Homework # 6 - [Due on November 1st, 2021]

1. Consider a cumulative distribution function $F(x)$ which first-order stochastically dominates $G(x)$.

   (a) Show that the mean of x under $G(x)$, $\int x\, dG(x)$, cannot exceed that under $F(x)$, $\int x\, dF(x)$, i.e.,
   $$\int xdF(x) \geqslant \int xdG(x)$$

   (b) Provide now an example where $\int xdF(x) \geqslant \int xdG(x)$ is satisfied, but $F(x)$ does not first order stochastically dominates $G(x)$.

2. A tax is to be levied on a commodity bought and sold in a competitive market. Two possible forms of tax may be used: In one case, a *per unit* tax is levied, where an amount t is paid per unit bought or sold. In the other case, an *ad valorem* tax is levied, where the government collects a tax equal to τ times the amount the seller receives from the buyer. Assume that a partial equilibrium approach is valid.

   (a) Show that, with a per unit tax, the ultimate cost of the good to consumers and the amounts purchased are independent of whether the consumers or the producers pay the tax. As a guidance, let us use the following steps:

      1. *Consumers:* Let p^c be the competitive equilibrium price when the *consumer* pays the tax. Note that when the consumer pays the tax, he pays $p^c + t$ whereas the producer receives p^c. State the equality of the (generic) demand and supply functions in the equilibrium of this competitive market when the consumer pays the tax.
      2. *Producers:* Let p^p be the competitive equilibrium price when the *producer* pays the tax. Note that when the producer pays the tax, he receives $p^p - t$ whereas the consumer pays p^p. State the equality of the (generic) demand and supply functions in the equilibrium of this competitive market when the producer pays the tax.

   (b) Show that if an equilibrium price p solves your equality in part (a), then $p + t$ solves the equality in (b). Show that, as a consequence, equilibrium amounts are independent of whether consumers or producers pay the tax.

   (c) Show that the result in part (b) is not generally true with an ad valorem tax. In this case, which collection method leads to a higher cost to consumers? [*Hint:* Use the same steps as above, first for the consumer and then for the producer, but taking into account that now the tax increases the price to $(1+\tau)p$. Then, construct the excess demand function for the case of the consumer and the producer.]

(d) Are there any special cases in which the collection method is irrelevant with an ad valorem tax? [*Hint:* Think about cases in which the tax introduces the same wedge on consumers and producers (inelasticity). Then prove your statement by using the above argument on excess demand functions.]

3. In our discussion of perfectly competitive markets, we considered that all firms produced a homogeneous good. However, our analysis can be easily extended to settings in which goods are heterogeneous. In particular, consider that every firm $i \in N$ faces a inverse demand function

$$p_i(q_i, q_{-i}) = \frac{\theta q_i^{\beta-1}}{\sum_{j=1}^{N} q_j^{\beta}}$$

where q_i denotes firm i's output, q_{-i} the output decisions of all other firms, i.e., $q_{-i} = (q_1, ..., q_{i-1}, q_{i+1}, ..., q_N)$, θ is a positive constant, and parameter $\beta \in (0, 1]$ captures the degree of substitutability. In addition, assume that every firm faces the same cost function $c(q_i) = F + cq_i$, where $F > 0$ denotes fixed costs and $c > 0$ represents marginal costs. Find the individual production level of every firm i, q_i^*, as a function of β. Interpret.