EconS 501 Final Exam - December 6th, 2021

Show all your work clearly and make sure you justify all your answers.

NAME_________________________

1. Consider a monopolist selling two different goods, q_1 and q_2, whose demands are interdependent and given by

$$q_i = a - bp_i + gp_j \quad \text{for } i = \{1,2\} \text{ and } j \neq i$$

where $b > 0$ (this guarantees that the demand for a particular good decreases in its own price). In addition, note that if $g > 0$, goods are substitutes, while if $g < 0$, they are complements (and if $g = 0$ the products are independent of each other). Assume also that $|g| < b$, which guarantees that the own-price effect, represented in parameter b, dominates the cross-price effect, embodied in parameter g. Intuitively, the demand for good i is more sensitive to a given increase in its own price than to the same increase in the price of a different good. Finally, consider that $a > c(b - g)$ which guarantees that output is strictly positive in equilibrium. In order to focus on the interdependence between both products' demands, let us assume that the marginal costs of production coincide across both products. That is, total costs are $TC(q_1, q_2) = cq_1 + cq_2$.

(a) Find the profit-maximizing price for good 1, p_1, and for good 2, p_2, for this monopolist. [*Hint:* Find the profit-maximizing prices rather than output levels.]

- The monopolist profit maximization problem is to choose prices p_1 and p_2 that solve

$$\max_{p_1,p_2} \ \pi = (a - bp_1 + gp_2)(p_1 - c) + (a - bp_2 + gp_1)(p_2 - c).$$

Taking first order condition respect to every price p_i, yields

$$\frac{\partial \pi}{\partial p_i} = a - 2bp_i + 2gp_j + c(b - g) = 0,$$

for every product $i, j = \{1,2\}$ and $i \neq j$. At the symmetric solution $p_1 = p_2 = p_m$, which implies

$$a - 2bp_m + 2gp_m + c(b - g) = 0,$$

and solving for p_m, we obtain

$$p_m = \frac{a + c(b - g)}{2(b - g)},$$

where $p_m > c$. In particular, recall that $a > c(b - g)$ holds by assumption, which implies that, if we add $c(b - g)$ on both sides of this inequality, we obtain

$$a + c(b - g) > 2c(b - g),$$

which must also hold. The last condition can alternatively be expressed as

$$\frac{a + c(b - g)}{2(b - g)} > c$$

thus entailing that $p_m > c$.

(b) Do prices increase or decrease in the parameter that reflects the cross-price effects, g?

- Taking the first order derivative of p_m with respect to g, yields,

$$\frac{\partial p_m}{\partial g} = \frac{a}{2(b-g)^2} > 0$$

  implying that as g increases in the interval $(-b, b)$, the price charged by the monopolist on both products also increases. [It is easy to check that p_m is concave in g, with its lowest possible value being $\frac{a+2bc}{4b}$, which occurs when $g \to -b$, and an asymptote as $g \to b$.]

- For illustrative purposes, figure 1 assumes parameters $a = 1$, $b = 1$ and $c = 0$, implying a price of $p_m = \frac{1}{2(1-g)}$, and allows for g to take values in the interval $(-b, b)$, i.e., $(-1, 1)$.

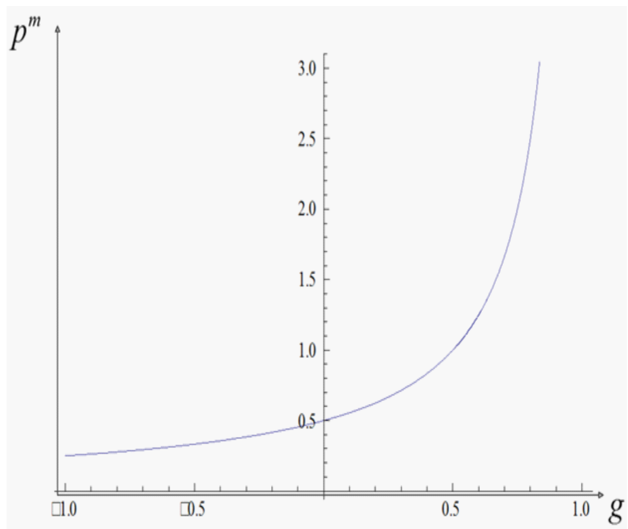

Figure 1. Monopoly price p_m increases in g.

