- Game Theory as the study of interdependence
  - "No man is an island"
- Definition:
  - Game Theory: a formal way to analyze **interaction** among a **group** of **rational** agents who behave strategically.

- Several important elemants of this definition help us understand what is game theory, and what is not:
- **Interaction:** If your actions do not affect anybody else, that is not a situation of interdependence.
- **Group:** we are not interested in games you play with your imaginary friend, but with other people, firms, etc.
- **Rational agents:** we assume that agents will behave rationally especially if the stakes are high and you allow them sufficient time to think about their available strategies.
  - Although we mention some experiments in which individuals do not behave in a completely rational manner...
  - these "anomalies" tend to vanish as long as you allow for sufficient repetitions, i.e., everybody ends up learning, or you raise stakes sufficiently (high incentives).

Examples (1):

- Output decision of two competing firms:
  - Cournot model of output competition.
- Research and Development expenditures:
  - They serve as a way to improve a firm's competitiveness in posterior periods.
- OPEC pricing, how to sustain collusion in the long run...

Examples (2):

- Sustainable use of natural resources *and* overexploitation of the common resource.
- Use of environmental policy as a policy to promote exports.
  - Setting tax emission fees in order to favor domestic firms.
- Public goods (everybody wants to be a "free-rider").
  - I have never played a public good game!
  - Are you sure? A group project in class. The slacker you surely faced was our "free-rider."

Rules of a General Game (informal):(WATSON CH.2,3)

The rules of a game seek to answer the following questions:

1. Who is playing ?$\leftarrow$ set of players (I)
2. What are they playing with ?$\leftarrow$ Set of available actions (S)
3. Where each player gets to play ?$\leftarrow$ Order, or time structure of the game.
4. How much players can gain (or lose) ?$\leftarrow$ Payoffs (measured by a utility function $U_i(s_i, s_{-i})$

1. We assume **Common knowledge** about the rules of the game.

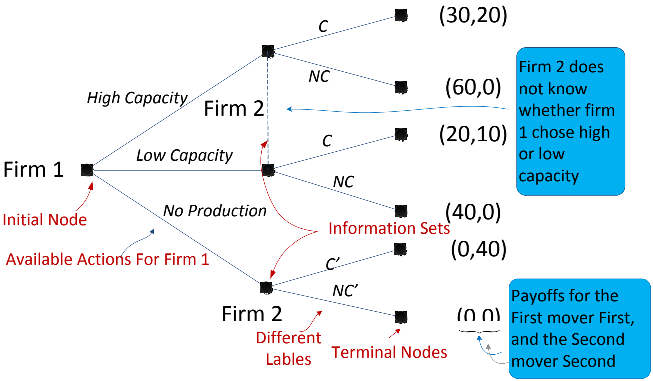
- As a player, I know the answer to the above four questions (rules of the game)
- In addition, I know that you know the rules, and...
- that you know that I know that you know the rules,.....(ad infinitum).

Two ways to graphically represent games

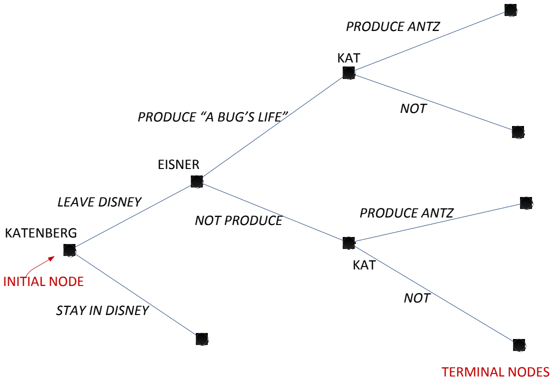
- Extensive form
  - We will use a game tree (next slide).
- Normal form (also referred as "strategic form").
  - We will use a matrix.

