Midterm #2 - EconS 424

Due on April 9th, 2021 at 4.00pm,

Name:________________________________

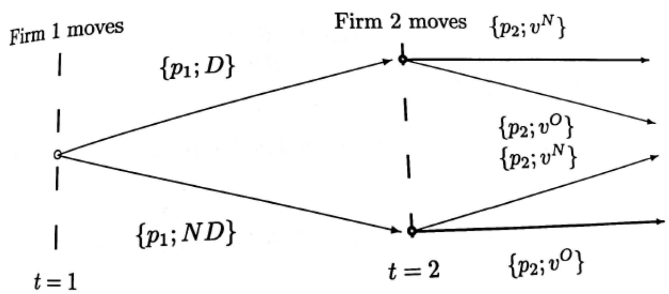
1. Analyze a two-period model for the market of computers in which two firms operate. Firm 1 only produces in period 1 and is endowed with an old technology providing a quality level v^O to consumers. Firm 2 is a potential entrant in period 2 and it is able to produce an old technology, v^O, and a new technology, v^N. However, the production of new technology requires an innovation cost of $I > 0$. Note that old and new technology can be nondurable (only last one period) or durable (it lasts for two periods). Hence, the cost of producing nondurable technology, $c^{ND} = 0$, is considerably lower than the cost of durable technology, $C^D = 3$. There is only one consumer in period 1 who seeks to buy a computer for the two periods of her life. In period 2, one additional consumer enters the market and seeks to buy a computer. Both consumers have the same gain from the quality of the technology embedded into the product they buy in period t. That is, $V^N = 7$ and $V^O = 5$ for new and old technology, respectively. The structure of the two-period, two-firm game is as follows: In period 1 firm 1 sells the old technology product and therefore has to decide which price to charge (p_1) and whether to produce a durable (D) or a nondurable (ND) product. In the second period, firm 2 obviously chooses to produce a nondurable good (since the world ends at the end of period 2) and, hence, has to decide whether to invest in adopting the newer technology and price (p_2).

   (a) Illustrate the extended form game of the two-period, two-firm game.

   (b) Describe the second-period pricing, first, for the case in which the first-period product is nondurable and, second, for the case in which it is durable.

   (c) Identify the first period durability choice.

Answer
Part (a)



Part (b)

Second-period pricing, given that the first-period production is nondurable

In the second period firm 2 offers either the old-technology v^O product for sale, or invests I for the adoption of its new-technology v^N product. The pricing and innovation decision of firm 2 are summarized by

$$p_2 = \begin{cases} v^N \text{ if } 2(v^N - v^O) \geq I \\ v^O \text{ if } 2(v^N - v^O) < I \end{cases}$$

$$\pi_2 = \begin{cases} 2v^N - I \text{ if } 2(v^N - v^O) \geq I \\ 2v^O \text{ if } 2(v^N - v^O) < I \end{cases}$$

That is, when firm 1 produces a ND good in period 1, then in period 2 both the old and the new consumers seek to purchase the product. If the innovation cost is sufficiently low, firm 2 invests in the improved technology and sells it to the old and new consumers. However, if I is high, firm 2 sells the old technology to both the old and the new consumers.

Second-period pricing, given that the first-period product is durable

Suppose that firm 1 sells a durable good in period 1. Then, in period 2, the old consumer already posses the v^O technology product. In this case, firm 2 has two possibilities: It can price its new-technology product low enough at $p_2^L = v^N - v^O$, which induces the old consumer to discard his old-technology durable product and purchase the new good v^N; in this case $\pi_2^L = 2(v^N - v^O) - I$. Or, it can price it high at $p_2^H = v^N$, so that only the new consumer purchases the new-technology product, while the old keeps using the old durable product. In this case, $\pi_2^H = v^N - I$. Comparing both profits yields:

If $v^N > 2v^O$ firm 2 sells its new-technology product to both the old and new consumers;
If $v^N < 2v^O$ firm 2 sells its new-technology product to new consumer only;.