(c) Compare prices if $g > 0$ and if $g < 0$ with those where $g = 0$. Explain

- *Complements.* Relative to the benchmark case where the two products are independent ($g = 0$), the monopolist reduces the price of its product when they are complements ($g < 0$), and it increases it when they are substitutes ($g > 0$). The intuition for this result is straightforward. When products are complements, they exercise a positive effect on each other and the monopolist internalizes such effect by decreasing its prices (i.e., a lower price of good 1 stimulates sales of good 2, and vice versa). In other words, if products 1 and 2 were sold by two distinct monopolists, consumers would pay more for them than when they are sold by the same firm. [This is a standard result in the literature on vertically integrated firms studied in industrial organization].

- *Substitutes.* When, instead, products are substitutes ($g > 0$), the external effect they exercise on each others' demands is negative, and the monopolist controls it by raising prices (a lower price of good 1 crowds out sales of good 2 and vice versa). Hence, if products 1 and 2 were sold by two distinct firms, consumers would pay less than when they are sold by the same firm.

- *Remark:* note that these insights extend to the case where the monopolist sells the same product in sequential markets, i.e., to a group of customers in the first period and to a different set of customers in the second period. In particular, we can re-interpret the demand relationship as one of intertemporal substitutability and complementarity, i.e., demand in the first period stimulates (reduces) demand in the second period when $g > 0$ ($g < 0$, respectively).

2. A utility function is additively separable if it has the form

$$u(x) = \sum_{k=1}^{L} u_k(x_k)$$

For instance, in the context of three goods, an additively separable function would be $u(x) = u_1(x_1) + u_2(x_2) + u_3(x_3)$, where function $u_k(x_k)$ can be linear or nonlinear in the units of good k, x_k.

   (a) For the context of two goods, provide at least two examples of utility functions that are additively separable, and two examples of utility functions that are not.

   - First, note that the utility of good k, $u_k(x_k)$, could be $u_k(x_k) = ax_k$, $u_k(x_k) = a \ln x_k$ or $u_k(x_k) = ax_k^2$ where $a > 0$; or more generally, functions of the form $u_k(x_k) = ax_k^{\beta}$, where $a, \beta \in \mathbb{R}$.
   - *Additively separable utility functions.* Consider, for instance, goods that are regarded as subsitutes, with utility function

$$u(x, y) = ax + by,$$

where $a, b \in \mathbb{R}$; or, more generally, utility functions such as

$$u(x, y) = ax^{\beta} + by^{\delta},$$

where $a, b, \beta, \delta \in \mathbb{R}$. Note that the last example includes quasi-linear utility functions of the form

$$u(x, y) = ax^{\beta} + by,$$

as a special case (when $\delta = 1$).

   - *Not additively separable utility functions.* Consider, for example, the Cobb-Douglas utility function

$$u(x, y) = ax^{\alpha}y^{\beta},$$

where $a, b, \alpha, \beta \in \mathbb{R}$; the utility function representing goods that are regarded as complements,

$$u(x, y) = \min \{ax, by\},$$

where $a, b \in \mathbb{R}$; or the Stone-Geary utility function

$$u(x, y) = a (x - \overline{x})^{\alpha} (y - \overline{y})^{\beta},$$

where $a, \alpha, \beta \in \mathbb{R}$ and $\overline{x}, \overline{y} > 0$.

(b) Show that the marginal utility of good k is a function of the units of good k alone. Interpret.

