

Homework # 2 EconS501 [Due on September 13th, 2021]

Instructor: Ana Espinola-Arredondo

1. Consider an individual with utility function

$$u(x_1, x_2) = \ln x_1 + x_2,$$

where x_1 and x_2 denote the amounts consumed of non-organic and organic goods, respectively. The prices of these goods are $p_1 > 0$ and $p_2 > 0$, respectively; and this individual's wealth is $w > 0$.

- (a) Find this consumer's uncompensated demand for every good $x_i(p, w)$, where $i = \{1, 2\}$. [For compactness, we use p to denote the price vector $p \equiv (p_1, p_2)$.] Under which conditions the consumer demands positive amounts of both goods? Interpret your results.
 - (b) Find the indirect utility function, $v(p, w)$.
 - (c) Find this consumer's expenditure function, $e(p, v)$, and her compensated demand for every good $h_i(p, w)$, where $i = \{1, 2\}$.
 - (d) Solve parts (a)-(c) of the exercise again, but considering that the consumer's utility function is now $u(x_1, x_2) = (x_1 - a_1)(x_2 - a_2)$, where parameters a_1 and a_2 are both weakly positive, $a_1, a_2 \geq 0$.
2. Consider a consumer with utility function $u(x_1, x_2, x_3) = x_1 x_2 x_3$, and income w .
 - (a) Set up the consumer's utility maximization problem and find the Walrasian demands for each good.
 - (b) Let $x_1 + \frac{p_2}{p_1} x_2 = x_c$ denote the units of a composite good. Set up the consumer's utility maximization problem again, but now in terms of the composite good x_c . Find the Walrasian demand function for the composite good x_c .
 - (c) Show that the Walrasian demands you found in parts (a) and (b) are equivalent.
 3. Consider a consumer with quasilinear utility function $u(x, y, q) = v(x, q) + y$, where x denotes units of good x , q represents its quality, and y reflects the numeraire good (whose price is normalized to 1). The price of good x is $p > 0$, and the consumer's wealth is $w > 0$. Assume that $v_x, v_q > 0$ and $v_{xx} \leq 0$.
 - (a) Set up the consumer's utility maximization problem.

- (b) Show that the Walrasian demand $x(p, q)$ is: (1) decreasing in p ; and (2) increasing in q if $v_{xq} > 0$. Interpret your results.
- (c) Assume in this part of the exercise that $v_{xq} > 0$ so that $\frac{\partial x(p, q)}{\partial q} > 0$. We say that a Walrasian demand $x(p, q)$ is supermodular in (p, q) if the following property holds

$$\underbrace{x(p, q) \frac{\partial^2 x(p, q)}{\partial p \partial q}}_{\text{First term}} - \underbrace{\frac{\partial x(p, q)}{\partial p}}_{(-) \text{ from part (b)}} \underbrace{\frac{\partial x(p, q)}{\partial p}}_{(+ \text{ from part (b)}} > 0.$$

Second term, +

From part (b) we know that $\frac{\partial x(p, q)}{\partial p} < 0$ and that $\frac{\partial x(p, q)}{\partial q}$ is positive. Therefore, for Walrasian demand $x(p, q)$ to be supermodularity we only need that the cross-partial $\frac{\partial^2 x(p, q)}{\partial p \partial q}$ is either positive, entailing an unambiguous expression above, or not very negative, so the positive second term offsets the potentially negative first term. Show that supermodularity holds if $v_{xx}v_{xq} + x(v_{xxx}v_{xq} - v_{xxq}v_{xx}) < 0$. Interpret your results.

4. Consider utility function $u(x, y)$, where x and y represent the units of two goods. Assume that $u(\cdot)$ is twice continuously differentiable, strictly increasing and concave in both of its arguments, x and y . Assuming that the consumer's wealth is given by $w > 0$, and that he faces a price vector $p = (p_x, p_y) \gg 0$, denote his indirect utility function as $v(p, w)$.

- (a) Use the indirect utility function $v(p, w)$ to find the consumer willingness to pay for good y .
- (b) Identify under which condition is this willingness to pay for good y increasing or decreasing in income, w . Interpret.