Homework # 7 - [Due on November 10th, 2021]

1. Let us consider an individual with preferences $u(x, y) = x^{\frac{1}{2}}y^{\frac{1}{2}}$, where x and y denote the amounts consumed of essential goods and non-essential goods, respectively. The prices of these goods are $p_x > 0$ and $p_y > 0$, respectively, and this individual's wealth is $w > 0$. The government needs to collect a large amount of money to finance the development of a vaccine, and considers two options:

   (a) introduce an income tax equivalent to 25% of individuals'wealth; or

   (b) charge a sales tax over the price of the non-essential good which would imply an increase in the price from p_y to $p_y(1 + t)$, collecting the same dollar amount as with the income tax.

   Using the indirect utility function of this individual under option 1 (income tax) and option 2 (sales tax), explain which tax produces a smaller utility reduction.

2. Consider two consumers with utility functions over two goods, x_1 and x_2, given by

$$\begin{aligned} u_A &= \log(x_1^A) + x_2^A - \frac{1}{2}\log(x_1^B) \quad \text{for consumer } A\text{, and} \\ u_B &= \log(x_1^B) + x_2^B - \frac{1}{2}\log(x_1^A) \quad \text{for consumer } B. \end{aligned}$$

   where the consumption of good 1 by individual $i = \{A, B\}$ creates a negative externality on individual $j \neq i$ (see the third term, which enters negatively on each individual's utility function). For simplicity, consider that both individuals have the same wealth, m, and that the price for both goods is 1.

   (a) *Unregulated equilibrium.* Set up consumer A's utility maximization problem, and determine his demand for goods 1 and 2, as x_1^A and x_2^A. Then operate similarly to find consumer B's demand for good 1 and 2, as x_1^B and x_2^B.

   (b) *Social optimum.* Calculate the socially optimal amounts of x_1^A, x_2^A, x_1^B and x_2^B, considering that the social planner maximizes a utilitarian social welfare function, namely, $W = U_A + U_B$.

   (c) *Restoring efficiency.* Show that the social optimum you found in part (b) can be induced by a tax on good 1 (so the after-tax price becomes $1+t$) with the revenue returned equally to both consumers in a lump-sum transfer.

3. Let us consider a market with two firms, Firm A and Firm B, producing a homogeneous good. However, Firm A generates more pollution than Firm B during the production

process as explained below. Firm i's marginal production costs are given by c_i where $i = \{A, B\}$, where c_B is strictly higher than c_A. In addition, the social welfare function that the regulator uses to set emission fees on these firms is

$$SW = CS + PS + T - Env$$

where CS is the consumer surplus, PS is the producer surplus, $T = t\,(q_A + q_B)$ is the tax revenue from emission fees on both firms, and $Env = d_A(q_A)^2 + d_B(q_B)^2$ is the environmental damage from the production of both goods, where $d_A \geq d_B$. Finally, the inverse demand function of firm $i = \{A, B\}$ is

$$p_i(q_i, q_j) = 1 - q_i - q_j \;\text{ where } j = \{A, B\} \text{ and } j \neq i.$$

where q_i denotes output.

(a) *No regulation.* Find equilibrium output levels when firms do not face emission fees. Interpret.

(b) *Regulation.* Find equilibrium output levels when firms face any emission fee t. Interpret.

(c) Identify the socially optimal output level for firm A, q_A^{SO}, and for firm B, q_B^{SO}.

(d) Find the socially optimal fees (t) that induce firms to produce the socially optimal output levels found in part (c). Assume that $d_A = 2$ and $d_B = 0$,and $c_B = \frac{1}{4}$ and $c_A = 0$.