NM} \geq \pi_I^{M,L} - r_H$. Solving for θ_L, we find $\theta_L \leq \widehat{\theta}(R_H) = \widehat{\theta}_H$ which, solving for R_H, is equivalent to $R_H \geq \pi_I^{M,L} - k\pi_i^{NM}$. Combining the inequalities we found from the high- and low-type firms, we obtain that a separating PBE can be sustained if $\theta_H > \widehat{\theta}_H > \theta_L$, where the high-type firm total cost, R_H, satisfies $R_H \geq \pi_I^{M,L} - k\pi_i^{NM}$.

Among all these separating PBEs, however, only the least-costly separating PBE, where $R_H = \pi_I^{M,L} - k\pi_i^{NM}$, survives Cho and Kreps' Intuitive Criterion. Using the definition of the cost function, $R(r|\theta_i) = f + \frac{r^2}{\theta^\alpha}$, we obtain that $\pi_I^{M,L} - k\pi_i^{NM} = f + \frac{r_H^2}{\theta_H^\alpha}$ or, after solving for r_H, we find $r_H = \left[\theta_H^\alpha \left(\pi_I^{M,L} - k\pi_i^{NM} - f\right)\right]^{1/2}$. QED.

Proposition A2. *A pooling PBE can be supported if $\theta_i > \widehat{\theta}(R^{PE})$ for every $i = \{H, L\}$, where both types of the merging entity choose a common investment, $0 \leq R^{PE} \leq \pi_I^{M,L} - k\pi_i^{NM}$, and beliefs satisfy $\mu\left(\theta_H | R^{PE}\right) = p$ and $\mu\left(\theta_H | R'\right) = 0$ for all off-the-equilibrium investment levels $R' \neq R^{PE}$, and total cost R^{PE} is defined as $R^{PE} \equiv R(r^{PE})$. However, if $\theta_i \leq \widehat{\theta}(0)$ for every i, a pooling PBE can be sustained where no firm type invests, $r^{PE} = 0$.*

Proof of Proposition A2. *Updated beliefs.* In this pooling strategy profile, the CA cannot update its beliefs according to Bayes' rule. Therefore, upon observing r, its beliefs are $\mu\left(\theta_H | r\right) = p$ and $\mu\left(\theta_L | r\right) = 1 - p$, whereas upon receiving any off-the-equilibrium investment $r' \neq r$, its off-the-equilibrium beliefs are $\mu\left(\theta_H | r'\right) = 0$.

Receiver's response. Given the above beliefs, upon observing r, in equilibrium, the CA responds approving a merger upon observing a request if and only if

$$p\frac{1 - c + (n - k + 1)x_H}{n - k + 2} + (1 - p)\frac{1 - c + (n - k + 1)x_L}{n - k + 2} \geq k\frac{1 - c}{n + 1}.$$

As shown in the proof of Proposition 3, rearranging, yields $p > \frac{\overline{\theta} - \theta_L}{\theta_H - \theta_L} \equiv \widehat{p}$, which holds by assumption. In contrast, upon observing the off-the-equilibrium investment r', the CA responds blocking the merger since $\mu\left(\theta_H | r'\right) = 0$ and $\theta_L < \overline{\theta}$ by assumption.

Sender's messages. Anticipating these responses, the θ_H-type entity invests r, as prescribed in this pooling strategy profile, if $\pi_I^{M,H} - R(r|\theta_H) \geq k\pi_i^{NM}$, where $R(r|\theta_H) = f + \frac{r^2}{\theta_H^\alpha}$, and the right side of the inequality assumes that the high-type deviates to zero investment ($r' = 0$, no merger request) because any deviation to $r' \neq r$ guarantees a merger decline and $r' = 0$ minimizes the firm's submission cost. This inequality simplifies to $\pi_I^{M,H} - k\pi_i^{NM} \geq R(r|\theta_H)$ or, solving for θ_H, we know from Proposition 2 that this inequality yields $\theta_H > \widehat{\theta}(R(r|\theta_H)) = \widehat{\theta}\left(f + \frac{r^2}{\theta_H^\alpha}\right)$.

Similarly, the θ_L-type entity chooses r, instead of deviating to any other $r' \neq r$, which would guarantee a merger decline, if and only if $\pi_I^{M,L} - R(r|\theta_L) \geq k\pi_i^{NM}$. (The right side of this inequality follows a similar argument as for the high-type firm.). This inequality simplifies to $\pi_I^{M,L} - k\pi_i^{NM} \geq R(r|\theta_L)$ or, solving for θ_L, we know from Proposition 2 that this inequality yields $\theta_L > \widehat{\theta}(R(r|\theta_L)) = \widehat{\theta}\left(f + \frac{r^2}{\theta_L^\alpha}\right)$.

Combining the inequalities we found from the high- and low-type firms, we obtain that a pooling PBE can be sustained if $\theta_H > \widehat{\theta}\left(f + \frac{r^2}{\theta_H^\alpha}\right)$ and $\theta_L > \widehat{\theta}\left(f + \frac{r^2}{\theta_L^\alpha}\right)$, but since $\theta_H > \theta_L$ by definition, a sufficient condition for both inequalities to hold is $\theta_L > \widehat{\theta}\left(f + \frac{r^2}{\theta_L^\alpha}\right)$, which is equivalent to $\pi_I^{M,L} - k\pi_i^{NM} \geq R(r|\theta_L)$.

Finally, we apply Cho and Kreps' Intuitive Criterion. Consider, for instance, the most costly pooling PBE, where both firm types choose the investment, r, that solves $\pi_I^{M,L} - k\pi_i^{NM} \geq R(r|\theta_H) = f + \frac{r^2}{\theta_H^\alpha}$ with equality, or $r = \left[\theta_H^\alpha\left(\pi_I^{M,L} - k\pi_i^{NM} - f\right)\right]^{1/2}$. Consider a deviation to r', where $\widetilde{r} < r' < r$, where $\widetilde{r}$ solves $\pi_I^{M,L} - R(\widetilde{r}|\theta_L) = k\pi_i^{NM}$, that is $\widetilde{r} = \left[\theta_L^\alpha\left(\pi_I^{M,L} - k\pi_i^{NM} - f\right)\right]^{1/2}$. Therefore, $\widetilde{r}$ is the investment level that makes the low-type firm indifferent between submitting the merger request, and having it approved by the CA, and not submitting it. Any deviation to r', then, yields at most $\pi_I^{M,H} - R(r'|\theta_H)$ for the high-type firm (when the merger request is approved), which exceeds its equilibrium payoff $\pi_I^{M,H} - R(r|\theta_H)$ since $r' < r$ because the firm saves in submission costs relative to the pooling PBE where it invests r. Similarly, for the low-type firm this deviation yields at most $\pi_I^{M,L} - R(r'|\theta_L)$, which exceeds its equilibrium payoff $\pi_I^{M,L} - R(r|\theta_H)$ since $r' < r$. As a consequence, the CA cannot update its off-the-equilibrium beliefs in the first step of the Intuitive Criterion, implying that the pooling equilibrium survives this refinement criterion. QED.

