

# Recitation 10 (November 5th, 2021)

1. Consider an economy with two individuals  $i = \{1, 2\}$  with the following quasi-linear utility function

$$u_i(s^i, q^i) = v^i(s^i) + \alpha w^i$$

where  $s^i$  denotes the speed at which individual  $i$  drives his car,  $w^i$  is his wealth, and  $\alpha > 0$ . The utility that individual  $i$  obtains from driving fast is  $v^i(s^i)$ , which is increasing but concave in speed, whereby  $\frac{\partial v^i(s^i)}{\partial s^i} > 0$  and  $\frac{\partial^2 v^i(s^i)}{(\partial s^i)^2} < 0$ . Driving fast, however, increases the probability of suffering a car accident, represented by  $\gamma(s^i, s^j)$ . This probability is increasing both in the speed at which individual  $i$  drives,  $s^i$ , and the speed at which other individuals drive,  $s^j$ , where  $j \neq i$ . Hence, the speed of other individuals imposes a negative externality on driver  $i$ , since it increases his risk of suffering a car accident. If individual  $i$  suffers an accident, he bears a cost of  $c^i > 0$ , which intuitively embodies the cost of fixing his car, health-care expenses, etc.

- (a) *Unregulated equilibrium.* Set up individual  $i$ 's expected utility maximization problem. Take first-order conditions with respect to  $s^i$ , and denote the (implicit) solution to this first-order condition as  $\widehat{s}^i$ .
- (b) *Social optimum.* Set up the social planner's expected welfare maximization problem. Take first-order conditions with respect to  $s^1$  and  $s^2$ . Denote the (implicit) solution to this first-order condition as  $\bar{s}^i$ .
- (c) *Comparison.* Show that drivers have individual incentives to drive too fast, relative to the socially optimal speed, i.e., show that  $\widehat{s}^i > \bar{s}^i$ .
- (d) *Restoring the social optimum.* Let us now evaluate the effect of speeding tickets (fines) to individuals driving too fast, i.e., to those drivers with a speed  $\widehat{s}^i$  satisfying,  $\widehat{s}^i > \bar{s}^i$ . What is the dollar amount of the fine  $m^i$  that induces every individual  $i$  to fully internalize the externality he imposes onto others?
- (e) Let us now consider that individuals obtain a utility from driving fast,  $v^i(s^i)$ , only in the case that no accident occurs. Repeat steps (a)-(c), finding the optimal fine  $m^i$  that induces individuals to fully internalize the externality.