Recitation 1, Friday August 27th

1. **Lexicographic preference relation.** Let us define a lexicographic preference relation in a consumption set $X \times Y$, as follows:

$$(x_1, x_2) \succsim (y_1, y_2) \text{ if and only if } \begin{cases} x_1 > y_1, \text{ or if} \\ x_1 = y_1 \text{ and } x_2 \geqslant y_2 \end{cases}$$

Intuitively, the consumer prefers bundle x to y if the former contains more units of the first good than the latter, i.e., $x_1 > y_1$. However, if both bundles contain the same amounts of good 1, $x_1 = y_1$, the consumer ranks bundle x above y if the former has more units of good 2 than the latter, i.e., $x_2 \geq y_2$. For simplicity, assume that both components have been normalized to $X = [0, 1]$ and $Y = [0, 1]$.

(a) Show that the lexicographic preference relation satisfies rationality (i.e., it is complete and transitive).

(b) Show that the lexicographic preference relation $\succsim$ *cannot* be represented by a utility function $u : X \times Y \to \mathbb{R}$.

(c) Assume now that this preference relation is defined on a *finite* consumption set $X = X_1 \times X_2$, where $X_1 = \{x_{11}, x_{12}, ..., x_{1n}\}$ and $X_2 = \{x_{21}, x_{22}, ..., x_{2m}\}$. [*Hint*: You can define a function $N_i(x_{ij})$ as the number of elements in sequence X_i prior to element x_{ij}; that is,

$$N_i\left(x_{ij}\right) = \#\left\{y \in X_i | y < x_{ij}\right\}.$$

Then define a utility function

$$u(y_1, y_2) = mN_1(y_1) + N_2(y_2), \text{ where } m > 0,$$

and for any pair $(y_1, y_2) \in X_1 \times X_2$.]