Example of a game tree

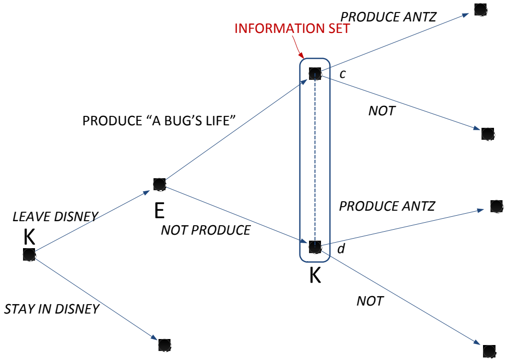
- Consider the following sequential-move game played by firms 1 and 2:
  - We will use a matrix



"ANTZ" vs. "A BUG'S LIFE"



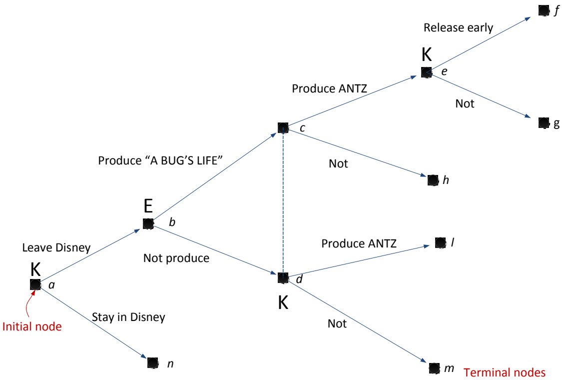
- In this example, Katsenberg observes whether Eisner produced the film "A BUG"S LIFE" or not before choosing to produce "ANTZ".



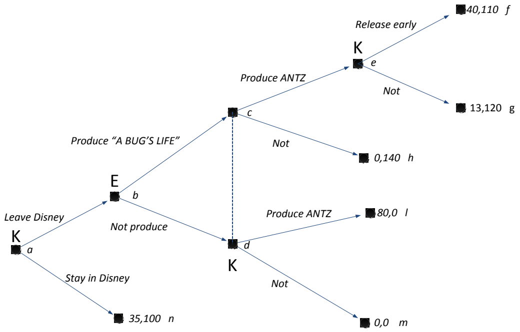
- When Katsenberg is at the move (either at node c or d), he knows that he is at one of these nodes,but he does not know at which one *and* the figure captures this lack of information with a dashed line connecting the nodes.:

The Bug Game

- We now add an additional stage at the end at which Katsenberg is allowed to release "Antz" early in case he produced the movie and Eisner also produced "A bug's life" (at node e).



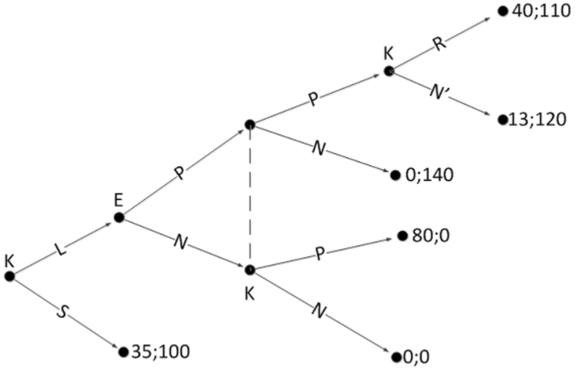
The Extensive Form of The Bug Game



- Let's define the payoff numbers as the profits that each obtains in the various outcomes, i.e.,in each terminal node.
- For example, in the event that Katzenberg stays at Disney, we assume he gets $35 million and Eisner gets $100 million (terminal node a).

The Bug Game Extensive Form (Abbreviating Labels)

- We often abbreviate labels in order to make the figure of the game tree less jammed, as we do next.



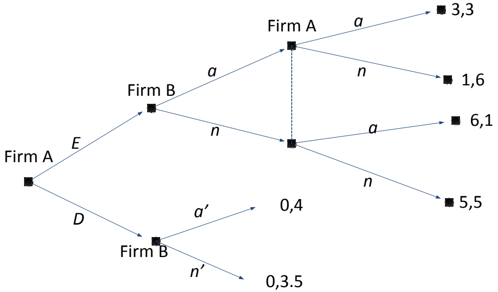
Information sets

- An information set is graphically represented with two or more nodes connected by a dashed line, (or a "sausage") including all these connected nodes.
- It represents that the player called to move at that information set cannot distinguish between the two or more actions chosen by his opponent before he is called to move.
- Hence, the set of available actions must be the same in all the nodes included on that information set (P and N in the previous game tree for Katsenberg).
  - Otherwise, Katsenberg, despite not observing Eisner's choice, would be able to infer it by analyzing which are the available actions he can choose from.

