Homework #3 (Due on September 20th, 2021)

1. After many years of hard work, your high school teacher has retired and she receives a fixed income $w > 0$ which does not adjust to inflation. Her expenditure function is

$$e(p_1, p_2, u) = (p_1 + p_2)^2 u,$$

where $p_1, p_2 > 0$ denote initial prices. Suppose that prices of goods 1 and 2 increase to $\tilde{p}_1$ and $\tilde{p}_2$, respectively.

   (a) You would like to give her a monetary gift so that she will not be affected by the above price increase. How much money should you give her? That is, find his compensating variation (CV).

   (b) Now find her equivalent variation (EV) from the price change.

   (c) Which is larger in this case, CV or EV?

   (d) Find her Walrasian demand for each good.

   (e) Find her utility function. What is this type of utility function called?

2. Show that the compensating and the equivalent variation coincide when the utility function is quasilinear with respect to the first good (and we fix $p_1 = 1$). [*Hint*: Use the definitions of the compensating and equivalent variations in terms of the expenditure function (not the hicksian demand). In addition, recall that if $u(x)$ is quasilinear with respect to good 1, then we can express it as

$$u(x) = x_1 + \phi\left(x_{-1}\right),$$

where x_{-1} represents all the remaining goods, $l = 2, 3, ..., L$.]

3. Consider a consumer with the following expenditure function

$$e(p, u^0) = g(p) + \left[u^0 \times f(p)\right]$$

where functions $g(p)$ and $f(p)$ depend on the price vector p alone. Show that a 1% increase in wealth leads to exactly a 1% increase in consumption (i.e., the income elasticity, $\varepsilon_{x_i,w}$) converges to one when the consumer's wealth level tends to infinity, i.e., $\lim_{w\to\infty} \varepsilon_{x_i,w} = 1$.

4. Consider an Italian household that is seen to purchase quantities of just two goods, spaghetti and cheese. Denote quantities of spaghetti by x and quantities of cheese by y.

The household comprises two individuals: Alessandro, whose preference relation can be represented by the utility function $u_A(x, y) = x$ (he is a spaghetti lover) and Beatrice, whose preference relation can be represented by the utility function $u_B(x, y) = y$ (she is a cheese lover).

(a) Find the Walrasian demand functions for both Alessandro and Beatrice.

(b) Consider that, upon receiving a household wealth level of w, Alessandro and Beatrice agree on evenly dividing the wealth between them (otherwise, a major Italian fight will ensue!). Suppose that you observe the aggregate demands of this household and you interpret it as if it originated from just a single representative consumer. Find the demands of the representative consumer.

(c) Assume that the Italian government considers introducing a tax on spaghetti in order to maintain the luxury parties some of their politicians are famous for, and they hire you to evaluate the welfare effects associated to this tax. Recall that the equivalent variation of a change in prices, from p^0 to p^1 is defined as:

$$EV = e(p^0, v(p^1, w^0)) - e(p^0, v(p^0, w^0))$$

If the change in prices is caused by the imposition of a commodity tax on spaghetti, then the deadweight loss (DWL), or excess burden of the tax, is given by:

$$DWL = EV - \sum_{l=1}^{L} t_l x_l(p^1, w^0)$$

where t_l represents the price change, i.e., $t_l = p_l^1 - p_l^0$, since the new price captures the initial price plus the tax, $p_l^1 = p_l^0 + t_l$. Briefly explain why this measure may be viewed as a deadweight loss for the Italian society.

(d) Suppose that this Italian household initially faces prices $p^0 = (1, 2)$ and has wealth $w^0 = 300$. In this context, a specific tax of 2 is imposed on spaghetti (i.e., good x) that leads to its price rising to $p_x = 3$ (with the price of cheese, i.e., good y, and the households' wealth both remaining unchanged). Calculate the DWL under the assumption that the household demands originate from one representative consumer.

(e) Without assuming the existence of a representative consumer, use the individuals' indirect utility functions derived in part (a) to calculate the two individual DWLs. Does its sum coincide with the deadweight loss you found in part (d) where we assumed the existence of a representative consumer?