- Differentiating the utility function $u(x)$ with respect to x_k, we obtain

$$MU_k = \frac{\sum_{l \neq k} u_l(x_l)}{\partial x_k} = \frac{\partial u_k(x_k)}{\partial x_k}$$

since all other components of the utility function, $\sum_{l \neq k} u_l(x_l)$, do not include x_k as arguments. Intuitively, when increasing the units of good k, the consumer only cares about the additional utility he obtains from this good, but ignores the number of units of other goods he consumes (that is, there is no interaction between the utilities of different goods, nor on their marginal utilities). Denoting the marginal utility of good k as $u_k'(x_k(p,w))$, we can more compactly express our above result as

$$MU_k = u_k'(x_k(p,w))$$

(c) Show that the Walrasian and Hicksian demand functions imply that all goods must be normal rather than inferior. (For simplicity, you can assume that the utility of every good is strictly concave for every good, differentiability, and interior solutions.)

- First, we know that the following tangency condition holds both in the UMP and in the EMP

$$MRS_{k,l} = \frac{MU_k}{MU_l} = \frac{u_k'(x_k(p,w))}{u_l'(x_l(p,w))} = \frac{p_k}{p_l}$$

(Recall that both the UMP and EMP have this tangency condition between the indifference curve and the budget line in common. However, the UMP inserts this result into its contraint, the budget line; whereas the EMP inserts the above result into its constraint, the utility level that the individual must reach.) Rearranging the above tangency condition, we obtain

$$u_k'(x_k(p,w)) = \frac{p_k}{p_l} u_l'(x_l(p,w)).$$

- Since we seek to show that no good can be inferior, we must consider a wealth change, leaving all prices unchanged. If wealth w increases, the demand for at least one good (say, good l) has to increase by Walras' law (otherwise, the individual would be buying fewer units of all goods, thus not exhausting his wealth). We seek to show that the demand for the remaining good k must also increase, thus implying that all goods are normal.
- To see this, first note that if the demand for good l increases, its marginal utility decreases. Graphically, a decrease in $u_l'(x_l(p,w))$ implies that the line representing $\frac{p_k}{p_l} u_l'(x_l(p,w))$ shifts downwards, yielding a new crossing point to the right-hand side of the initial crossing point depicted in the above figure. As a consequence, the consumer demands a larger amount of good k, i.e., $x_k(p,w)$ increases, ultimately implying that good k must be normal. Since our analysis applies to any good k, all goods must be normal.

3. Consider a setting with coupon rationing so that each commodity has two prices: a dollar price and a ration-coupon price. Assume that there are three commodities and that the consumer has a dollar income w and a ration-coupon allotment z. Also assume that this allotment is not so liberal that any commodity combination that he can afford to purchase with his dollar income can also be purchased with his coupons.

   (a) Set up this consumer's utility-maximization problem assuming a strictly concave utility function. Interpret your results. [*Hint*: You need to impose two constraints: a budget and a coupon constraint].

   - The consumer solves

$$\begin{aligned} \max_{x_1,x_2,x_3} \quad & u(x_1,x_2,x_3) \\ \text{s.t.} \quad & p_1x_1 + p_2x_2 + p_3x_3 \leq w \quad \text{(Budget constraint)} \\ & c_1x_1 + c_2x_2 + c_3x_3 \leq z \quad \text{(Coupon constraint)} \end{aligned}$$

     where $p_1 - p_3$ denote good prices, while $c_1 - c_3$ represent ration-coupon prices for each good. The associated Lagrangian is

$$L = u(x_1,x_2,x_3) + \lambda(w - p_1x_1 - p_2x_2 - p_3x_3) + \mu(z - c_1x_1 - c_2x_2 - c_3x_3)$$

     where λ (μ) is the Lagrange multiplier for the budget (coupon) constraint.

   (b) Find first-order conditions and interpret them.

   - Differentiating with respect to x_1, x_2, x_3, λ and μ, we find

$$\begin{aligned} \frac{\partial L}{\partial x_1} &= u_1 - \lambda p_1 - \mu c_1 \leq 0 & x_1 \geq 0 \\ \frac{\partial L}{\partial x_2} &= u_2 - \lambda p_2 - \mu c_2 \leq 0 & x_2 \geq 0 \\ \frac{\partial L}{\partial x_3} &= u_3 - \lambda p_3 - \mu c_3 \leq 0 & x_3 \geq 0 \\ \frac{\partial L}{\partial \lambda} &= w - p_1x_1 - p_2x_2 - p_3x_3 \geq 0 & \lambda \geq 0 \\ \frac{\partial L}{\partial \mu} &= z - c_1x_1 - c_2x_2 - c_3x_3 \geq 0 & \mu \geq 0 \end{aligned}$$

