

**Midterm #2 - EconS 424**  
**Due on April 9th, 2021 at 4.00pm,**

Name: \_\_\_\_\_

1. Analyze a two-period model for the market of computers in which two firms operate. Firm 1 only produces in period 1 and is endowed with an old technology providing a quality level  $v^O$  to consumers. Firm 2 is a potential entrant in period 2 and it is able to produce an old technology,  $v^O$ , and a new technology,  $v^N$ . However, the production of new technology requires an innovation cost of  $I > 0$ . Note that old and new technology can be nondurable (only last one period) or durable (it lasts for two periods). Hence, the cost of producing nondurable technology,  $c^{ND} = 0$ , is considerably lower than the cost of durable technology,  $C^D = 3$ . There is only one consumer in period 1 who seeks to buy a computer for the two periods of her life. In period 2, one additional consumer enters the market and seeks to buy a computer. Both consumers have the same gain from the quality of the technology embedded into the product they buy in period  $t$ . That is,  $V^N = 7$  and  $V^O = 5$  for new and old technology, respectively. The structure of the two-period, two-firm game is as follows: In period 1 firm 1 sells the old technology product and therefore has to decide which price to charge ( $p_1$ ) and whether to produce a durable ( $D$ ) or a nondurable ( $ND$ ) product. In the second period, firm 2 obviously chooses to produce a nondurable good (since the world ends at the end of period 2) and, hence, has to decide whether to invest in adopting the newer technology and price ( $p_2$ ).
  - (a) Illustrate the extended form game of the two-period, two-firm game.
  - (b) Describe the second-period pricing, first, for the case in which the first-period product is nondurable and, second, for the case in which it is durable.
  - (c) Identify the first period durability choice.
2. Consider a two-stage game in which at  $t = 1$ , firms determine (first noncooperatively and then cooperatively) how much to invest in cost-reducing R&D and at  $t = 2$ , firms are engaged in a Cournot quantity game. Firms produce a homogeneous product, where the demand function is given by  $p = 125 - Q$ . In addition,  $x_i$  denotes the amount of R&D undertaken by firm  $i$ ,  $i = 1, 2$ , and the unit production cost of firm  $i$  is  $c_i(x_1, x_2) \equiv 30 - x_i - 0.6x_j$ . Finally, R&D is costly to firms  $TC(x_i) = \frac{x_i^2}{2}$ .
  - (a) Identify the noncooperative equilibrium R&D level invested by each firm when firms do not cooperate.
  - (b) Identify the cooperative equilibrium R&D level invested by each firm when firms cooperate
  - (c) Compare the industry's R&D levels under noncooperative and cooperative R&D. Discuss how your results change with different values of  $\beta$ .
3. Let us consider an industry composed by three firms  $x$ ,  $y$  and  $z$ . Assume that firms have the same cost structure, that is,  $C(q_i) = 10 + 4q_i$ , where  $q_i$  denotes firm  $i$ 's output and  $i = \{x, y, z\}$ . In addition, the industry demand is  $P(Q) = 20 - Q$ , where  $Q$  denotes the aggregate output, where  $Q = q_x + q_y + q_z$ . The structure of the game is the following: (i) Firm  $x$  is the first mover of the game and it chooses its output level; (ii) firm  $y$  is the second mover of this game, and after observing  $q_x$ , firm  $y$  chooses its own output,  $q_y$ ; (iii) finally, after observing both firms output, firm  $z$  chooses its output  $q_z$ . The timing of production, industry demand and cost function are common knowledge among firms. Find values of output level  $q_x, q_y, q_z$  in the SPNE of the game.
4. Two friends (Johar and Zichen) are trying to split \$1,000. In the first round of bargaining, Zichen makes an offer at cost  $z$  (to herself), proposing to keep  $x_Z$  and give the remaining to Johar,  $x_J = 1,000 - x_Z$ . Johar either accepts her offer (ending the game) or rejects it. In round 2, Johar makes an offer of  $(y_Z, y_J)$ , at a cost of 5 to himself, which Zichen accepts or rejects. If Zichen accepts the offer, the game ends; but if she rejects it, the game proceeds to the third round, in which Zichen makes an offer  $(z_Z, z_J)$ , at a cost  $z$  to herself. If Johar accepts her offer in the third round, the game ends and payoffs are accrued to each player; whereas if he rejects it, the money is lost. Assume that players are risk-neutral (utility is equal to money obtained minus any costs), the discount factor is denoted by  $\delta$  and  $z = 0$ .

- (a) Graphically represent this bargaining game.
  - (b) What is the subgame-perfect equilibrium outcome?
5. Consider the case in which you are trying to buy a house and you are bargaining with the current owner over the sale price. The house is of value \$200,000 to you and \$100,000 to the current owner. Hence, if the price is between \$100,000 and \$200,000 then you would both be better off with the sale. Assume that bargaining takes place with alternative offers and that each stage of bargaining (an offer and response) takes a full day to complete. If agreement is not reached after 10 days of bargaining, then the opportunity for the sale disappears. Suppose that you and the current owner discount the future according to the discount factor  $\delta$  per day. The real estate agent has allowed you to decide whether you will make the first offer.
- (a) Suppose that  $\delta$  is small; in particular  $\delta < 0.5$ . Should you make the first offer or let the current owner make the first offer? Why?
  - (b) Suppose  $\delta$  is close to 1; in particular  $\delta > 0.93$ . Should you make the first offer or let the current owner make the first offer? Why?
6. Discuss under which cases the SPNE concept is superior than the NE concept. Provide an example supporting your argument.

**GOOD LUCK!**