Part (c)
In period $t = 1$, firm 1 chooses a price p_1 and whether to produce a durable or a nondurable. If firm 1 sells a nondurable, the maximum price it can charge for selling a one period of the product service is $p_1^{ND} = v^O$. In this case, $\pi_1^{ND} = v^O - c^{ND} = v^O$. In contrast, if firm 1 sells durable, the maximum it can charge is $p_1^D = 2v^O$, since the product provides a service for two periods. In this case, $\pi_1^D = 2v^O - c^D$. Comparing both profits yields

Firm 1 produces a durable if $v^O > c^D$. Otherwise, it produces a nondurable.

2. Consider a two-stage game in which at $t = 1$, firms determine (first noncooperatively and then cooperatively) how much to invest in cost-reducing R&D and at $t = 2$, firms are engaged in a Cournot quantity game. Firms produce a homogeneous product, where the demand function is given by $p = 125 - Q$. In addition, x_i denotes the amount of R&D undertaken by firm i, $i = 1, 2$, and the unit production cost of firm i is $c_i(x_1, x_2) \equiv 30 - x_i - 0.6x_j$. Finally, R&D is costly to firms $TC(x_i) = \frac{x_i^2}{2}$.
   (a) Identify the noncooperative equilibrium R&D level invested by each firm when firms do not cooperate.
   (b) Identify the cooperative equilibrium R&D level invested by each firm when firms cooperate
   (c) Compare the industry's R&D levels under noncooperative and cooperative R&D. Discuss how your results change with different values of β.

Answer
Part a. *Noncooperative R&D.*
Second Period. The Cournot profit levels are given by

$$\pi_i(c_1, c_2) = \frac{(a - 2c_i + c_j)^2}{3b} = \frac{(125 - 2c_i + c_j)^2}{9}$$

First Period. Each firm noncooperatively chooses its level of R&D given the R&D of the rival firm.

$$\begin{aligned} \max_{x_i} \pi_i &= \frac{1}{9}\left[125 - 2(30 - x_i - 0.6x_j) + 30 - x_j - 0.6x_i\right]^2 - \frac{x_i^2}{2} \\ &= \frac{1}{9}\left[125 - 30 + 1.4x_i + 0.2x_j\right]^2 - \frac{x_i^2}{2} \end{aligned}$$

The F.O.C yields

$$0 = \frac{\partial \pi_i}{\partial x_i} = \frac{2}{9}\left[95 + 1.4x_i + 0.2x_j\right] 1.4 - x_i$$

Given that the payoff functions are symmetric between firms, we look for a symmetric NE where $x_1 = x_2 \equiv x^{nc}$

$$x^{nc} = \frac{95(1.4)}{4.5 - (1.4)(1.6)} = 58.85$$

Part b. *Cooperative R&D.*
The firms seek to jointly choose x_1 and x_2 to

$$\max_{x_1, x_2} (\pi_1 + \pi_2)$$

F.O.C are given by

$$0 = \frac{2}{9}\left[95 + 1.4x_i + 0.2x_j\right]1.4 - x_i + \frac{2}{9}\left[95 + 1.4x_j + 0.2x_i\right]0.2$$

Assuming that second order conditions for a maximum are satisfied, the F.O.C yield the cooperative R&D level:

$$x_1^c = x_2^c = x^c = \frac{95(1.6)}{4.5 - (1.6)^2} = 78.35$$

Part c. From our above results we know that $x^c > x^{nc}$. However, if the R&D spillover effect is small (lower values of, i.e. $\beta < 0.5$), then $x^c < x^{nc}$.

3. Let us consider an industry composed by three firms x, y and z. Assume that firms have the same cost structure, that is, $C(q_i) = 10 + 4q_i$, where q_i denotes firm i's output and $i = \{x, y, z\}$. In addition, the industry demand is $P(Q) = 20 - Q$, where Q denotes the aggregate output, where $Q = q_x + q_y + q_z$. The structure of the game is the following: (i) Firm x is the first mover of the game and it chooses its output level; (ii) firm y is the second mover of this game, and after observing q_x, firm y chooses its own output, q_y; (iii) finally, after observing both firms output, firm z chooses its output q_z. The timing of production, industry demand and cost function are common knowledge among firms. Find values of output level q_x, q_y, q_z in the SPNE of the game.