8.2 Appendix 2 - Semiseparating equilibria

In order to check for a semiseparating equilibrium, we consider that the high-type firm randomizes between submitting and not submitting with probability σ_H and $(1-\sigma_H)$, respectively, while the low-type firm randomizes with probabilities σ_L and $(1-\sigma_L)$. The CA approves the merger with probability σ_{CA} and blocks it with probability $1-\sigma_{CA}$. Importantly, note that if the low-type firm is indifferent between submitting and not submitting, then the high-type firm must strictly prefer to submit, that is,

$$\sigma_{CA}\left[\left(\frac{1-c+(n-k+1)x_L}{n-k+2}\right)^2 - R\right] + (1-\sigma_{CA})k\left(\frac{1-c}{n+1}\right)^2 = k\left(\frac{1-c}{n+1}\right)^2$$

entails that

$$\sigma_{CA}\left[\left(\frac{1-c+(n-k+1)x_H}{n-k+2}\right)^2 - R\right] + (1-\sigma_{CA})k\left(\frac{1-c}{n+1}\right)^2 > k\left(\frac{1-c}{n+1}\right)^2$$

which implies that $\sigma_H = 1$.

First step. For the CA to mix, his beliefs μ must satisfy

$$\mu\frac{1-c+(n-k+1)x_H}{n-k+2} + (1-\mu)\frac{1-c+(n-k+1)x_L}{n-k+2} = k\frac{1-c}{n+1}$$

where the left side represents the expected output when the CA approves the merger and the right side indicates its certain output when the CA blocks the merger. Rearranging, yields

$$\frac{\mu x_H + (1-\mu)x_L}{1-c} = \frac{k-1}{1+n} \equiv \overline{\theta}$$

which we can also express as

$$E[\theta] \equiv \mu\frac{x_H}{1-c} + (1-\mu)\frac{x_L}{1-c} = \mu\theta_H + (1-\mu)\theta_L = \overline{\theta}.$$

or, after solving for μ, we obtain that the CA's beliefs must satisfy $\mu = \frac{\overline{\theta}-\theta_L}{\theta_H-\theta_L} \equiv \widehat{\mu}$; otherwise, it would not be mixing between approving the merger (if $\mu > \widehat{\mu}$) and blocking it (if $\mu < \widehat{\mu}$).

Second step. Given the CA's beliefs, we find from Bayes' rule that

$$\mu = \frac{\overline{\theta} - \theta_L}{\theta_H - \theta_L} = \frac{p}{p + (1-p)\sigma_L}$$

where the numerator captures the probability that the high-type firm submits a merger request (since $\sigma_H = 1$), while the denominator reflects the probability that the CA receives a merger request from any firm type. Solving for probability σ_L, yields

$$\sigma_L^* = \frac{p}{1-p}\frac{\theta_H - \overline{\theta}}{\overline{\theta} - \theta_L} \tag{1}$$

which is unambiguously positive, and less than 1 if $\frac{p}{1-p} < \frac{\overline{\theta}-\theta_L}{\theta_H-\overline{\theta}}$.

Third step. Given our above results about μ and σ_L, we can now find σ_{CA}. The low-type firm mixes if and only if

$$\sigma_{CA}\left[\left(\frac{1-c+(n-k+1)x_L}{n-k+2}\right)^2 - R\right] + (1-\sigma_{CA})k\left(\frac{1-c}{n+1}\right)^2 = k\left(\frac{1-c}{n+1}\right)^2$$

where the left (right) side denotes the expected (certain) profit from submitting (not submitting) a merger request, which is approved with probability σ_{CA}. However, after rearranging, we find that

$$\sigma_{CA}\left[\left(\frac{1-c+(n-k+1)x_L}{n-k+2}\right)^2 - R - k\left(\frac{1-c}{n+1}\right)^2\right] = 0.$$

If $\theta_L > \widehat{\theta}$, the term in brackets is positive, entailing that the CA's probability, σ_{CA}, becomes $\sigma_{CA} = 0$. In other words, the CA blocks all merger requests, implying that no firm type would have incentives to spend R into the submission process, that is, $\sigma_H = \sigma_L = 0$, as in the pooling PBE where no firm type submits a merger request (see Proposition 3). Therefore, a semiseparating PBE cannot be sustained when $\theta_L > \widehat{\theta}$.

If, instead, $\theta_L < \widehat{\theta}$ holds, the term in brackets is negative, implying that probability σ_{CA} would have to be negative too, which cannot occur, entailing that a semiseparating PBE cannot be supported in this case either.

Finally, if $\theta_L = \widehat{\theta}$, the term in brackets is exactly zero, implying that probability σ_{CA} is undefined, $\sigma_{CA} \in [0,1]$. In this context, a semiseparating PBE can be sustained, where the firm randomizes with probability $\sigma_H^* = 1$ and $\sigma_L^* = \frac{p}{1-p}\frac{\theta_H-\overline{\theta}}{\overline{\theta}-\theta_L}$ and the CA responds approving mergers with any probability $\sigma_{CA} \in [0,1]$, if and only if $\theta_H > \theta_L = \widehat{\theta}$ holds.

8.3 *Proof of Lemma 1*

In a case of no mergers, every firm solves,

$$\max_{q_i \geq 0} \quad (1 - q_i - Q_{-i})q_i - cq_i$$

where Q_{-i} denotes the aggregate output of firm i's rivals. Differentiating with respect to q_i, and solving for q_i, we find firm i's best response function $q(Q_i) = \frac{1-c}{2} - \frac{1}{2}Q_{-i}$ In a symmetric equilibrium, $q_i = q_j = q$ for every two firms $i \neq j$, which entails $Q_{-i} = (N-1)q$. Therefore, the equilibrium output in this setting is

$$q_i^{NM} = \frac{1-c}{n+1}$$

and equilibrium profits become $\pi_i^{NM} = \left(\frac{1-c}{n+1}\right)^2 = \left(q_i^{NM}\right)^2$.

