

Recitation #9 [10/29/2021]

1. Consider a competitive market in which the government will be imposing an ad valorem tax of τ . Aggregate demand curve is $x(p) = Ap^\varepsilon$, where $A > 0$ and $\varepsilon < 0$, and aggregate supply curve is $q(p) = \alpha p^\gamma$, where $\alpha > 0$ and $\gamma > 0$. Denote $\kappa = 1 + \tau$, and assume that a partial equilibrium analysis is valid.

- (a) Evaluate how the equilibrium price is affected by a marginal increase in the tax, that is, by a marginal increase in κ .

Imposing the ad valorem tax τ , the aggregate demand curve now becomes

$$x(p, \tau) = A [p(\tau)(1 + \tau)]^\varepsilon$$

Whereas, the aggregate supply curve becomes

$$q(p, \tau) = \alpha [p(\tau)]^\gamma$$

At equilibrium, the aggregate supply curve intersects the aggregate demand curve so that the market clears, that is,

$$A [p(\tau)(1 + \tau)]^\varepsilon = \alpha [p(\tau)]^\gamma$$

Rearranging, we can express the price $p(\tau)$ that the producers receive in terms of tax rate τ .

$$p(\tau) = \left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma-\varepsilon}} (1 + \tau)^{\frac{\varepsilon}{\gamma-\varepsilon}}$$

Differentiating the above expression with respect to τ ,

$$p'(\tau) = \frac{\varepsilon}{\gamma - \varepsilon} \left(\frac{A}{\alpha} \right)^{\frac{1}{\gamma - \varepsilon}} (1 + \tau)^{\frac{2\varepsilon - \gamma}{\gamma - \varepsilon}}$$

We evaluate the first order condition at $\tau = 0$ for a marginal increase in the tax, so that

$$p'(0) = \frac{\varepsilon}{\gamma - \varepsilon} \left(\frac{A}{\alpha} \right)^{\frac{1}{\gamma - \varepsilon}} (1 + 0)^{\frac{2\varepsilon - \gamma}{\gamma - \varepsilon}}$$

so that the tax burden borne by the producers is $p'(0) = \frac{c\varepsilon}{\gamma - \varepsilon}$, where $c \equiv \left(\frac{A}{\alpha} \right)^{\frac{1}{\gamma - \varepsilon}}$ is a constant independent of τ . Essentially, c captures the responsiveness of the equilibrium price to the steepness of the aggregate demand and supply curves. In fact, for a higher A which represents a steeper aggregate demand curve, the price received by the producer is more responsive to a marginal change in the ad valorem tax τ , and the opposite is said for a higher α which represents a steeper aggregate supply curve.

We are also interested in the price paid by the consumers, given by

$$\hat{p}(\tau) \equiv p(\tau) \cdot (1 + \tau) = \left(\frac{A}{\alpha} \right)^{\frac{1}{\gamma - \varepsilon}} (1 + \tau)^{\frac{\gamma}{\gamma - \varepsilon}}$$

Differentiating the above expression with respect to τ ,

$$\hat{p}'(\tau) = \frac{\gamma}{\gamma - \varepsilon} \left(\frac{A}{\alpha} \right)^{\frac{1}{\gamma - \varepsilon}} (1 + \tau)^{\frac{\varepsilon}{\gamma - \varepsilon}}$$

We evaluate the first order condition at $\tau = 0$ for a marginal increase in the tax, so that

$$\hat{p}'(0) = \frac{\gamma}{\gamma - \varepsilon} \left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma - \varepsilon}} (1 + 0)^{\frac{\varepsilon}{\gamma - \varepsilon}}$$

so that the tax burden borne by the consumers is $\hat{p}'(0) = \frac{c\gamma}{\gamma - \varepsilon}$, where $c \equiv \left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma - \varepsilon}}$.

(b) Describe the incidence of the tax when $\gamma = 0$.

$$p'(\tau = 0 | \gamma = 0) = - \left(\frac{A}{\alpha}\right)^{-\frac{1}{\varepsilon}} \frac{\varepsilon}{\varepsilon}$$

$$\hat{p}'(\tau = 0 | \gamma = 0) = - \left(\frac{A}{\alpha}\right)^{-\frac{1}{\varepsilon}} \frac{0}{\varepsilon}$$

Therefore, when the supply is perfectly inelastic, as given by $\gamma = 0$, the tax is entirely borne by the producers, and the price received by each one of them is reduced by a factor of $\left(\frac{A}{\alpha}\right)^{-\frac{1}{\varepsilon}}$ on the ad valorem tax τ . Whereas, the price paid by the consumers is unaffected by τ .

(c) What is the tax incidence when, instead, $\varepsilon = 0$?

$$p'(\tau = 0 | \varepsilon = 0) = \left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma}} \frac{0}{\gamma}$$

$$\hat{p}'(\tau = 0 | \varepsilon = 0) = \left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma}} \frac{\gamma}{\gamma}$$

Therefore, when the demand is perfectly inelastic, as given by $\varepsilon = 0$, the tax is entirely borne by the consumers, and the price paid by each one of them is increased by a factor of $\left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma}}$ on the ad valorem tax τ . Whereas, the price received by the producers is unaffected by τ .