Answer

Firm z. Using backward induction, we first analyze the production decision of the last mover firm 3, which maximizes profits by solving:

$$\max(20 - (q_x + q_y + q_z))q_z - (10 + 4q_z)$$

Taking first order conditions with respect to , we obtain

$$20 - q_x - q_y - 2q_z - 4 = 0$$

Hence, solving for q_z we obtain firm z's best response function (which decreases in both q_x and q_y)

$$q_z(q_x, q_y) = 8 - \frac{1}{2}(q_x + q_y)$$

Firm y. Given this $BRF_z(q_x, q_y)$, we can now examine firm y's production decision (the second mover in the game), which chooses an output level q_y to solve

$$\max(20 - (q_x + q_y + [8 - \frac{1}{2}(q_x + q_y)]))q_y - (10 + 4q_y)$$

where we inserted firm z's best response function, since firm y can anticipate firm z's optimal response at the subsequent stage of the game. Simplifying this profit, we find

$$\max(12 - (\frac{1}{2}(q_x + q_y))q_y - (10 + 4q_y)$$

Taking first order conditions with respect to q_y, yields

$$12 - \frac{1}{2}q_x - q_y - 4 = 0$$

And solving for q_y we find firm y's best response function (which only depends on the production that occurs before its decision, i.e.,),

$$q_y(q_x) = 8 - \frac{1}{2}q_x$$

Firm x. We can finally analyze firm x's production decision (the first mover). Taking into account the output level with which firm y and z will respond, as described by $BRF_z(q_x, q_y)$ and $BRF_y(q_x)$, respectively, firm x maximizes

$$\max(20-(q_x+[8-\frac{1}{2}q_x]+[8-\frac{1}{2}(q_x+[8-\frac{1}{2}q_x])]))q_x-(10+4q_x)$$

which simplifies to

$$\max(8-\frac{1}{4}q_x)q_x-(10+4q_x)$$

Taking first order conditions with respect to q_x, we obtain

$$8-\frac{1}{2}q_x-4=0$$

Solving for q_x we find the equilibrium production level for the leader (firm x), . Therefore, we can plug into firm y's and z's best response function to obtain

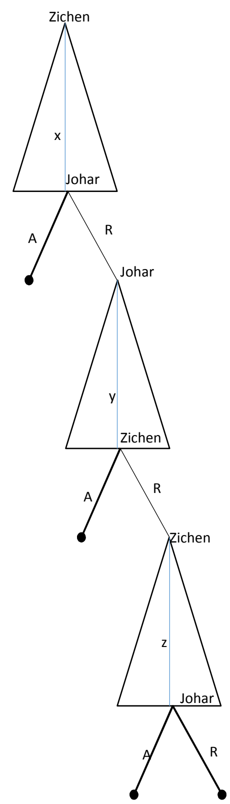
$$q_x^*=8,\ q_z^*=4 \text{ and } q_z^*=2$$

However, the SPNE of this Stackelberg game must specify optimal actions for all players, both along the equilibrium path (for instance, after firm y observes firm x producing $q_x^S=8$ units) and off-the-equilibrium path (for instance, when firm y observes firm x producing $q_x^S\neq 8$ units). We can accurately represent this optimal action at every point in the game in the following SPNE:

$$(q_x^S,q_y(q_x),q_z(q_x,q_y))=(8,8-\frac{1}{2}q_x,8-\frac{1}{2}(q_x+q_y))$$

4. Two friends (Johar and Zichen) are trying to split \$1,000. In the first round of bargaining, Zichen makes an offer at cost z (to herself), proposing to keep x_Z and give the remaining to Johar, $x_J=1,000-x_Z$. Johar either accepts her offer (ending the game) or rejects it. In round 2, Johar makes an offer of (y_Z,y_J), at a cost of 5 to himself, which Zichen accepts or rejects. If Zichen accepts the offer, the game ends; but if she rejects it, the game proceeds to the third round, in which Zichen makes an offer (z_Z,z_J), at a cost z to herself. If Johar accepts her offer in the third round, the game ends and payoffs are accrued to each player; whereas if he rejects it, the money is lost. Assume that players are risk-neutral (utility is equal to money obtained minus any costs), the discount factor is denoted by δ and $z=0$.