     For compactness, $u_i \equiv \frac{\partial u}{\partial x_i}$ denotes the marginal utility from good i. In the case of interior solutions, the first-order conditions hold with equality and yield

$$\underbrace{\frac{u_i}{u_j}}_{\text{MRS}} = \underbrace{\frac{\lambda p_i + \mu c_i}{\lambda p_j + \mu c_j}}_{\text{"Generalized" price ratio}}$$

     where $j \neq i$. Intuitively, the left-hand side represents the ratio of marginal utilities between goods i and j, that is, marginal rate of substitution; as in standard consumer problems without coupons or rationing. The right-hand side, however, is not necessarily equal to the standard price ratio $\frac{p_i}{p_j}$, since it also includes good prices and ration-coupon prices, weighted by the corresponding marginal utilities (the Lagrange multipliers λ and μ). For compactness, this price ratio is often referred to as "Generalized price ratio" since it embodies the standard price ratio $\frac{p_i}{p_j}$ as a special case, when $\mu = 0$.

(c) Find a sufficient condition guaranteeing that the imposition of rationing does not alter the consumer's optimal bundle.

- As suggested in part (b) of the exercise, the imposition of rationing yields the standard optimality condition $MRS = \frac{p_i}{p_j}$ when $\mu = 0$. Intuitively, this occurs when the marginal utility of additional coupons is close to zero, which could happen when the consumer receives a large amount of coupons z. For instance, if z is weakly larger than the total amount of goods $x_1 + x_2 + x_3$ the consumer purchases under no rationing, then rationing has no effect on his purchases, making the standard optimality condition $MRS = \frac{p_i}{p_j}$ still valid.

(d) *Numerical Example*: Consider now that the consumer exhibits Cobb-Douglas utility function $u(x_1, x_2, x_3) = x_1 x_2 x_3$, income $w = 100$, and price vector $p = (1, 4, 2)$. Assume that, under rationing, the price vector is $c = (1/2, 3, 1)$ and the total coupon amount is $z = 80$. Find the optimal bundle with and without rationing in this context.

- *No rationing.* When no rationing is in effect, the consumer solves a standard utility maximization problem

$$\max_{x_1, x_2, x_3} \quad x_1 x_2 x_3$$
$$\text{s.t.} \quad p_1 x_1 + p_2 x_2 + p_3 x_3 \leq w$$

with associated Lagrangian

$$L = x_1 x_2 x_3 + \lambda(w - p_1 x_1 - p_2 x_2 - p_3 x_3)$$

Taking first-order conditions, we obtain

$$\begin{aligned}
\frac{\partial L}{\partial x_1} &= x_2 x_3 - \lambda p_1 \leq 0, \text{ with equality if } x_1^* > 0 \\
\frac{\partial L}{\partial x_2} &= x_1 x_3 - \lambda p_2 \leq 0, \text{ with equality if } x_2^* > 0 \\
\frac{\partial L}{\partial x_3} &= x_1 x_2 - \lambda p_3 \leq 0, \text{ with equality if } x_3^* > 0 \\
\frac{\partial L}{\partial \lambda} &= w - p_1 x_1^* - p_2 x_2^* - p_3 x_3^* = 0
\end{aligned}$$

In the case of interior solutions, solving for λ yields the following relations

$$\begin{aligned}
\frac{x_2}{x_1} &= \frac{p_1}{p_2} \Longleftrightarrow \frac{p_2 x_2}{p_1} = x_1 \\
\frac{x_3}{x_2} &= \frac{p_2}{p_3} \Longleftrightarrow x_3 = \frac{p_2 x_2}{p_3} \\
\frac{x_2 x_3}{x_1 x_2} &= \frac{p_1}{p_3}
\end{aligned}$$

Substituting the above conditions into the budget constraint gives

$$\begin{aligned} p_1x_1 + p_2x_2 + p_3x_3 &= \\ p_1\frac{p_2x_2}{p_1} + p_2x_2 + p_3\frac{p_2x_2}{p_3} &= w \end{aligned}$$

