# Recitation #6 (October 1st, 2021)

1. Consider a firm with production function $q = \sqrt{z}$, using one input (e.g., labor) to produce units of output $q$. The price of every unit of input is $w > 0$, and the price of every unit of output is $p > 0$.

   (a) Set up the firm's profit-maximization problem (PMP), and solve for its unconditional factor demand $z(w, p)$.

   (b) What is the output level that arises from using the amount of inputs $z(w, p)$? Label this output level $q(w)$.

   (c) Set up the firm's cost-minimization problem (CMP), and solve for its conditional factor demand $z(w, q)$ for any output level $q$. (For now, we write the constraint of the CMP to be $f(z) \geq q$, where the output level $q$ that the firm seeks to reach does not necessarily coincide with that found in part (b), $q(w)$.)

   (d) Evaluate the conditional factor demand $z(w, q)$ at output level $q = q(w)$, to obtain $z(w, q(w))$. Show that it coincides with the unconditional factor demand $z(w, p)$ found in part (a), that is,

   $$z(w, q(w)) = z(w, p).$$

   (e) *Shephard's lemma.* Evaluate the CMP's objective function, $w \cdot z$, at the conditional factor demand $z(w, q)$, to obtain the cost function, that is, find $c(w, q) = w \cdot z(w, q)$. Differentiate the cost function with respect to $w$, and show that your result coincides with the conditional factor demand $z(w, q)$.

   (f) *Substitution and output effects.* Let us now consider that the firm faces cheaper wages (lower $w$). Differentiate the unconditional factor demand $z(w, p)$ found in part (a) with respect to $w$ to find the total effect of this price change.

   (g) Differentiate the conditional factor demand $z(w, q)$ found in part (c) with respect to $w$ to obtain the substitution effect of this price change.

   (h) Compare your results in parts (f) and (g). Which is the output effect of the change in $w$?