Now consider a merger of k firms is approved. Insider firms solve

$$\max_{q_I^M \geq 0} \quad (1 - q_I^M - Q_{-i})q_I - (c-x)q_I$$

where q_I^M denotes the insiders' output, and Q_{-i} represents the aggregate output of all outsiders combined. Differentiating with respect to q_I^M, and solving for q_I^M, we find the best response function $q_I^M(Q_{-i}) = \frac{1-c+x}{2} - \frac{1}{2}Q_{-i}$. Similarly, every outsider firm i solves

$$\max_{q_i^M \geq 0} \quad (1 - q_i^M - q_I^M - Q_{-i}^M)q_i^M - cq_i^M$$

where Q_{-i}^M denotes the aggregate production level of all other $(n-k)-1$ firms that are outsiders in the merger. Differentiating with respect to q_i^M, and solving for q_i^M, we obtain the best response function $q_i^M(q_I^M, Q_{-i}^M) = \frac{1-c}{2} - \frac{1}{2}\left(q_I^M + Q_{-i}^M\right)$. In a symmetric equilibrium, all outsiders produce the same output, $q_i^M = q_i^M = q_O^M$ for all $n-k$ firms, implying that $Q_{-i}^M = [(n-k)-1]q_O^M$. Inserting this result in the above best response functions, and rearranging, yields equilibrium output levels

$$q_I^M = \frac{1-c+(n-k+1)x}{n-k+2} \quad \text{and} \quad q_O^M = \frac{1-c-x}{n-k+2}$$

Finally, equilibrium profits for the merger insiders are

$$\pi_I^M = (1 - q_I^M - (n-k)q_O^M)q_I^M - cq_I^M = \left(\frac{1-c+(n-k+1)x}{n-k+2}\right)^2 = \left(q_I^M\right)^2$$

whereas every outsider earns

$$\pi_O^M = (1 - q_I^M - [(n-k)-1]q_O^M)q_O^M - cq_O^M = \left(\frac{1-c-x}{n-k+2}\right)^2 = \left(q_O^M\right)^2.$$

8.4 *Proof of Lemma 2*

In this setting, an increase in consumer surplus is equivalent to an increase in output. In particular, $q_I^M \geq kq_i^{NM}$ holds if and only if

$$\frac{1-c+(n-k+1)x}{n-k+2} \geq k\frac{1-c}{n+1}.$$

Rearranging, and solving for yields $x \geq \frac{(1-c)(k-1)}{n+1}$ or, alternatively,

$$\theta \equiv \frac{x}{1-c} \geq \frac{k-1}{1+n} \equiv \overline{\theta}$$

8.5 *Proof of Proposition 1*

A merger between k out of n firms is profitable if the post-merger profits exceed the pre-merger profits (for all firms that form the merger, as an entity), that is, $\pi_I^M - R \geq k\pi_i^{NM}$, which holds if

$$\left(\frac{1-c+(n-k+1)x}{n-k+2}\right)^2 - R = \left(q_I^M\right)^2 - R \geq k\left(\frac{1-c}{n+1}\right)^2$$

After simplifying, we obtain

$$\frac{1-c+(n-k+1)x}{n-k+2} \geq \sqrt{k\left(\frac{1-c}{n+1}\right)^2 + R}$$

and upon further rearranging, we find

$$\theta \equiv \frac{x}{1-c} \geq \frac{n-k+2}{(1-c)(n-k+1)}\sqrt{k\left(\frac{1-c}{n+1}\right)^2 + R} - \frac{1}{n-k+1} \equiv \widehat{\theta}$$

In the absence of submission costs, $R = 0$, this cutoff simplifies to

$$\begin{aligned} \theta &\geq \frac{n-k+2}{(1-c)(n-k+1)}\sqrt{k\left(\frac{1-c}{n+1}\right)^2} - \frac{1}{n-k+1} \\ &= \frac{(n-k+2)\sqrt{k}}{(n-k+1)(n+1)} - \frac{n+1}{(n-k+1)(n+1)} \\ &= \frac{(n-k+2)\sqrt{k}-(n+1)}{(n-k+1)(n+1)} \equiv \theta_F. \end{aligned}$$

Comparing cutoffs, we find that $\widehat{\theta} > \theta_F$ for all $R > 0$. Comparing now cutoffs $\overline{\theta}$ and θ_F yields, $\overline{\theta} > \theta_F$ since

$$\frac{k-1}{n+1} > \frac{(n-k+2)\sqrt{k}-(n+1)}{(n-k+1)(n+1)}$$

simplifies to

$$(n-k+1)(k-1) + (n+1) = k(n-2+k) > (n-k+2)\sqrt{k}$$

which is clearly satisfied. However, $\widehat{\theta} > \overline{\theta}$ holds if and only if

$$\frac{n-k+2}{(1-c)(n-k+1)}\sqrt{k\left(\frac{1-c}{n+1}\right)^2 + R} - \frac{1}{n-k+1} > \frac{k-1}{1+n}$$

or, rearranging, and solving for R,

$$R > \left(\frac{1-c}{n+1}\right)^2 \left[\left(\frac{(n+1)+(n-k+1)(k-1)}{(n-k+2)}\right)^2 - k\right] \equiv \widehat{f}.$$

Therefore, $\widehat{\theta} > \overline{\theta} > \theta_F$ holds when $R > \widehat{R}$; otherwise, $\overline{\theta} > \widehat{\theta} > \theta_F$ holds. Finally, it is easy to check that cutoff $\widehat{\theta}$ increases in R, as it enters positively, and in c since

$$\frac{\partial \widehat{\theta}}{\partial c} = \frac{f(n-k+2)}{(1-c)^2(n-k+1)\sqrt{k\left(\frac{1-c}{n+1}\right)^2 + R}} > 0.$$

8.6 *Proof of Proposition 2*

Updated beliefs. In this separating strategy profile, the CA updates its beliefs according to Bayes' rule, obtaining $\mu(\theta_H|M) = 1$ and $\mu(\theta_L|M) = 0$.

Receiver's response. Given the above beliefs, the CA is convinced of facing a high-type firm, and responds approving a merger upon observing one since $\theta_H > \overline{\theta}$ by definition.

Sender's messages. Anticipating these beliefs, the θ_H-type entity submits a merger approval, as prescribed in this separating strategy profile, if and only if $\pi_I^M - f \geq k\pi_i^{NM}$, since it anticipates that the request will be approved by the CA. This inequality entails that

$$\left(\frac{1-c+(n-k+1)x_H}{n-k+2}\right)^2 - f \geq k\left(\frac{1-c}{n+1}\right)^2$$

Rearranging and solving for θ_H yields

$$\theta_H \equiv \frac{x_H}{1-c} \geq \frac{n-k+2}{(1-c)(n-k+1)}\sqrt{k\left(\frac{1-c}{n+1}\right)^2 + f} - \frac{1}{n-k+1} \equiv \widehat{\theta}.$$

Therefore, the θ_H-type entity submits a merger approval if and only if $\theta_H > \widehat{\theta}$. In contrast, the θ_L-type entity does not submit a merger approval, as required in this separating strategy profile, if and only if $\pi_I^M - f \leq k\pi_i^{NM}$. Note that a merger request would have been approved by the CA (left side of the inequality). This inequality yields

$$\left(\frac{1-c+(n-k+1)x_L}{n-k+2}\right)^2 - f \leq k\left(\frac{1-c}{n+1}\right)^2$$

Rearranging and solving for θ_L, we obtain $\theta_L \leq \widehat{\theta}$.

