# EconS 424 – Spring 2021

# Midterm exam #1

**Name ________________________________________ Student ID#____________________________**

**Instructions:**

This exam has three exercises for a total of 100 points, and a bonus exercise. The bonus exercise is voluntary. However, answering it right can add up to 15 additional points to your exam's grade, with a maximum of 100 (answering the bonus exercise will never harm your grade). **Submission is March 5th at 12.10pm.**

**Exercise 1 – Applying IDSDS in three-player games [25 points]**

Consider the following anti-coordination game in Figure 1 played by three potential entrants seeking to enter into a new industry, such as the development of software applications for smartphones. Every firm (labeled as A, B, and C) has the option of entering or staying out (i.e., remain in the industry they have been traditionally operating, e.g., software for personal computers). The normal form game in figure 1 depicts the market share that each firm obtains, as a function of the entering decision of its rivals. Firms simultaneously and independently choose whether or not to enter. As usual in simultaneous-move games with three players, the triplet of payoffs describes the payoff for the row player (firm A) first, for the column player (firm B) second, and for the matrix player (firm C) third.

| Firm A \ Firm B | Enter | Stay Out |
|---|---|---|
| Enter | 14,24,32 | 8,30,27 |
| Stay Out | 30,16,24 | 13,12,50 |

| Firm A \ Firm B | Enter | Stay Out |
|---|---|---|
| Enter | 16,26,30 | 31,16,24 |
| Stay Out | 31,23,14 | 14,26,32 |

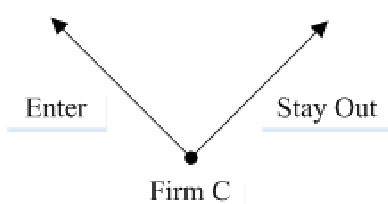


Figure 1. Normal-form representation of a three-players game

a) Find the set of strategy profiles that survive the iterative deletion of strictly dominated strategies (IDSDS).

b) Is the equilibrium you found using this solution concept unique?

**Exercise 2 – Tournaments [38 points]**

Several strategic settings can be modeled as a tournament, whereby the probability of winning a certain prize not only depends on how much effort you exert, but also on how much effort other participants in the tournament exert. For instance, wars between countries, or R&D competitions between different firms in order to develop a new product, not only depend on a participant's own effort, but on the effort put by its competitors. Let's analyze equilibrium behavior in these settings. Consider that the benefit that firm 1 obtains from being the first company to launch a new drug is $36 million. However, the probability of winning this R&D competition against its rival (i.e., being the first to launch the drug) is

$$\frac{x_1}{x_1+x_2},$$

which it increases with this firm's own expenditure on R&D, $x_1$, relative to total expenditure, $x_1 + x_2$. Intuitively, this suggests that, while spending more than its rival, i.e., $x_1 > x_2$, increases firm 1's chances of being the winner, the fact that $x_1 > x_2$ does not guarantee that firm 1 will be the winner. That is, there is still some randomness as to which firm will be the first to develop the new drug, e.g., a firm can spend more resources than its rival but be "unlucky" because its laboratory exploits a few weeks before being able to develop the drug. For simplicity, assume that firms' expenditure cannot exceed 25, i.e., $x_i \in [0, 25]$.The cost is simply $x_i$, so firm 1's profit function is

$$\pi_1(x_1, x_2) = 36\left(\frac{x_1}{x_1 + x_2}\right) - x_1$$

and there is an analogous profit function for country 2:

$$\pi_2(x_1, x_2) = 36\left(\frac{x_2}{x_1 + x_2}\right) - x_2$$

You can easily check that these profit functions are concave in a firm's own expenditure, i.e., $\frac{\partial^2 \pi_i(x_{i,} x_j)}{\partial x_i^2} \leq 0$ for every firm $i$={1, 2} where $j \neq i$. Intuitively, this indicates that, while profits increase in the firm's R&D, the first million dollar is more profitable than the 10th million dollar, e.g., the innovation process is more exhausted.

a. Find each firm's best-response function.

b. Find a symmetric Nash equilibrium, i.e., $x_1^* = x_2^* = x^*$.

**Exercise 3 - Game of Chicken [37 points]**

Consider the game of Chicken (depicted in figure 2), in which two players driving their cars against each other must decide whether or not to swerve.

| | | *Player 2* | |
|---|---|---|---|
| | | Straight | Swerve |
| *Player 1* | Straight | 0,0 | 3,1 |
| | Swerve | 1,3 | 2,2 |

Figure 2. Game of Chicken

a. Is there any strictly dominated strategy for Player 1? And for Player 2?

b. What are the best responses for Player 1? And for Player 2?

c. Can you find any pure strategy Nash Equilibrium (psNE) in this game?

d. Find the mixed strategy Nash Equilibrium (msNE) of the game.
[*Hint:* denote by $p$ the probability that Player 1 chooses Straight and by $(1-p)$ the probability that he chooses to Swerve. Similarly, let $q$ denote the probability that Player 2 chooses Straight and $(1-q)$ the probability that she chooses to Swerve.]

e. Show you result of part (d) by graphically representing every player $i$'s best response function $BRF_i(s_j)$, where $s_j = \{Swerve, Straight\}$ is the strategy selected by player $j \neq i$.

**BONUS EXERCISE - Cournot game [15 points]**

Consider two neighboring wineries in fierce competition over the production of their specialty wine (where their grapes come from the same vineyard, so the wines are exactly the same), One owned by Jaclyn (J), the other by Roey (R). Each winery produces their wine the same way and have the symmetric total cost function $TC_i(q_i)=3+0.5q_i$ where i=J,R. Inverse market demand for wine is $p=50-2(q_J + q_R)$.

a. Cournot competition. Write down the profit-maximization problem for each firm if they compete in quantities (à la Cournot).

b. Using the profit-maximization problems you wrote in part (a), find each firm's best response function. Interpret.

c. Using the best response functions you found in part (b), what is each winery's equilibrium output and price?