

## Homework #5 (Due on October 25th, 2021)

1. Consider an individual with the following utility function, where  $x$  denotes income.

$$u(x) = \begin{cases} 2x & \text{if } x \leq \frac{5}{2} \\ \frac{5}{2} + x & \text{if } x > \frac{5}{2} \end{cases}$$

- (a) Depict the utility function with  $u(x)$  on the vertical axis and income,  $x$ , on the horizontal axis. Show that this individual is (weakly) risk averse.
- (b) Suppose that there are three states of the world, each equally likely. There are two assets,  $x$  and  $y$ . The asset  $x$  is the random variable with payoffs (1, 5, 9) and the asset  $y$  is the random variable with payoffs (2, 3, 10). (Note that assets specify a payoff triple, to indicate the payoff arising in each of the three equally likely states of the world.) Calculate the expected utility of asset  $x$  and of asset  $y$ . Which asset, hence, would be preferred by this individual, if both of them were offered at the same price?
- (c) Calculate the expected *value* of each asset (you previously found the expected *utility*). Calculate the variance of both assets. Which asset would be chosen by this individual if he were variance averse?
- (d) From your previous answers, comment on the validity of the following statement: “Every risk-averse individual is also variance averse”.
2. Consider Tony playing the following lotteries,

$$L = (0.3, 0.5, 0.2)$$

$$L' = (0.6, 0.3, 0.1)$$

$$L'' = (0.25, 0.4, 0.35)$$

which are the probabilities on outcomes 1, 2, and 3 respectively, subject to  $\sum_{i=1}^3 p_i = 1$ .

- (a) Suppose Tony weakly prefers lottery  $L$  to  $L'$  if and only if  $\max_{i \in \{1,2,3\}} p_i \geq \max_{i \in \{1,2,3\}} p'_i$ . Does this extreme preference for certainty violate the IA?
- (b) Suppose Tony weakly prefers lottery  $L$  to  $L'$  if and only if

$$\begin{aligned} p_1 &> p'_1 && \text{or} \\ p_1 &= p'_1 && \text{and } p_2 > p'_2, && \text{or} \\ p_2 &= p'_2 && \text{and } p_3 > p'_3 \end{aligned}$$

Does this lexicographic preference violate the IA?

3. Consider an individual with the following lexicographic preference relation over lotteries: He strictly prefers lottery  $L$  to  $L'$ ,  $L \succ L'$ , if and only if

$$\begin{aligned} p_1 &> p'_1 && \text{or} \\ p_1 &= p'_1 && \text{and } p_2 > p'_2, && \text{or} \\ p_1 &= p'_1, && p_2 = p'_2, && \text{and } p_3 > p'_3, && \text{or} \\ &&& \dots \end{aligned}$$

where outcomes  $i \in \{1, \dots, N\}$  are ranked lexicographically. Illustrate with a numerical example that the lexicographic preference relation satisfies the IA.

4. Consider an individual with the following utility function,

$$u(x) = x^\alpha$$

where  $0 \leq \alpha \leq 1$ , and  $x > 0$  represents the outcome of the individual.

- (a) Find the Arrow-Pratt coefficient of absolute risk aversion,  $r_A(x)$ .
- (b) Suppose the individual has equal chances of obtaining  $x$  and  $3x$ . Find the certainty equivalent of this lottery (you should obtain an expression in terms of  $\alpha$  and  $x$ ).
- (c) Suppose the individual assigns probabilities  $w$  and  $1 - w$  to outcomes  $x$  and  $3x$ , respectively. What are the probabilities that make him enjoy the same utility as the utility of expected value of the lottery presented in part (b)?
- (d) Let  $\alpha = \frac{1}{2}$  and  $x = 1$ . Evaluate the certainty equivalent in part (b) and the probability weights in part (c). Interpret your results.