8.7 *Proof of Proposition 3*

Pooling PBE where both firm types submit a merger request. *Updated beliefs.* In this pooling strategy profile, the CA cannot update its beliefs according to Bayes' rule, keeping its prior probabilities unaffected, $\mu(\theta_H|M) = p$ and $\mu(\theta_L|M) = 1 - p$.

Receiver's response. Given the above beliefs, the CA responds approving a merger upon observing a request if and only if

$$p\frac{1-c+(n-k+1)x_H}{n-k+2} + (1-p)\frac{1-c+(n-k+1)x_L}{n-k+2} \geq k\frac{1-c}{n+1}.$$

Rearranging, yields $px_H + (1-p)x_L \geq \frac{(1-c)(k-1)}{n+1}$ or, alternatively,

$$\frac{px_H + (1-p)x_L}{1-c} \geq \frac{k-1}{1+n} \equiv \overline{\theta}$$

which we can also express as

$$E[\theta] \equiv p\frac{x_H}{1-c} + (1-p)\frac{x_L}{1-c} = p\theta_H + (1-p)\theta_L \geq \overline{\theta}.$$

or, after solving for p, we obtain $p > \frac{\overline{\theta}-\theta_L}{\theta_H-\theta_L} \equiv \widehat{p}$, which holds by assumption.

Sender's messages. The θ_H-type entity submits a merger approval, as prescribed in this pooling strategy profile, if and only if $\pi_I^M - f \geq k\pi_i^{NM}$, since it anticipates that the request will be approved by the CA. This inequality entails that

$$\left(\frac{1-c+(n-k+1)x_H}{n-k+2}\right)^2 - f \geq k\left(\frac{1-c}{n+1}\right)^2$$

Rearranging and solving for θ_H yields $\theta_H \geq \widehat{\theta}$. Similarly, the θ_L-type entity submits a merger approval, as required in this pooling strategy profile, if and only if $\pi_I^M - R \geq k\pi_i^{NM}$, which yields

$$\left(\frac{1-c+(n-k+1)x_L}{n-k+2}\right)^2 - f \geq k\left(\frac{1-c}{n+1}\right)^2$$

Rearranging and solving for θ_L, we obtain $\theta_L \geq \widehat{\theta}$.

Pooling PBE where both firm types do not submit a merger request. *Updated beliefs.* In this pooling strategy profile, the CA's information set is not reached since no merger approval request is submitted. Then, the CA holds off-the-equilibrium beliefs $\mu(\theta_H|M) = \mu$ and $\mu(\theta_L|M) = 1 - \mu$.

Receiver's response. Given the above beliefs, the CA responds approving a merger upon observing one (which can only happen off-the-equilibrium path) if and only if

$$\mu\theta_H + (1-\mu)\theta_L > \overline{\theta}$$

or, alternatively, $\mu > \frac{\overline{\theta}-\theta_L}{\theta_H-\theta_L} \equiv \widehat{p}$. We next separately analyze the case in which $\mu > \widehat{p}$ (and the CA responds approving merger, if one is submitted off the equilibrium) and that in which $\mu \leq \widehat{p}$ (and the CA blocks the merger).

Sender's messages. If $\mu > \widehat{p}$, the θ_H-type entity, does not submit a merger approval, as prescribed in this pooling strategy profile, if and only if $\pi_I^M - f < k\pi_i^{NM}$, since it anticipates that a deviation towards sending a request will be approved by the CA (given its off-the-equilibrium beliefs). This inequality entails $\theta_H < \widehat{\theta}$. Similarly, the θ_L-type entity, does not submit a merger approval, as required in this pooling strategy profile, if and only if $\pi_I^M - R_L < k\pi_i^{NM}$. This inequality yields $\theta_L < \widehat{\theta}$.

If, instead, $\mu \leq \widehat{p}$ holds, both entity types anticipate that the CA will respond blocking merger approval requests (off-the-equilibrium path). In this context, the θ_H-type entity does not submit a request, as prescribed in this pooling strategy profile, if and only if $\pi_I^{NM} - f < k\pi_i^{NM}$, which simplifies to $-f < (k-1)\,\pi_i^{NM}$, which holds since $k \geq 2$ by definition. A similar argument applies to the θ_L-type entity.

In summary, the pooling strategy profile where no merging entity types submits a merger request can be sustained as a PBE if the CA's off-the-equilibrium beliefs satisfy $\mu \leq \widehat{p}$ under all parameter values; and if $\mu > \widehat{p}$ if, in addition, $\theta_i < \widehat{\theta}$ for all types $i = \{H, L\}$.

Applying the Cho and Kreps' Intuitive Criterion. We first consider the pooling PBE where both firm types submit a merger request. In this case, if any firm i deviates towards not submitting a merger request (off-the-equilibrium path), the CA is not called on to move, implying that this player does not hold off-the-equilibrium beliefs. As a consequence, we cannot further restrict the set of types that could have sent such an off-the-equilibrium message, ultimately implying that we cannot restrict the CA's beliefs either. As a consequence, this pooling PBE survives the Intuitive Criterion.

We next consider the pooling PBE where both firm types do not submit a merger request. In this setting, if the CA observes a merger request (off-the-equilibrium path), we must evaluate which firm type (if any) can potentially gain from this deviation. The high-type can increase its equilibrium payoff, $k\pi_i^{NM}$, to $\pi_I^M - f$, which happens if the merger request is approved, if and only if $\theta_H \geq \widehat{\theta}$. A similar argument applies for the low-type, which can increase its equilibrium payoff, $k\pi_i^{NM}$, to $\pi_I^M - f$ if the CA responds approving its merger request. Therefore, both firm types would benefit from the deviation if $\theta_i \geq \widehat{\theta}$ for every firm type i; only the high-type does when $\theta_H \geq \widehat{\theta}$ and $\theta_L < \widehat{\theta}$; only the low-type does when $\theta_H < \widehat{\theta}$ and $\theta_L \geq \widehat{\theta}$ (which cannot hold since $\theta_H > \theta_L$ by assumption); and no firm type would benefit from the deviation if $\theta_i < \widehat{\theta}$ holds for every i. This means that the pooling PBE supported when $\mu \leq \widehat{p}$ under all values of θ_L and θ_H violates the Intuitive Criterion, except when $\theta_i < \widehat{\theta}$ holds since no firm type would have incentives to deviate even when they anticipate that a merger request will be approved by the CA. Similarly, the pooling PBE supported when $\mu > \widehat{p}$ and $\theta_i < \widehat{\theta}$ survives the Intuitive Criterion.