(d) What happens when each of these elasticities approach ∞ in absolute value?

$$\begin{aligned}\lim_{\gamma \rightarrow \infty} p'(0) &= \lim_{\gamma \rightarrow \infty} \left[\left(\frac{A}{\alpha} \right)^{\frac{1}{\gamma - \varepsilon}} \frac{\varepsilon}{\gamma - \varepsilon} \right] \\ &= \left(\lim_{\gamma \rightarrow \infty} \left(\frac{A}{\alpha} \right)^{\frac{1}{\gamma - \varepsilon}} \right) \cdot \left(\lim_{\gamma \rightarrow \infty} \frac{\varepsilon}{\gamma - \varepsilon} \right) \quad \text{by the continuity in the limits} \\ &= 1 \cdot 0 = 0\end{aligned}$$

$$\begin{aligned}\lim_{\gamma \rightarrow \infty} \hat{p}'(0) &= \lim_{\gamma \rightarrow \infty} \left[\left(\frac{A}{\alpha} \right)^{\frac{1}{\gamma - \varepsilon}} \frac{\gamma}{\gamma - \varepsilon} \right] \\ &= \left(\lim_{\gamma \rightarrow \infty} \left(\frac{A}{\alpha} \right)^{\frac{1}{\gamma - \varepsilon}} \right) \left(\lim_{\gamma \rightarrow \infty} \frac{1}{1 - \frac{\varepsilon}{\gamma}} \right) \quad \text{by the continuity in the limits} \\ &= 1 \cdot 1 = 1\end{aligned}$$

Therefore, when the supply is perfectly elastic, as given by $\gamma \rightarrow \infty$, the tax is entirely borne by the consumers by a factor of 1, and the price received by the producers is unaffected by the ad valorem tax τ .

$$\begin{aligned}\lim_{\varepsilon \rightarrow -\infty} p'(0) &= \lim_{\varepsilon \rightarrow -\infty} \left[\left(\frac{A}{\alpha} \right)^{\frac{1}{\gamma - \varepsilon}} \frac{\varepsilon}{\gamma - \varepsilon} \right] \\ &= \left(\lim_{\varepsilon \rightarrow -\infty} \left(\frac{A}{\alpha} \right)^{\frac{1}{\gamma - \varepsilon}} \right) \cdot \left(\lim_{\varepsilon \rightarrow -\infty} \frac{1}{\frac{\gamma}{\varepsilon} - 1} \right) \quad \text{by the continuity in the limits} \\ &= -1 \cdot 1 = 1\end{aligned}$$

$$\begin{aligned}\lim_{\varepsilon \rightarrow -\infty} \hat{p}'(0) &= \lim_{\varepsilon \rightarrow -\infty} \left[\left(\frac{A}{\alpha} \right)^{\frac{1}{\gamma - \varepsilon}} \frac{\gamma}{\gamma - \varepsilon} \right] \\ &= \left(\lim_{\varepsilon \rightarrow -\infty} \left(\frac{A}{\alpha} \right)^{\frac{1}{\gamma - \varepsilon}} \right) \left(\lim_{\varepsilon \rightarrow -\infty} \frac{\gamma}{\gamma - \varepsilon} \right) \quad \text{by the continuity in the limits} \\ &= 1 \cdot 0 = 0\end{aligned}$$

Therefore, when the demand is perfectly elastic, as given by $\varepsilon \rightarrow -\infty$, the tax is entirely borne by the producers by a factor of 1, and the price received by the consumers is unaffected by the ad valorem tax τ .

- (e) Redo your analysis in part (a) the relative change in price due to ad valorem tax τ . How would you compare your results to the effects of a unit tax t in example 6.3?

The relative change in price that the producers receive is

$$\frac{d \ln p(\tau)}{d\tau} = \frac{p'(\tau)}{p(\tau)} = \frac{\varepsilon}{\gamma - \varepsilon} \frac{1}{1 + \tau}$$

We evaluate the above first order condition at $\tau = 0$,

$$\frac{p'(0)}{p(0)} = \frac{\varepsilon}{\gamma - \varepsilon}$$

The relative change in price that the consumers pay is

$$\frac{d \ln \hat{p}(\tau)}{d\tau} = \frac{\hat{p}'(\tau)}{\hat{p}(\tau)} = \frac{\gamma}{\gamma - \varepsilon} \frac{1}{1 + \tau}$$

We evaluate the above first order condition at $\tau = 0$,

$$\frac{\hat{p}'(0)}{\hat{p}(0)} = \frac{\gamma}{\gamma - \varepsilon}$$

Therefore, the tax burden borne by the producers and consumers relative to the pre-tax price are $\frac{\varepsilon}{\gamma-\varepsilon}$ and $\frac{\gamma}{\gamma-\varepsilon}$ respectively for an ad valorem tax τ , which coincide with the absolute tax burden borne by them for a unit tax t .