Finally, solving for x_2 yields the Walrasian demand for good x_2, $x_2(w, p_1, p_2, p_3) = \frac{w}{3p_2}$. Similar manipulations gives the Walrasian demands for goods x_1 and x_3, $x_1(w, p_1, p_2, p_3) = \frac{w}{3p_1}$ and $x_3(w, p_1, p_2, p_3) = \frac{w}{p_3}$. Substituting income $w = 100$ and price vector $p = (1, 4, 2)$ entails Walrasian demands

$$\begin{aligned} x_1^* &= \frac{100}{3} \\ x_2^* &= \frac{100}{12} \\ x_3^* &= \frac{100}{6} \end{aligned}$$

- *Rationing.* When rationing is in effect, the consumer solves

$$\begin{aligned} &\max_{x_1,x_2,x_3} \quad x_1x_2x_3 \\ \text{s.t.} \quad & p_1x_1 + p_2x_2 + p_3x_3 \leq w \quad \text{(Budget constraint)} \\ & c_1x_1 + c_2x_2 + c_3x_3 \leq z \quad \text{(Coupon constraint)} \end{aligned}$$

with associated Lagrangian

$$L = x_1x_2x_3 + \lambda(w - p_1x_1 - p_2x_2 - p_3x_3) + \mu(z - c_1x_1 - c_2x_2 - c_3x_3)$$

Taking first-order conditions, we obtain

$$\begin{aligned} \frac{\partial L}{\partial x_1} &= x_2x_3 - \lambda p_1 - \mu c_1 \leq 0, \text{ with equality if } x_1^* > 0 \\ \frac{\partial L}{\partial x_2} &= x_1x_3 - \lambda p_2 - \mu c_2 \leq 0, \text{ with equality if } x_2^* > 0 \\ \frac{\partial L}{\partial x_3} &= x_1x_2 - \lambda p_3 - \mu c_3 \leq 0, \text{ with equality if } x_3^* > 0 \\ \frac{\partial L}{\partial \lambda} &= w - p_1x_1 - p_2x_2 - p_3x_3 \geq 0 \quad \lambda \geq 0 \\ \frac{\partial L}{\partial \mu} &= z - c_1x_1 - c_2x_2 - c_3x_3 \geq 0 \quad \mu \geq 0 \end{aligned}$$

We then require

$$\begin{aligned} \lambda(w - p_1x_1 - p_2x_2 - p_3x_3) &= 0 \\ \mu(z - c_1x_1 - c_2x_2 - c_3x_3) &= 0 \end{aligned}$$

We solve this problem considering several cases:

- **Case 1**, $\lambda = 0, \mu > 0$. If the coupon constraint binds but the budget constrain does not, the first-order conditions become

$$\begin{aligned}\frac{\partial L}{\partial x_1} &= x_2x_3 - \mu c_1 = 0\\ \frac{\partial L}{\partial x_2} &= x_1x_3 - \mu c_2 = 0\\ \frac{\partial L}{\partial x_3} &= x_1x_2 - \mu c_3 = 0\\ \frac{\partial L}{\partial \mu} &= z - c_1x_1 - c_2x_2 - c_3x_3 = 0\end{aligned}$$

Solving for μ yields the following relations

$$\begin{aligned}\frac{x_2}{x_1} &= \frac{c_1}{c_2} \Longleftrightarrow \frac{c_2x_2}{c_1} = x_1\\ \frac{x_3}{x_2} &= \frac{c_2}{c_3} \Longleftrightarrow x_3 = \frac{c_2x_2}{c_3}\\ \frac{x_3}{x_1} &= \frac{c_1}{c_3}\end{aligned}$$

Substituting the above conditions into the coupon constraint gives

$$\begin{aligned}c_1x_1 + c_2x_2 + c_3x_3 &=\\ c_1\frac{c_2x_2}{c_1} + c_2x_2 + c_3\frac{c_2x_2}{c_3} &= z\end{aligned}$$

which yields the following Walrasian demands:

$$\begin{aligned}x_1^* &= \frac{z}{3c_1} = \frac{160}{3}\\ x_2^* &= \frac{z}{3c_2} = \frac{80}{9}\\ x_3^* &= \frac{z}{3c_3} = \frac{80}{3}\end{aligned}$$

However, if we substitute these demands into the budget constraint, we find that they violates the budget constraint since

$$\frac{160}{3} + 4\frac{80}{9} + 2\frac{80}{3} = 142 > 100$$

implying that the above Walrasian demands cannot be an optimal solution.

