Recitation #3 - September 10th, 2021

1. [**Relationship between WARP and CLD**] Figure 1 illustrates the change in a decrease in the price of good 1, thus producing an outward pivoting effect on the consumer's budget line, from $B_{p,w}$ to $B_{p',w}$, where the price of good 2 and wealth remain constant. This corresponds to the case where the consumer receives a wealth compensation (changing his wealth level from w to w') that guarantees he can still afford his initial consumption bundle, $x(p,w)$. (This type of wealth compensation is often referred to as the "Slutsky wealth compensation.") Assuming that the Walrasian demand satisfies the Weak Axiom of Revealed Preference (WARP), answer the following questions.

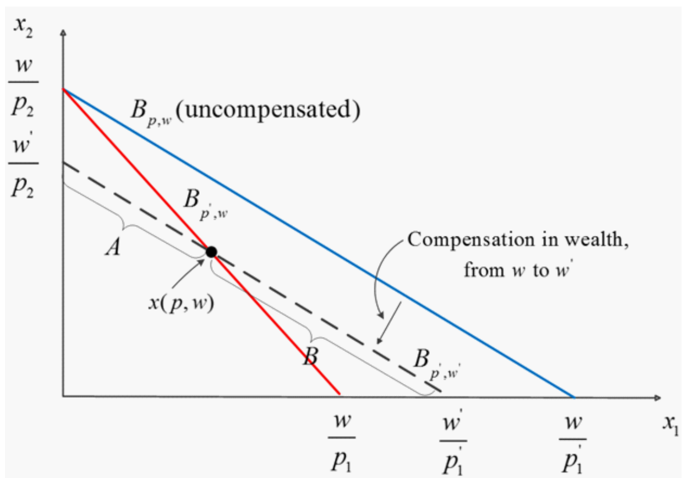

Figure 1. WARP and the Compensated Law of Demand.

(a) Bundle $x(p',w')$ cannot lie on segment A, which is to the left-hand side of bundle $x(p,w)$, but it must lie on segment B, which is to the right-hand side of bundle $x(p,w)$.

- In order to check whether bundle $x(p',w')$ being in segment A or B is compatible with WARP, let us separately assume that it lies in each segment:
- *Segment A.* Let us start checking that $x(p',w')$ cannot lie on segment A. Applying WARP, first note that both bundles $x(p,w)$ and $x(p',w')$ are affordable under initial prices and wealth, $B_{p,w}$, i.e., graphically they both lie on or below budget line $B_{p,w}$ in figure 2.15. However, in the second step of WARP, we see that $x(p,w)$ is affordable under $B_{p',w'}$, i.e., it lies on the dashed budget line $B_{p',w'}$, which constitutes a violation of WARP. Hence, $x(p',w')$ cannot lie on segment A.
- *Segment B.* Let us now check if $x(p',w')$ can lie on segment B. In the first step of WARP, we see that $x(p,w)$ is affordable under initial prices and wealth,

$B_{p,w}$, but $x(p', w')$ is not, i.e., bundle $x(p', w')$ lies strictly above budget line $B_{p,w}$ in figure 2.8. Hence, the premise of WARP does not hold, and as a consequence WARP is not violated if $x(p', w')$ lies on segment B.

(b) What conclusions can you infer from your results in part (a) about the slope of the Walrasian demand function? And the slope of the Hicksian demand function?

- *Hicksian demand.* From the previous result, we can conclude that $x(p', w')$ must contain more of good 1 (note that graphically, bundle $x(p', w')$ lies in segment B, which is to the right-hand side of bundle $x(p, w)$). Then, a decrease in the price of good 1 (when we appropriately compensate for wealth effects) leads to an increase in the quantity demanded. This is the Compensated Law of Demand (CLD), and it implies that the Hicksian (compensated) demand curve must be negatively sloped (a decrease in prices leads to an increase in the consumption of that good).
- *Walrasian demand.* From this result, however, we cannot guarantee that the uncompensated law of demand (ULD) is satisfied. Therefore, we cannot conclude that the Walrasian demand curve (in which wealth effects are left uncompensated) is also negatively sloped. It can be positively or negative sloped, depending on whether the good is Giffen or not, respectively.

2. **[Slutsky matrix]** Consider a setting with three goods ($L = 3$) and a consumer with Walrasian demand function $x(p, w)$ given by

$$x_1(p, w) = \frac{p_2}{p_3}; \quad x_2(p, w) = -\frac{p_1}{p_3}; \quad \text{and} \quad x_3(p, w) = \frac{w}{p_3}$$

(a) Show that the Walrasian demand is homogeneous of degree zero in prices and wealth, (p, w).

- Increasing prices and wealth by a common factor λ, we obtain
  - $x_1(\lambda p, \lambda w) = \frac{\lambda p_2}{\lambda p_3} = \frac{p_2}{p_3} = x_1(p, w)$
  - $x_2(\lambda p, \lambda w) = -\frac{\lambda p_1}{\lambda p_3} = -\frac{p_1}{p_3} = x_2(p, w)$
  - $x_3(\lambda p, \lambda w) = \frac{\lambda w}{\lambda p_3} = \frac{w}{p_3} = x_3(p, w)$

  That is, increasing both prices and wealth by the same factor λ does not change this consumer's demand. Intuitively, if we double the price of all goods but also double his income, the individual's demand is unaffected.

(b) Show that $x(p, w)$ satisfies Walras' law.

- Recall that Walras' Law states that for a strictly positive price vector ($p >> 0$) and a positive wealth level ($w > 0$), $p \cdot x = w$, or alternatively, $\sum_{i=1}^{3} p_i x_i = w$.