8.8 *Proof of Corollary 2*

The region where a pooling PBE can be sustained is given by the shaded triangle in figure 2, $PE = \frac{\left(1-\widehat{\theta}\right)\left(\overline{\theta}-\widehat{\theta}\right)}{2}$. Differentiating with respect to $\widehat{\theta}$, yields $\frac{\partial PE}{\partial \widehat{\theta}} = \frac{2\widehat{\theta}-\left(1+\overline{\theta}\right)}{2}$, which is negative for all $\overline{\theta} > 2\widehat{\theta} - 1$, which holds since $\overline{\theta} > \widehat{\theta}$ by definition. (Graphically, $2\widehat{\theta} - 1$ originates at -1 when

$\widehat{\theta}=0$, increases in $\widehat{\theta}$, and reaches a height of 1 when $\widehat{\theta}=1$, implying that $\overline{\theta}=2\widehat{\theta}-1$ lies below the 45-degree line, $\overline{\theta}=\widehat{\theta}$, for all admissible values.) Therefore, PE unambiguously shrinks as cutoff $\widehat{\theta}$ increases.

The region where a separating PBE can be supported is given by the rectangle $SE=\left(1-\widehat{\theta}\right)\widehat{\theta}$. Differentiating with respect to $\widehat{\theta}$, we obtain $\frac{\partial SE}{\partial\widehat{\theta}}=1-2\widehat{\theta}$, which is positive for all $\widehat{\theta}<1/2$.

Comparing $\frac{\partial PE}{\partial\widehat{\theta}}$ against $\frac{\partial SE}{\partial\widehat{\theta}}$, we find that

$$\frac{\partial PE}{\partial\widehat{\theta}}-\frac{\partial SE}{\partial\widehat{\theta}}=\frac{6\widehat{\theta}-3-\overline{\theta}}{2}$$

which is positive for all $\overline{\theta}<6\widehat{\theta}-3$. Graphically, $6\widehat{\theta}-3$ originates at -3 when $\widehat{\theta}=0$, increases in $\widehat{\theta}$, and reaches a height of 3 when $\widehat{\theta}=1$. In addition, $6\widehat{\theta}-3$ crosses the horizontal axis at $6\widehat{\theta}-3=0$, or $\widehat{\theta}=1/2$; and crosses the 45-degree line at $6\widehat{\theta}-3=\widehat{\theta}$, or $\widehat{\theta}=3/5$. Therefore, the line $\overline{\theta}=6\widehat{\theta}-3$ divides the region of admissible values into two areas, both above the 45-degree line where $\overline{\theta}=\widehat{\theta}$: (a) $6\widehat{\theta}-3>\overline{\theta}>\widehat{\theta}$, where an increase in $\widehat{\theta}$ produces a larger reduction in the parameters supporting the separating than the pooling PBE; and (b) $\overline{\theta}>\max\{6\widehat{\theta}-3,\widehat{\theta}\}$, where an increase in $\widehat{\theta}$ produces a larger reduction in the parameters sustaining the pooling than the separating PBE.

8.9 *Proof of Proposition 4*

Updated beliefs. In this separating strategy profile, the CA updates its beliefs according to Bayes' rule, obtaining $\mu\left(\theta_H|R_H\right)=1$ and $\mu\left(\theta_H|R_L\right)=0$. For simplicity, we consider that, upon observing an off-the-equilibrium message $R\neq R_H\neq R_L$, the CA's off-the-equilibrium beliefs satisfy $\mu\left(\theta_H|R\right)=0$.

Receiver's response. Given the above beliefs, the CA is convinced of facing a high-type firm upon observing R_H, and responds approving a merger since $\theta_H>\overline{\theta}$ by definition. In contrast, upon observing R_L, the CA is convinced of facing a low-type firm, thus blocking the merger since $\theta_L<\overline{\theta}$ by assumption. A similar argument applies upon observing any other $R\neq R_H$, leading to a merger blocking decision.

Sender's messages. Anticipating these responses, the θ_H-type entity invests R_H, as prescribed in this separating strategy profile, if $\pi_I^M-R_H\geq k\pi_i^{NM}$, where the right side assumes that the high-type deviates to zero (no merger request). This inequality simplifies to $\pi_I^{M,H}-k\pi_i^{NM}\geq R_H$ or, solving for θ_H, we know from Proposition 2 that this inequality yields $\theta_H>\widehat{\theta}_H$, where recall that cutoff $\widehat{\theta}_H$ is evaluated at R_H, i.e., $\widehat{\theta}_H=\widehat{\theta}(R_H)$. Alternatively, the high-type firm could deviate to R_L, but doing so would yield even a lower payoff on the right-side of the above inequality, that is, $\pi_I^{M,H}-R_H\geq k\pi_i^{NM}-R_L$, implying that $\theta_H>\widehat{\theta}_H$ is a sufficient condition for $\pi_I^{M,H}-R_H\geq k\pi_i^{NM}-R_L$.

In contrast, the θ_L-type entity chooses R_L, instead of deviating to R_H, if and only if $k\pi_i^{NM}-R_L\geq\pi_I^{M,L}-R_H$, as the CA denies the merger upon observing R_L but approves it upon observing R_H. Among all the values of R_L that lead to a merger decline, the most profitable is, of course,

$R_L = 0$ (minimizing submission costs), so the above inequality becomes $k\pi_i^{NM} \geq \pi_I^{M,L} - R_H$. Solving for θ_L, we find $\theta_L \leq \widehat{\theta}(R_H) = \widehat{\theta}_H$ which, solving for R_H, is equivalent to $R_H \geq \pi_I^{M,L} - k\pi_i^{NM}$. Combining the inequalities we found from the high- and low-type firms, we obtain that a separating PBE can be sustained if $\theta_H > \widehat{\theta}_H > \theta_L$, where the high-type firm invests $R_H \geq \pi_I^{M,L} - k\pi_i^{NM}$.

Among all these separating PBEs, however, only the least-costly separating PBE, where $R_H = \pi_I^{M,L} - k\pi_i^{NM}$, survives Cho and Kreps' Intuitive Criterion.

8.10 *Proof of Proposition 5*

Pooling PBE where both firm types invest a positive amount. *Updated beliefs.* In this pooling strategy profile, the CA cannot update its beliefs according to Bayes' rule. Therefore, upon observing R, where $R \geq f$, its beliefs are $\mu(\theta_H|R) = p$ and $\mu(\theta_L|R) = 1-p$, whereas upon receiving any off-the-equilibrium message $R' \neq R$, where $R' \geq f$, its off-the-equilibrium beliefs are $\mu(\theta_H|R') = 0$.