Guided exercise (page 19-20 in Watson)

- **Lets practice how to depict a game tree of a strategic situation on an industry:**
- Firm A decides whether to enter firm B's industry. Firm B observes this decision.
  - If firm A stays out, firm B alone decides whether to advertise. In this case, firm A obtains zero profits, and firm B obtains $4 million if it advertises and $3.5 million if it does not.
  - If firm A enters, both firms simultaneously decide whether to advertise, obtaining the following payoffs.
    - If both advertise, both firms earn $3 million.
    - If none of them advertise, both firms earn $5 million.
    - If only one firm advertises, then it earns $6 million and the other firm earns $1 million.

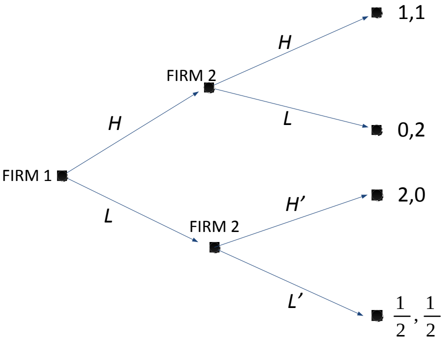
Guided Exercise, (continued)



- Let E and D denote firm A's initial alternatives of entering and not entering B's industry.
- Let a and n stand for "advertise" and "not advertise", respectively.
- Note that simultaneous advertising decisions are captured by firm A's information set.

Strategy: Definition of Strategy

- Lets practice finding the strategies of firm 1 and 2 in the following game tree:
  - We will use a matrix



Strategies for firm 1 : H and L.
Strategies for firm 2 : H. H';H. L';L. H;L

Strategy space and Strategy profile

- **Strategy space:** It is a set comprising each of the possible strategies of player i.
  - From our previous example:
    - $S_1 = \{H, L\}$ for firm 1
    - $S_2 = \{HH', HL', LH', LL'\}$ for firm 2.
- **Strategy profile**
  - It is a vector (or list) describing a particular strategy for every player in the game. For instance, in a two-player game

$$s = (s_1, s_2)$$

    where s_1 is a specific strategy for firm 1.(for instance, $s_1 = H$), and s_2 is a specific strategy for firm 2, e.g., $s_2 = LH'$.
  - More generally, for N players, a strategy profile is a vector with N components,

$$s = (s_1, s_2, s_3, ..., s_n)$$

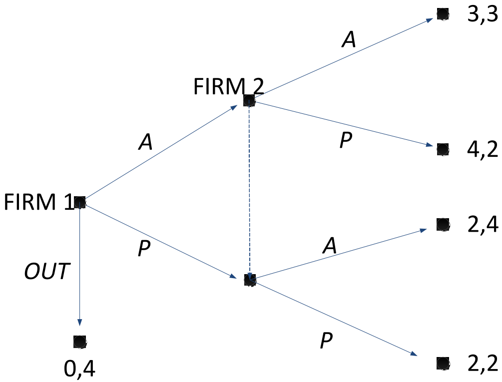
Strategy profile:

- In order to represent the strategies selected by all players except player i, we write:

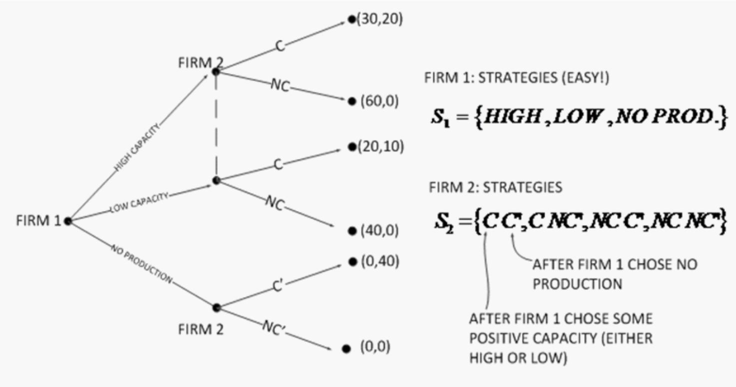
$$s_{-i} = (s_1, s_2, ..., s_{i-1}, s_{i+1}, ..., s_n)$$

  (Note that these strategies are potentially different)
- We can hence write, more compactly, as strategy profile with only two elements:
  The strategy player i selects, s_i, and the strategies chosen by everyone else, s_{-i}, as : $s = (s_i, s_{-i})$
- **Example:**
  - Consider a strategy profile s which states that player 1 selects B, player 2 chooses X, and player 3 selects Y, i.e., $s = (B, X, Y)$.Then,
    - $s_{-1} = (X, Y)$,
    - $s_{-2} = (B, Y)$, and
    - $s_{-3} = (B, X)$.