(a) Graphically represent this bargaining game.



(b) What is the subgame-perfect equilibrium outcome?

Answer

Third period. Operating by backward induction, we start in the last round of the negotiation (third round). In , Zichen will offer herself the maximum split that still guarantees that it is accepted by Johar:

$$\begin{aligned} u_J(Acceptt_3) &= u_J(rejectt_3) \\ 1000 - z_Z - 5 &= -5,\ z_Z = 1000 \end{aligned}$$

That is, Zichen offers $(z_Z, z_J) = (1000, 0)$ in $t = 3$.

Second period. In $t = 2$, Johar needs to offer Zichen a split that makes her indifferent between accepting in $t = 2$ and rejecting in order to offer herself the entire pie in $t = 3$(which we showed to be her equilibrium behavior in the last round of play). In particular, Zichen is indifferent between accepting and rejecting if

$$\begin{aligned} u_Z(Acceptt_2) &= u_Z(rejectt_2) \\ 1000 - y_J &= \delta 1000,\ y_J = 1000(1 - \delta) \end{aligned}$$

That is, Johar offers $(y_Z, y_J) = (1 - 1000(1 - \delta), 1000(1 - \delta))$ in $t = 2$.

First period. In $t = 1$, Alice needs to offer Johar a split that makes him indifferent between accepting in $t = 1$ and rejecting (which entails that he will offer himself $y_J = 1000(1 - \delta)$ in the subsequent period, $t = 2$):

$$\begin{aligned} u_J(Acceptt_1) &= u_J(rejectt_1) \\ 1000 - x_Z - 5 &= 1000\delta(1 - \delta) - 5,\ x_Z = 1000(1 - \delta(1 - \delta)) \end{aligned}$$

That is, Zichen offers $(x_Z, x_J) = (1000(1-\delta(1-\delta)), (1-1000(1-\delta(1-\delta))))$ in $t = 1$. So the agreement is reached immediately.

5. Consider the case in which you are trying to buy a house and you are bargaining with the current owner over the sale price. The house is of value $200,000 to you and $100,000 to the current owner. hence, if the price is between $100,000 and $200,000 then you would both better off with the sale. Assume that bargaining takes place with alternative offers and that each stage of bargaining (an offer and response) takes a full day to complete. If agreement is not reached after 10 days of bargaining, then the opportunity for the sale disappears. Suppose that you and the current owner discount the future according to the discount factor δ per day. The real estate agent has allowed you to decide wether you will make the first offer.

   (a) Suppose that δ is small; in particular $\delta < 0.5$. Should you make the first offer or let the current owner make the first offer? Why?

   (b) Suppose δ is close to 1; in particular $\delta > 0.93$. Should you make the first offer or let or let the current owner make the first offer? Why?

Answer

Part (a). Here you should make the first offer, because the current owner is very impatient and will be quite willing to accept a low offer in the first period. More precisely, since $\delta < 0.5$ the responder in the first period prefers accepting less than one-half of the surplus in the second period. Thus, the offer in the first period will get more than half of the surplus.

Part (b). In this case, you should make the second offer, because you are patient and would be willing to wait until the last period rather than accepting a small amount at the beginning of the game. More precisely, in the least, you can wait until the last period, at which point you can get the entire surplus (the owner will accept anything then). Discounting to the first period, this will give you more than one-half of the surplus available in the first period.

6. Discuss under which cases the SPNE concept is superior than the NE concept. Provide and example supporting your argument.

GOOD LUCK!