Receiver's response. Given the above beliefs, upon observing R, in equilibrium, the CA responds approving a merger upon observing a request if and only if

$$p\frac{1-c+(n-k+1)x_H}{n-k+2} + (1-p)\frac{1-c+(n-k+1)x_L}{n-k+2} \geq k\frac{1-c}{n+1}.$$

Rearranging, yields $px_H + (1-p)x_L \geq \frac{(1-c)(k-1)}{n+1}$ or, alternatively,

$$E[\theta] \equiv p\frac{x_H}{1-c} + (1-p)\frac{x_L}{1-c} = p\theta_H + (1-p)\theta_L \geq \overline{\theta}.$$

or, after solving for p, we obtain $p > \frac{\overline{\theta}-\theta_L}{\theta_H-\theta_L} \equiv \widehat{p}$, which holds by assumption. In contrast, upon observing the off-the-equilibrium message R', the CA responds blocking the merger since $\mu(\theta_H|R') = 0$ and $\theta_L < \overline{\theta}$ by assumption.

Sender's messages. Anticipating these responses, the θ_H-type entity invests R, as prescribed in this pooling strategy profile, if $\pi_I^M - R \geq k\pi_i^{NM}$, where the right side assumes that the high-type deviates to zero investment (no merger request) because any deviation to $R' \neq R$ guarantees a merger decline and $R' = 0$ minimizes the firm's submission cost. This inequality simplifies to $\pi_I^{M,H} - k\pi_i^{NM} \geq R_H$ or, solving for θ_H, we know from Proposition 2 that this inequality yields $\theta_H > \widehat{\theta}(R)$.

Similarly, the θ_L-type entity chooses R, instead of deviating to any other $R' \neq R$, which guarantees a merger decline, if and only if $\pi_I^{M,L} - R \geq k\pi_i^{NM}$. (The right side of this inequality follows a similar argument as for the high-type firm.). This inequality simplifies to $\pi_I^{M,L} - k\pi_i^{NM} \geq R$ or, solving for θ_L, we know from Proposition 2 that this inequality yields $\theta_L > \widehat{\theta}(R)$.

Combining the inequalities we found from the high- and low-type firms, we obtain that a pooling PBE can be sustained if $\theta_H > \widehat{\theta}(R)$ and $\theta_L > \widehat{\theta}(R)$, but since $\theta_H > \theta_L$ by definition, a sufficient condition for both inequalities to hold is $\theta_L > \widehat{\theta}(R)$, which is equivalent to $\pi_I^{M,L} - k\pi_i^{NM} \geq R$.

Applying the Cho and Kreps' Intuitive Criterion does not have a bite in this case. Specifically, both firm types have incentives to deviate from R to R', where $R > R' \geq f$, if the merger request is still approved. As a consequence, the CA cannot update its off-the-equilibrium beliefs in the first step of the Intuitive Criterion.

Pooling PBE where both firm types invest zero. *Updated beliefs.* In this pooling strategy profile, where $R = 0$ for both firm types, the CA's information set is not reached since no merger approval request is submitted. Then, the CA holds off-the-equilibrium beliefs $\mu(\theta_H|M) = \mu$ and $\mu(\theta_L|M) = 1 - \mu$.

Receiver's response. Given the above beliefs, the CA responds approving a merger upon observing one (which can only happen off-the-equilibrium path) if and only if

$$\mu\theta_H + (1-\mu)\theta_L > \overline{\theta}$$

or, alternatively, $\mu > \frac{\overline{\theta}-\theta_L}{\theta_H-\theta_L} \equiv \widehat{p}$. We next separately analyze the case in which $\mu > \widehat{p}$ (and the CA responds approving merger, if one is submitted off the equilibrium) and that in which $\mu \leq \widehat{p}$ (and the CA blocks the merger).

Sender's messages. If $\mu > \widehat{p}$, the θ_H-type entity, does not invest, $R = 0$, as prescribed in this pooling strategy profile, if and only if $\pi_I^M - R' < k\pi_i^{NM}$, where $R' \geq f$, since it anticipates that a deviation towards sending a request will be approved by the CA (given its off-the-equilibrium beliefs). This inequality entails $\theta_H < \widehat{\theta}(R')$. Similarly, the θ_L-type entity, does not invest, as required in this pooling strategy profile, if and only if $\pi_I^M - R' < k\pi_i^{NM}$, which yields $\theta_L < \widehat{\theta}(R')$.

If, instead, $\mu \leq \widehat{p}$ holds, both entity types anticipate that the CA will respond blocking merger approval requests (off-the-equilibrium path). In this context, the θ_H-type entity does not invest, as prescribed in this pooling strategy profile, if and only if $\pi_I^{NM} - R' < k\pi_i^{NM}$, which simplifies to $-R' < (k-1)\pi_i^{NM}$, which holds since $k \geq 2$ by definition. A similar argument applies to the θ_L-type entity.

In summary, the pooling strategy profile where no merging entity types submits a merger request can be sustained as a PBE if the CA's off-the-equilibrium beliefs satisfy $\mu \leq \widehat{p}$ under all parameter values; and if $\mu > \widehat{p}$ if, in addition, $\theta_i < \widehat{\theta}(R')$ for all types $i = \{H, L\}$.

We next apply Cho and Kreps' Intuitive Criterion to this pooling PBE. In this setting, if the CA observes a merger request (off-the-equilibrium path), we must evaluate which firm type (if any) can potentially gain from this deviation. The high-type can increase its equilibrium payoff, $k\pi_i^{NM}$, to $\pi_I^M - R'$, where $R' \geq f$, which happens if the merger request is approved, if and only if $\theta_H \geq \widehat{\theta}(R')$. A similar argument applies for the low-type, which can increase its equilibrium payoff, $k\pi_i^{NM}$, to $\pi_I^M - R'$ if the CA responds approving its merger request. Therefore, both firm types would benefit from the deviation if $\theta_i \geq \widehat{\theta}(R')$; only the high-type does when $\theta_H \geq \widehat{\theta}(R')$ and $\theta_L < \widehat{\theta}(R')$; only the low-type does when $\theta_H < \widehat{\theta}(R')$ and $\theta_L \geq \widehat{\theta}(R')$ (which cannot hold since $\theta_H > \theta_L$ by assumption); and no firm type would benefit from the deviation if $\theta_i < \widehat{\theta}(R')$ holds. This means that the pooling PBE supported when $\mu \leq \widehat{p}$ under all values of θ_L and θ_H violates the Intuitive Criterion, except when $\theta_i < \widehat{\theta}(R')$ holds since no firm type would have incentives to deviate even when they anticipate that a merger request will be approved by the CA. Similarly, the pooling PBE supported when $\mu > \widehat{p}$ and $\theta_i < \widehat{\theta}(R')$ survives the Intuitive Criterion.