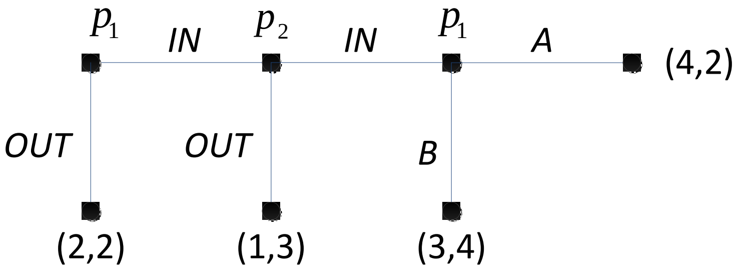
- Lets practice finding strategy sets in the following game tree:



- Let's define firm 1 and 2's available strategies in the first example of a game tree we described a few minutes ago:



ANOTHER EXAMPLE: THE CENTIPEDE GAME:



- Strategy set for player 2 : $S_2 = \{IN, OUT\}$
- Strategy set for player 1 : $S_1 = \{IN\ A, IN\ B, OUT\ A, OUT\ B\}$
- More examples on page 27 (Watson)

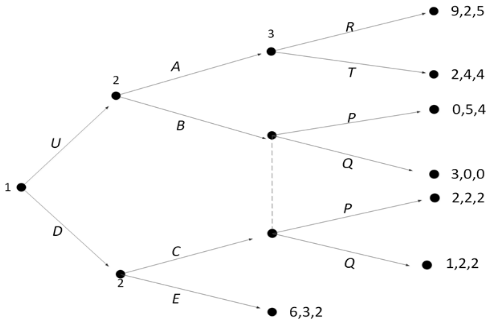
One second...

- Why do we have to specify my future actions after selecting "out" ? Two reasons:

1. Because of potential mistakes:
    - Imagine I ask you to act on my behalf, but I just inform you to select "out" at the initial node. However, you make a mistake (i.e., you play "In"), and player 2 responds with "In" as well. What would you do now??
    - With a strategy (complete contingent plan) you would know what to do even in events that are considered off the equillibrium path.
2. Because player 1's action later on affects player 2's actions, and ...
    - ultimately player 2's actions affects player 1's decision on whether to play "In" or "Out" at the beginning of the game.
    - This is related with the concept of backwards induction that we will discuss when solving sequential-move games.)

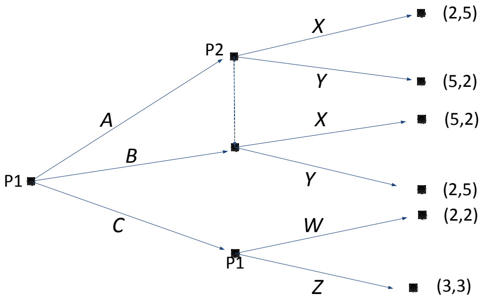
Some extensive-form games

- Let's now find the strategy spaces of a game with three players:



- S_1 = {U, D}
- S_2 = {AC, AE, BC, BE}; and
- S_3 ={RP,RQ,TP,TQ}

Some extensive-form games (Cont'l)



- $S_1 = \{AW, BW, CW, AZ, BZ, CZ\}$
- $S_2 = \{X, Y\}$

- **When a game is played simultaneously, we can represent it using a matrix**
  - *Example:* Prisoners' Dilemma game.

		Prisoner 2	
		Confess	Don't Confess
Prisoner 1	Confess	$-5, -5$	$0, -15$
	Don't Confess	$-15, 0$	$-1, -1$

- **Another example of a simultaneous-move game**
    - The "battle of the sexes" game.(I know the game is sexist, but please don't call it the "battle of the sexist" game !)