- **Case 2**, $\lambda > 0, \mu = 0$. In this case, the coupon constraint does not bind while the budget constrain binds. The first-order conditions then become

$$\begin{aligned}\frac{\partial L}{\partial x_1} &= x_2x_3 - \lambda p_1 = 0\\ \frac{\partial L}{\partial x_2} &= x_1x_3 - \lambda p_2 = 0\\ \frac{\partial L}{\partial x_3} &= x_1x_2 - \lambda p_3 = 0\\ \frac{\partial L}{\partial \lambda} &= w - p_1x_1 - p_2x_2 - p_3x_3 = 0\end{aligned}$$

which yields the same list of FOCs as without rationing. Therefore, Walrasian demands coincide with those under no rationing:

$$x_1^* = \frac{100}{3}$$
$$x_2^* = \frac{100}{12}$$
$$x_3^* = \frac{100}{6}$$

Substituting this solution into the coupon constraint, we find that the coupon constraint is met, that is, the consumer does not exhaust his coupons, since

$$\frac{1}{2}\frac{100}{3} + 3\frac{100}{12} + \frac{100}{6} = 58.33 < 80$$

4. Consider an economy with 2 consumers, Anna and Bob, $i = \{A, B\}$, one private good x, and one public good G. Let each consumer have an income of M. For simplicity, let the prices of both the public and private good be 1. In addition, the utility functions of consumer A and B are:

$$U^A = \log(x^A) + \log(G), \quad \text{for individual } A\text{, and}$$
$$U^B = \log(x^B) + \log(G), \quad \text{for individual } B$$

Assume that the public good G is only provided by the contributions of these two individuals, that is, $G = g^A + g^B$.

(a) Find Anna's best response function. Depict it in a figure with his contribution, g^A, on the vertical axis and Bob's contribution, g^B, on the horizontal axis.

- The utility maximization problem of Anna is that of selecting his consumption of private good, x, and his contribution to the public good, g^A, to solve

$$\max_{x, g^A} \log x^A + \log G$$

$$\text{subject to } \; x^A + g^A = M \; \text{ and } g^A + g^B = G$$

Taking into account that $x^A = M - g^A$, the above problem can be more compactly expressed as a program with a single choice variable,

$$\max_{g^A} \log(M - g^A) + \log(g^A + g^B)$$

Taking first order condition with respect to g^A yields

$$-\frac{1}{M - g^A} + \frac{1}{g^A + g^B} = 0$$

and solving for g^A we obtain:

$$g^A(g^B) = \frac{M}{2} - \frac{g^B}{2}$$

which represents individual A's best response function (see figure 1).

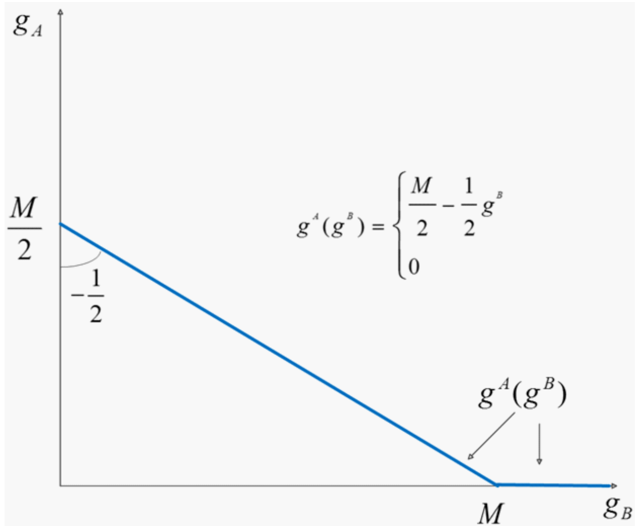

Figure 1. Alessandro's best response function.