8.11 *Proof of Corollary 3*

The ranking of cutoff $\widehat{\theta} \equiv \widehat{\theta}(f)$ against $\widehat{\theta}(R_H^{SE})$ in figure 3a is irrelevant for identifying inefficient mergers since $\widehat{\theta}(R_H^{SE})$ determines the region of separating PBEs, where socially efficient mergers occur in equilibrium. However, the ranking of cutoff $\widehat{\theta}$ against that identifying pooling PBEs in Proposition 5, $\widehat{\theta}(R^{PE})$, matters, as socially excessive mergers arise in this equilibrium. Comparing cutoffs $\widehat{\theta}$ and $\widehat{\theta}(R^{PE})$, we find that either: (1) $R^{PE} < f$, which entails $\widehat{\theta}(R^{PE}) < \widehat{\theta}$; or (2) $f < R^{PE}$, which yields $\widehat{\theta} < \widehat{\theta}(R^{PE})$. The first case, however, is not admissible since R^{PE} must satisfy $R^{PE} \geq f$. The second case is admissible, where $\widehat{\theta} < \widehat{\theta}(R^{PE})$, implying that the pooling PBE under endogenous submission costs (Proposition 5) sustains socially inefficient mergers under more restrictive parameter conditions than the pooling PBE under exogenous costs (Proposition 3).

8.12 *Proof of Corollary 4*

Differentiating ∇MI with respect to f, we obtain that it is negative if

$$\frac{[(n+1)\theta_L + k - 1]\left[\sqrt{f + \frac{(1-c)^2 k}{(n+1)^2}} - \sqrt{f + r + \frac{(1-c)^2 k}{(n+1)^2}}\right][3 + n + \overline{\theta}(n+1) - k(\overline{\theta}+1)]}{4(n+1)(n-k+1)^2\sqrt{f + \frac{(1-c)^2 k}{(n+1)^2}}\sqrt{f + r + \frac{(1-c)^2 k}{(n+1)^2}}} < 0$$

which, solving for $\overline{\theta}$, yields $\overline{\theta} > -1 - \frac{2}{n-k+1}$. Since $-1 - \frac{2}{n-k+1} < 0$ under all admissible parameters, condition $\overline{\theta} > -1 - \frac{2}{n-k+1}$ always holds, implying that ∇MI unambiguously decreases in f.

8.13 *Proof of Proposition 6*

Beliefs. If the CA does not update its beliefs according to Bayes' rule, holding beliefs $\mu(\theta_H|R) = p$ and $\mu(\theta_L|R) = 1 - p$ for all $R \geq 0$.

Receiver's response. Given the above beliefs, the CA responds approving a merger, upon observing any R, if

$$E[\theta] \equiv p\frac{x_H}{1-c} + (1-p)\frac{x_L}{1-c} = p\theta_H + (1-p)\theta_L \geq \overline{\theta}.$$

or, after solving for p, if $p > \frac{\overline{\theta}-\theta_L}{\theta_H-\theta_L} \equiv \widehat{p}$, which holds by assumption. Therefore, the CA approves all merger requests, independently on the firm's submission cost R.

Sender's messages. Anticipating these responses, the θ_H-type entity invests zero in submission costs, as it can anticipate that the merger is approved for all $R \geq 0$, if and only if $\pi_I^M \geq k\pi_i^{NM}$, which holds if $\theta_H \geq \widehat{\theta}(0)$. Similarly, the θ_L-type entity chooses a zero submission cost if and only $\pi_I^M \geq k\pi_i^{NM}$, which is satisfied if $\theta_L \geq \widehat{\theta}(0)$.

8.14 *Proof of Corollary 5*

Under low priors, $p \leq \widehat{p}$, a separating equilibrium can still be sustained as long as $\theta_H > \widehat{\theta}(R_H)$ and $\theta_L < \widehat{\theta}(0)$. Under these conditions, the CA can still update its beliefs based on the firm's type-dependent actions, that is, submit a merger request if high type but not submit it otherwise. However, if it is profitable for both (neither) firm to submit a merger request, the CA cannot update its beliefs and will not approve a merger proposal since $p \leq \widehat{p}$. Thus, a pooling equilibrium where both types submit a proposal cannot be sustained in this context, as it is not sequentially rational for firms to do so as long as the submission cost satisfies $R > 0$. On the other hand, a pooling equilibrium where neither firm submits will always be sustained, even when it is profitable for a merger to happen for either or both types.

8.15 *Proof of Proposition 7*

Updated beliefs. In this pooling strategy profile, the CA cannot update its beliefs according to Bayes' rule. Therefore, upon observing R, where $R \geq f$, its beliefs are $\mu(\theta_H|R) = p$ and $\mu(\theta_L|R) = 1 - p$, whereas upon receiving any off-the-equilibrium message $R' \neq R$, where $R' \geq f$, its off-the-equilibrium beliefs are $\mu(\theta_H|R') = 0$.

Receiver's response. Given the above beliefs, upon observing R, in equilibrium, the CA responds exerting a challenging effort α, upon observing R, that solves

$$\max_{\alpha\in[0,1]} \alpha\left(p\frac{1-c+(n-k+1)x_H}{n-k+2} + (1-p)\frac{1-c+(n-k+1)x_L}{n-k+2}\right) + (1-\alpha)\left(k\frac{1-c}{n+1}\right) - \frac{1}{2}\lambda\alpha^2.$$

which, differentiating with respect to α, yields

$$\left(p\frac{1-c+(n-k+1)x_H}{n-k+2} + (1-p)\frac{1-c+(n-k+1)x_L}{n-k+2}\right) - \left(k\frac{1-c}{n+1}\right) - \lambda\alpha = 0.$$

and, solving for α, we obtain the CA's optimal response after observing the pooling submission cost R,

$$\alpha^* = \frac{1-c}{\lambda}\left[\frac{1+E[\theta](n-k+1)}{n-k+2} - \left(\frac{k}{n+1}\right)\right]$$

where $E[\theta] \equiv p\theta_H + (1-p)\theta_L$. In addition, α^* satisfies $\alpha^* > 0$ if

$$\frac{1+E[\theta](n-k+1)}{n-k+2} > \frac{k}{n+1}$$

or, after rearranging, $E[\theta] > \frac{k-1}{n+1} \equiv \overline{\theta}$, which is true by assumption; and α^* satisfies $\alpha^* < 1$ if

$$E[\theta] < \frac{1}{n-k+1}\left[\left(\frac{\lambda(n+1)+k(1-c)}{(1-c)(n+1)}\right)(n-k+2) - 1\right]$$

In contrast, upon observing the off-the-equilibrium message R', the CA responds blocking the merger since $\mu(\theta_H|R') = 0$ and $\theta_L < \overline{\theta}$ by assumption.