		Wife	
		Opera	Movie
Husband	Opera	1, 2	0, 0
	Movie	0, 0	2, 1

- **Yet, another example of a simultaneous-move game**
  - Pareto-coordination game.

		Firm 2	
		Superior tech.	Inferior tech.
Firm 1	Superior tech.	2, 2	0, 0
	Inferior tech.	0, 0	1, 1

- **Yet, another example of a simultaneous-move game**
  - The game of "chicken."

		Dean	
		Straight	Swerve
James	Straight	0, 0	3, 1
	Swerve	1, 3	2, 2

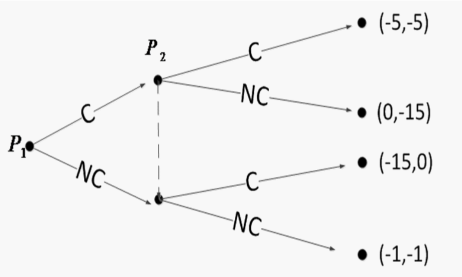
Other examples of the "Chickengame"

Mode	Description
Trackors	Footloose, (1984,Movie)
Bulldozers	Buster and Gob in Arrested Development (2004,TV)
Wheelchairs	Two old ladies with motorized wheelchairs in Banzai(2003,TV)
Snowmobiles	"[Two adult males] died in a head-on collision, earning a tie in the game of chicken they were playing with their snowmobiles" <www.seriouslyinternet.com/278.0.html>
Film Release Dates	Dreamworks and Disney-PIxar (2004)
Nuclear Weapons	Cuban Missile Crisis (1963)

Normal (Strategic) Form

- We can alternatively represent simultaneous-move games using a game tree, as long as we illustrate that players choose their actions without observing each others' moves, i.e., using information sets, as we do next for the prisoner's dilemma game:
- Extensive form representation of the Prisoner's Dilemma game :

		P_2	
		C	NC
P_1	C	-5,-5	0,-15
	NC	-15,0	-1,-1



- **Practice** :Using a game tree, depict the equivalent extensive - form representation of the following matrix representing the "Battle of the Sexes" game.

		Wife	
		Opera	Movie
Husband	Opera	1,2	0,0
	Movie	0,0	2,1

Strategic Dominance

Strategic Dominance

- The first solution concept: equilibrium dominance.
- Strict dominance. Player i finds that strategy s_i *strictly dominates* another strategy s_i' if choosing s_i provides her with a strictly higher payoff than selecting s_i', regardless of her rivals' strategies.
  - s_i is a "strictly dominant strategy" when strictly dominates s_i'.
    - A strictly dominant strategy provides player i with an unambiguously higher payoff than any other available strategy.
  - s_i' is "strictly dominated" by s_i.
    - A strictly dominated strategy gives player i a strictly lower payoff.

Strategic Dominance

- Tool 12.1. *How to find a strictly dominated strategy*:
    1. Focus on the row player by fixing attention on one strategy of the column player.
        a) Cover with your hand all columns not being considered.
        b) Find the highest payoff for the row player by comparing, across rows, the first component of every pair.
        c) Underline this payoff.
    2. Repeat step 1, but fix you attention on a different column.
    3. If, after repeating step 1 enough times, the highest payoff for the row player always occurs at the same row, this row becomes her dominant strategy.
    4. For the column player, the method is analogous, but now fix your attention on one strategy of the row player.

Strategic Dominance

- *Example 12.1: Finding strictly dominant strategies.*
  - Consider matrix 12.2a with 2 firms simultaneously and independently choosing a technology:

		Firm 2	
		Tech a	Tech b
Firm 1	Tech A	5,5	2,0
	Tech B	3,2	1,1

Matrix 12.2a

  - Technology A is strictly dominant for firm 1 because it yields a higher payoff than B, both
    - when firm 2 chooses a because $5 > 3$; and
    - when it selects b given that $2 > 1$.

Strategic Dominance

- *Example 12.1* (continued):

		Firm 2	
		Tech a	Tech b
Firm 1	Tech A	5,5	2,0
	Tech B	3,2	1,1

Matrix 12.2a

- Technology a is strictly dominant for firm 2 because it provides a higher payoff than b, both
  - when firm 1 chooses A because $5 > 0$; and
  - when it selects B given that $2 > 1$.
- The equilibrium of this game is (A, a).