- Intuitively, when Bob does not contribute to the public good, $g^B = 0$, Anna contributes $g^A = \frac{M}{2}$, but as Bob increases his contribution (rightward movements in figure 1), Anna responds by decreasing her own donation. In the extreme, when Bob donates all his wealth to the public good, i.e., $g^B = M$, Anna refrains from contributing, $g^A = 0$ for all $g^B \geqslant M$, as represented in the segment of her best response function, $g^A(g^B)$, that overlaps the horizontal axis in the right-hand side of figure 1.

(b) Identify Bob's best response function. Depict it in a figure with her contribution, g^B, on the horizontal axis and Anna's contribution, g^A, on the vertical axis.

- Similarly as for Anna, the utility maximization decision of Bob is that of selecting a contribution to the public good, g^B, that solves

$$\max_{g^B} \log(M - g^B) + \log(g^A + g^B)$$

with first order condition

$$-\frac{1}{M - g^B} + \frac{1}{g^A + g^B} = 0$$

solving for g^B yields

$$g^B(g^A) = \frac{M}{2} - \frac{g^A}{2}$$

which represents Bob's best response function (see figure 2). Note that we use the same axes as in the best response function of Ana, so we can afterwards superimpose both best response functions, $g^A(g^B)$ and $g^B(g^A)$, in the same figure to find their crossing point.

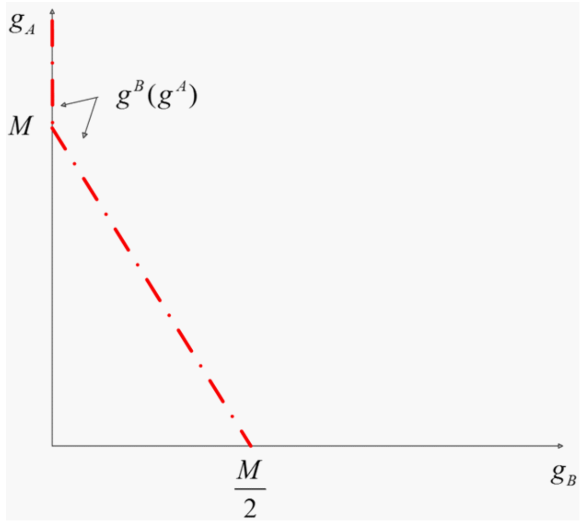

Figure 2. Beatrice's best response function.

(c) *Unregulated equilibrium.* Find the equilibrium contributions to the public good by Anna and Bob.

- Plugging Bob's best response function into Anna's best response function,

$$g^A = \frac{M}{2} - \frac{\left(\frac{M}{2} - \frac{g^A}{2}\right)}{2}$$

and solving for g^A, we obtain

$$g^A = \frac{M}{3}$$

which identifies the crossing point between both individuals' best response function, as depicted in figure 3. (A similar equilibrium contribution arises for Bob, $g^B = \frac{M}{3}$.)

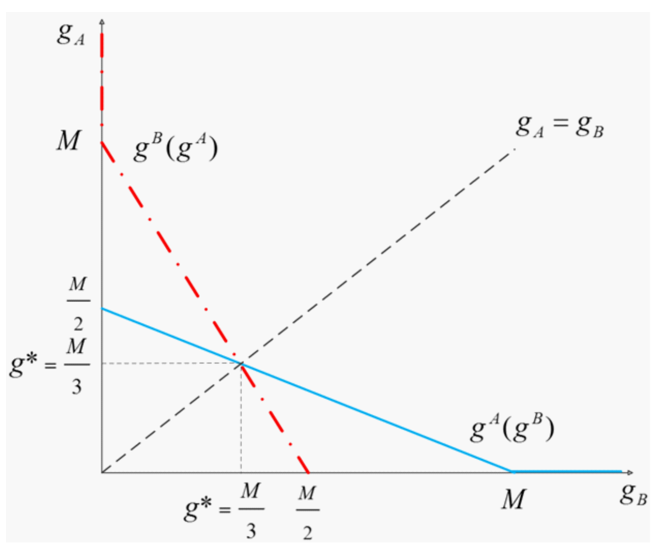

Figure 3. Equilibrium contributions to the public good.