Hence, in this context,

$$\sum_{i=1}^{3} p_i x_i = p_1 x_1(p,w) + p_2 x_2(p,w) + p_3 x_3(p,w) =$$
$$= p_1 \frac{p_2}{p_3} + p_2\left(-\frac{p_1}{p_3}\right) + p_3 \frac{w}{p_3}$$

and further rearranging, we obtain

$$\sum_{i=1}^{3} p_i x_i = \frac{p_1 p_2 - p_2 p_1 + p_3 w}{p_3} = w$$

Therefore, Walras' Law is satisfied, confirming that the individual spends all his income on goods 1, 2 and 3.

(c) Show that $x(p,w)$ violates the weak axiom of revealed preference (WARP).

- Let us use a counterexample.

$$\begin{array}{lll} w = 1 & p = (1,1,1) & \text{which yields a demand of } x(p,w) = (1,-1,1) \\ w' = 2 & p' = (1,1,2) & \text{which yields a demand of } x(p',w') = \left(\frac{1}{2}, \frac{-1}{2}, 1\right) \end{array}$$

We know that WARP is satisfied if for any pair of prices and wealth (p,w) and (p',w'),

$$p \cdot x(p',w') \leq w \text{ and } x(p',w') \neq x(p,w) \text{ then } p' \cdot x(p,w) > w'$$

In our example, the bundle that the consumer selects at the final price-wealth pair is affordable under initial prices and wealth,

$$p \cdot x(p',w') = [1,1,1] \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{bmatrix} = \frac{1}{2} - \frac{1}{2} + 1 = 1 \leq w \text{ (since } w = 1)$$

However, the consumption bundle at initial prices and wealth, $x(p,w)$, is affordable under final prices and wealth. In particular,

$$p' \cdot x(p,w) = [1,1,2] \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = 1 - 1 + 2 = 2$$

Hence, since $w' = 2$, bundle $x(p,w)$ is exactly affordable at final prices and wealth, implying that the conclusion of WARP, $p' \cdot x(p, w) > w'$ is *not* satisfied. Therefore, WARP is violated.

(d) Find the Slutsky matrix $S(p, w)$.

- Let us first recall the Slutsky matrix:

$$S(p,w) = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1N} \\ s_{21} & s_{22} & \cdots & s_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ s_{N1} & s_{N2} & \cdots & s_{NN} \end{bmatrix}$$

where every component s_{ik} is defined as

$$s_{ik} = \frac{\partial x_i(p,w)}{\partial p_k} + \frac{\partial x_i(p,w)}{\partial w} x_k(p,w) \qquad \text{(Slutsky equation)}$$

The Slutsky equation informs us about the change in the demand for good i that results from a change in the price of good k once the consumer's wealth is appropriately compensated. Let us now find each of the components of the Slutsky matrix for this particular exercise.

$$\begin{aligned}
s_{11} &= \frac{\partial x_1(p,w)}{\partial p_1} + \frac{\partial x_1(p,w)}{\partial w} x_1(p,w) = 0 + 0 = 0 \\
s_{12} &= \frac{\partial x_1(p,w)}{\partial p_2} + \frac{\partial x_1(p,w)}{\partial w} x_2(p,w) = \frac{1}{p_3} + 0 = \frac{1}{p_3} \\
s_{13} &= \frac{\partial x_1(p,w)}{\partial p_3} + \frac{\partial x_1(p,w)}{\partial w} x_3(p,w) = -\frac{p_2}{p_3^2} + 0 = -\frac{p_2}{p_3^2} \\
s_{21} &= \frac{\partial x_2(p,w)}{\partial p_1} + \frac{\partial x_2(p,w)}{\partial w} x_1(p,w) = -\frac{1}{p_3} + 0 = -\frac{1}{p_3} \\
s_{22} &= \frac{\partial x_2(p,w)}{\partial p_2} + \frac{\partial x_2(p,w)}{\partial w} x_2(p,w) = 0 + 0 = 0 \\
s_{23} &= \frac{\partial x_2(p,w)}{\partial p_3} + \frac{\partial x_2(p,w)}{\partial w} x_3(p,w) = \frac{p_1}{p_3^2} + 0 = \frac{p_1}{p_3^2} \\
s_{31} &= \frac{\partial x_3(p,w)}{\partial p_1} + \frac{\partial x_3(p,w)}{\partial w} x_1(p,w) = 0 + \frac{1}{p_3}\frac{p_2}{p_3} = \frac{p_2}{p_3^2} \\
s_{32} &= \frac{\partial x_3(p,w)}{\partial p_2} + \frac{\partial x_3(p,w)}{\partial w} x_2(p,w) = 0 + \frac{1}{p_3}\left(-\frac{p_1}{p_3}\right) = -\frac{p_1}{p_3^2} \\
s_{33} &= \frac{\partial x_3(p,w)}{\partial p_3} + \frac{\partial x_3(p,w)}{\partial w} x_3(p,w) = -\frac{w}{p_3^2} + \frac{1}{p_3}\frac{w}{p_3} = 0
\end{aligned}$$

Therefore, the Slutsky matrix is

$$S(p,w) = \begin{bmatrix} 0 & \frac{1}{p_3} & -\frac{p_2}{p_3^2} \\ -\frac{1}{p_3} & 0 & \frac{p_1}{p_3^2} \\ \frac{p_2}{p_3^2} & -\frac{p_1}{p_3^2} & 0 \end{bmatrix}$$