Recitation #2 (September 3rd, 2021)

1. **[Checking properties of the Cobb-Douglas utility function.]** Consider the utility function
$$u(x) = \prod_{i=1}^{n} x_i^{\alpha_i},$$
where x denotes a vector of n different goods $x \in \mathbb{R}_+^n$, and $1 > \alpha_i > 0$. Check if this utility function satisfies: additivity, homogeneity of degree k, and homotheticity.

2. **[Finding Walrasian demands]** Determine the Walrasian demand $x(p, w) = (x_1(p, w), x_2(p, w))$ and the indirect utility function $v(p, w)$ for each of the following utility functions in $\mathbb{R}_+^2$. Briefly describe the indifference curves of each utility function and find the marginal rate of substitution, $MRS_{1,2}(x)$. Please consider the following two points in your analysis.

   - *Existence:* First, in all three cases the budget set is compact (it is closed, since the bundles in the frontier are available for the consumer, and bounded). Additionally, all utility functions are continuous. Therefore, we can apply Weierstrass theorem to conclude that each of the utility maximization problems (UMPs) we consider has at least one solution.
   - *Binding constraints:* We know that, if preferences are locally non-satiated, then the budget constraint will be binding, i.e., the consumer will be exhausting all his wealth. We can easily check that these utility functions are increasing in both x_1 and x_2, which implies monotonicity and, in turn, entails local non-satiation. Hence, we can assume thereafter that the budget constraint is binding.

   (a) Cobb-Douglas utility function, $u(x) = x_1^3 x_2^4$.

   (b) Preferences for substitutes (linear utility function), $u(x) = 3x_1 + 4x_2$.