Sender's messages. Anticipating these responses, the θ_H-type entity invests R, as prescribed in this pooling strategy profile, if

$$\alpha\left(\pi_I^{M,H}-R\right)+(1-\alpha)k\pi_i^{NM}\geq k\pi_i^{NM},$$

where the right side assumes that the high-type deviates to zero investment (no merger request) because any deviation to $R'\neq R$ guarantees a merger decline and $R'=0$ minimizes the firm's submission cost. Inserting the CA's optimal response, α^* identified above, into this inequality, we obtain

$$\alpha^*\left(\pi_I^{M,H}-R-k\pi_i^{NM}\right)\geq 0.$$

Since $\alpha^*>0$, the above inequality simplifies to $\pi_I^{M,H}-R-k\pi_i^{NM}\geq 0$. After solving for R, we find $R\leq\pi_I^{M,H}-k\pi_i^{NM}$. Solving for θ_H, we know from Proposition 2 that this inequality yields $\theta_H>\widehat{\theta}(R)$.

Similarly, the θ_L-type entity chooses R, instead of deviating to any other $R'\neq R$, which guarantees a merger decline, if and only if

$$\alpha\left(\pi_I^{M,L}-R\right)+(1-\alpha)k\pi_i^{NM}\geq k\pi_i^{NM},$$

(The right side of this inequality follows a similar argument as for the high-type firm.). Inserting α^* into this inequality, yields

$$\pi_I^{M,L}-R-k\pi_i^{NM}\geq 0$$

which simplifies to $R\leq\pi_I^{M,L}-k\pi_i^{NM}$. Solving for θ_L, we know from Proposition 2 that this inequality yields $\theta_L>\widehat{\theta}(R)$. Combining the inequalities we found from the high- and low-type firms, we obtain that a pooling PBE can be sustained if $R\leq\pi_I^{M,L}-k\pi_i^{NM}$. $\theta_H>\widehat{\theta}(R)$ and $\theta_L>\widehat{\theta}(R)$, but since $\theta_H>\theta_L$ by definition, a sufficient condition for both inequalities to hold is $\theta_L>\widehat{\theta}(R)$, which is equivalent to $\pi_I^{M,L}-k\pi_i^{NM}\geq R$.

Intuitive Criterion. Applying the Cho and Kreps' Intuitive Criterion does not have a bite in this case. Specifically, both firm types have incentives to deviate from R to R', where $R>R'\geq f$, if the CA responds to the merger request in the most positive way ($\alpha^*=1$). In particular, α^* is not a function of R, so the merging entity's deviation does not produce a direct effect on the CA's response, only an indirect effect via a potential change in the CA's off-the-equilibrium beliefs. As a consequence, the CA cannot update its off-the-equilibrium beliefs in the first step of the Intuitive Criterion, as both firm types have incentives to deviate to R', entailing that the pooling PBE survives the Intuitive Criterion..

Divinity Criterion. Applying the Divinity Criterion, we start considering a deviation from R to R', where $R>R'\geq f$. Upon observing R', the set of CA responses, α, that weakly improve the high-type firm's equilibrium payoff satisfies

$$\alpha\left(\pi_I^{M,H}-R'\right)+(1-\alpha)k\pi_i^{NM}\geq\alpha^*\left(\pi_I^{M,H}-R\right)+(1-\alpha^*)k\pi_i^{NM}$$

where the left (right) side denotes this firm's payoff from deviating to R' (from choosing its equilibrium submission cost R, respectively). Solving for α, yields

$$\alpha \geq \alpha^* \left[\frac{\pi_I^{M,H} - R - k\pi_i^{NM}}{\pi_I^{M,H} - R' - k\pi_i^{NM}} \right].$$

Similarly, the set of CA responses, α, that weakly improve the low-type firm's equilibrium payoff satisfies

$$\alpha \left(\pi_I^{M,L} - R' \right) + (1-\alpha) k\pi_i^{NM} \geq \alpha^* \left(\pi_I^{M,L} - R \right) + (1-\alpha^*) k\pi_i^{NM}$$

which, solving for α, yields

$$\alpha \geq \alpha^* \left[\frac{\pi_I^{M,L} - R - k\pi_i^{NM}}{\pi_I^{M,L} - R' - k\pi_i^{NM}} \right].$$

Comparing cutoffs $\alpha^* \left[\frac{\pi_I^{M,H} - R - k\pi_i^{NM}}{\pi_I^{M,H} - R' - k\pi_i^{NM}} \right]$ and $\alpha^* \left[\frac{\pi_I^{M,L} - R - k\pi_i^{NM}}{\pi_I^{M,L} - R' - k\pi_i^{NM}} \right]$, we obtain that

$$\alpha^* \left[\frac{\pi_I^{M,H} - R - k\pi_i^{NM}}{\pi_I^{M,H} - R' - k\pi_i^{NM}} \right] > \alpha^* \left[\frac{\pi_I^{M,L} - R - k\pi_i^{NM}}{\pi_I^{M,L} - R' - k\pi_i^{NM}} \right]$$

simplifies to

$$\frac{R' - R}{\pi_I^{M,H} - R' - k\pi_i^{NM}} > \frac{R' - R}{\pi_I^{M,L} - R' - k\pi_i^{NM}}$$

and, since $R > R'$, this expression further reduces to $\pi_I^{M,L} < \pi_I^{M,H}$, which holds by definition. Therefore, R' is more likely to originate from the low-type firm, implying that the CA restricts its off-the-equilibrium beliefs to $\mu(R') = 0$. As a consequence, upon observing R', the CA responds with $\alpha = 0$.

In the second step of the Divinity Criterion, we check if either firm type has incentives to deviate anticipating this response. The high-type deviates from R if R' if

$$0 \left(\pi_I^{M,H} - R' \right) + k\pi_i^{NM} \geq \alpha^* \left(\pi_I^{M,H} - R \right) + (1-\alpha^*) k\pi_i^{NM}$$

which simplifies to

$$0 \geq \alpha^* \left(\pi_I^{M,H} - R - k\pi_i^{NM} \right)$$

which is incompatible with the condition for the high-type firm to behave as prescribed in this pooling PBE, $\alpha^* \left(\pi_I^{M,H} - R - k\pi_i^{NM} \right) \geq 0$.

Similarly, the low-type firm deviates if

$$0 \left(\pi_I^{M,L} - R' \right) + k\pi_i^{NM} \geq \alpha^* \left(\pi_I^{M,L} - R \right) + (1-\alpha^*) k\pi_i^{NM}$$

which simplifies to

$$0 \geq \alpha^* \left(\pi_I^{M,L} - R - k\pi_i^{NM} \right)$$

which is incompatible with the condition for the low-type firm to behave as prescribed in this pooling PBE, $\alpha^* \left(\pi_I^{M,L} - R - k\pi_i^{NM}\right) \geq 0$. In conclusion, no firm type has incentives to deviate from R, implying that the pooling PBE survives the Divinity Criterion.

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