Strategic Dominance

- The definition of strict dominance does not allow for ties in the payoffs that firm i earns.
- Weak dominance. Player i finds that strategy s_i *weakly dominates* another strategy s_i' if choosing s_i provides her with a strictly higher payoff than selecting s_i' for at least one of her rivals' strategies, but provides the same payoff as s_i' for the remaining strategies of her rivals.
  - A weakly dominant strategy yields the same payoff as other available strategies, but a strictly higher payoff against at least one strategy of the player's rivals.

Strategic Dominance

- Consider matrix 12.2b:

		Firm 2 Tech a	Tech b
Firm 1	Tech A	5,5	2,0
	Tech B	3,1	2,1

Matrix 12.2b

- Firm 1 finds that technology A weakly dominates B because
  - A yields a higher payoff than B against a, $5 > 3$; but
  - provides firm 1 with exactly the same payoff as B, \$2, against b.
- Firm 2 finds that technology a weakly dominates b because
  - a yields a higher payoff than b against A, $5 > 0$; but
  - generates the same payoff as b, \$1, when firm 1 chooses B.

Strategic Dominance

- In matrices with more than 2 rows and/or columns, finding strictly dominated strategies is helpful.
- We can delete those strategies (rows or columns) because the player would not choose them.
- Once we have deleted the dominated strategies for one player, we can move to another player and do the same, and subsequently move on to another player.

Strategic Dominance

- This process is known as Deletion of Strictly Dominated Strategies (IDSDS).
- Once we cannot find any more strictly dominated strategies for either player, we are left with the equilibrium prediction.
- IDSDS can yield to multiple equilibria.

Strategic Dominance

- *Example 12.2: When IDSDS does not provide a unique equilibrium.*
  - Consider matrix 12.3 representing the price decision of two firms:

		Firm 2		
		High	Medium	Low
Firm 1	High	2,3	1,4	3,2
	Medium	5,1	2,3	1,2
	Low	3,7	4,6	5,4

Matrix 12.3

  - For firm 1, High is strictly dominated by Low because High yields a lower payoff, regardless of the price chosen by firm 2. We can delete High from firm 1's rows, resulting in the reduced matrix 12.4.

Strategic Dominance

- *Example 12.2* (continued):

Firm 2

		High	Medium	Low
Firm 1	Medium	5,1	2,3	1,2
	Low	3,7	4,6	5,4

Matrix 12.4

  - For firm 2, Low is strictly dominated by Medium because Low yields a strictly than Medium, regardless of the row that firm 1 selects.
  - After deleting the Low column from firm 2's strategies, we are left with a further reduced matrix (matrix 12.5).
  - We can now move again to analyze firm 1.

Strategic Dominance

- *Example 12.2* (continued):

		Firm 2	
		High	Medium
Firm 1	Medium	5,1	2,3
	Low	3,7	4,6

Matrix 12.5

- We cannot find any more strictly dominated strategies for firm 1 because there is no strategy (no row) yielding a lower payoff, regardless of the column player 2 plays.
    - Firm 1 prefers Medium to Low if firm 2 chooses High because $5 > 3$; but
    - it prefers Low if firm 2 chooses Medium given that $4 > 2$.
- A similar argument applies to firm 2.

Strategic Dominance

- *Example 12.2* (continued):

Firm 2

Firm 1		High	Medium
	Medium	5,1	2,3
	Low	3,7	4,6

Matrix 12.5

- The remaining four cells in this matrix constitute the most precise equilibrium prediction after applying IDSDS.
- This is one of the disadvantages of IDSDS as solution concept.
- In some games IDSDS "does not have a bite" because it does not help to reduce the set of strategies that a rational player would choose in equilibrium.

Strategic Dominance

- *Example 12.3: When IDSDS does not have a bite.*
  - Matrix 12.6 represents the Matching Pennies game.

Player 2

		Heads	Tails
Player 1	Heads	$1,-1$	$-1,1$
	Tails	$-1,1$	$1,-1$

Matrix 12.6

  - Player 1 does not find any strategy strictly dominated:
    - She prefers Heads when player 2 chooses Heads, but Tails when player 2 chooses Tails.
  - A similar argument applies to player 2.
  - No player has strictly dominated strategies. IDSDS has "no bite."