Thus, the aggregate contribution to the public good in the Nash equilibrium is

$$g^A + g^B = \hat{G} = \frac{2M}{3}.$$

(d) *Social optimum.* Find the efficient (socially optimal) contribution to the public good by Anna and Bob.

- Recall that the utilitarian social welfare is $W = U^A + U^B$. The social planner must therefore choose individual contributions g^A and g^B to solve

$$\max_{g^A, g^B} \log\left(M - g^A\right) + \log(g^A + g^B) + \log\left(M - g^B\right) + \log(g^A + g^B)$$

Since individuals are symmetric, their optimal contributions must coincide, i.e., $g^A = g^B = g$, implying that we can simplify the above problem to

$$\max_{g} \ \log\left(M - g\right) + \log(g + g) + \log\left(M - g\right) + \log(g + g)$$

or

$$\max_{G} 2\log\left(M - g\right) + 2\log(2g)$$

and since $g + g = G$, we can further simplify the above problem to

$$\max_{G} \ 2\log\left(M - \frac{G}{2}\right) + 2\log(G)$$

Taking first order condition with respect to G, we obtain

$$-\frac{1}{2}\frac{2}{M - \frac{G}{2}} + \frac{2}{G} = 0.$$

Solving for G we find that the optimal aggregate contribution is $\widetilde{G} = M$. Hence, the sum of both individuals' contributions must add up to M. In a symmetric outcome this implies that each individual contributes half of this socially optimal level, that is

$$\widetilde{g}^A = \widetilde{g}^B = \frac{M}{2}.$$

- *Comparison.* Comparing $\widetilde{G} = M$ with $\hat{G} = \frac{2M}{3}$ shows that total provision at the Nash equilibrium, where every donor independently selects his own contribution, is below the socially optimal level, i.e., $\hat{G} < \widetilde{G}$.

(e) Use a figure to contrast the Pareto efficient level of private provision and the Nash equilibrium level of provision.

- Figure 4 compares individual contributions in the Nash equilibrium, $\hat{g} = \frac{M}{3}$, and socially optimal (Pareto efficient) contributions, $\frac{M}{2}$, which lie on the middle of the set of allocations satisfying $g^A + g^B = M$.

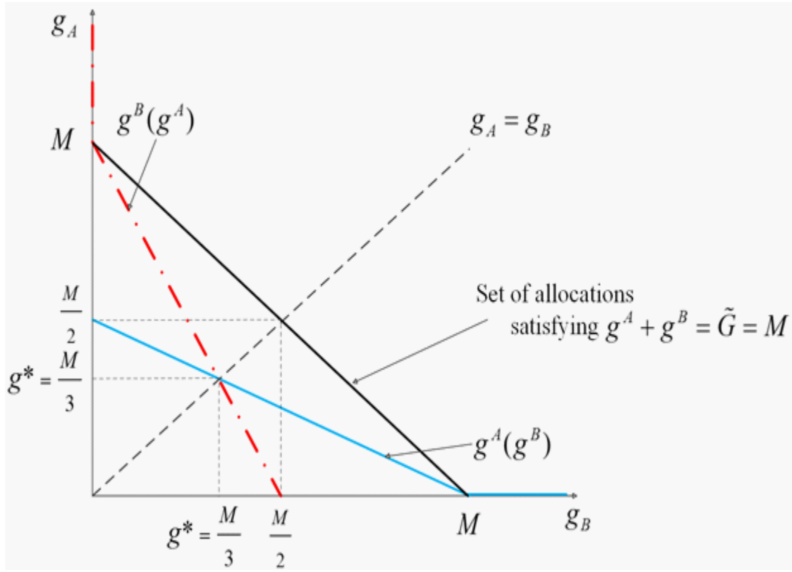

Figure 4. Equilibrium and socially optimal donations.