EconS 424- Strategy and Game Theory
Homework #3 - Due date: Friday, March 26th.

1. Consider the following matrix based on the movie "Friday the 13th!"

Jason (Front):

		Beth: Front	Beth: Back
Tommy	Front	$-1,-1,2$	$-1,1,1$
	Back	$1,-1,1$	$2,2,0$

Jason (Back):

		Beth: Front	Beth: Back
Tommy	Front	$2,2,0$	$1,-1,1$
	Back	$-1,1,2$	$-1,-1,2$

(a) Is there any strictly dominated strategy for Jason? And for Tommy?

There no strictly dominated strategies for Jason or Tommy.

1. (b) Can you find any pure strategy Nash Equilibrium (psNE) in this game?

Jason (Front):

		Beth: Front	Beth: Back
Tommy	Front	$-1,-1,\underline{2}$	$-1,\underline{1},\underline{1}$
	Back	$\underline{1},-1,1$	$\underline{2},\underline{2},0$

Jason (Back):

		Beth: Front	Beth: Back
Tommy	Front	$\underline{2},\underline{2},0$	$\underline{1},-1,\underline{1}$
	Back	$-1,\underline{1},\underline{2}$	$-1,-1,\underline{2}$

Hence, there is no psNE

1. (c) Find the mixed strategy Nash Equilibrium (msNE) of the game.

Assume that the probability that Tommy (Beth, Jason) chooses Front is t (b and j, respectively) and the probability that Tommy (Beth, Jason) chooses Back is $(1-t)$ ($(1-b)$ and $(1-j)$, respectively). Let's first focus on Jason:

$$\begin{aligned} EU_J(F) &= 2tb + 1(1-t)b + 1(1-b)t + 0(1-t)(1-b) = b + t \\ EU_J(B) &= 0tb + 1t(1-b) + 2(1-t)b + 2(1-t)(1-b) = 2 - t - tb \end{aligned}$$

Hence,

$$\begin{aligned} EU_J(F) &= EU_J(B) && \text{(Condition 1)} \\ b + t &= 2 - t - tb \\ b + 2t + tb &= 2 && (1) \end{aligned}$$

Next, let's analyze Tommy:

$$\begin{aligned} EU_T(F) &= -1bj - 1(1-b)j + 2b(1-j) + 1(1-b)(1-j) = bj + b - 2j + 1 \\ EU_T(B) &= 1bj + 2(1-b)j - 1b(1-j) - 1(1-b)(1-j) = -bj + 3j - 1 \end{aligned}$$

Hence,

$$\begin{aligned} EU_T(F) &= EU_T(B) && \text{(Condition 2)} \\ bj + b - 2j + 1 &= -bj + 3j - 1 \\ 5j - b &= 2 && (2) \end{aligned}$$

Finally, we examine Beth:

$$\begin{aligned} EU_B(F) &= -1tj - 1(1-t)j + 2t(1-j) + 1(1-t)(1-j) = t - tj - 2J + 1 \\ EU_B(B) &= 1tj + 2(1-t)j - 1t(1-j) - 1(1-t)(1-j) = 3j - tj - 1 \end{aligned}$$

Hence,

$$\begin{aligned} EU_B(F) &= EU_B(B) && \text{(Condition 3)} \\ t - tj - 2j + 1 &= 3j - tj - 1 \\ 5j - t &= 2 && (3) \end{aligned}$$

using condition 3 and 2 we have that

$$\begin{aligned} 5j - b &= 5j - t \\ b &= t \end{aligned}$$

substituting this into condition 1 we obtain

$$\begin{aligned} b + 2b + b^2 &= 2 \\ b^2 + 3b - 2 &= 0 \\ \mathbf{b} \simeq \mathbf{0.56} & \quad \text{or } b = -3.56 \end{aligned}$$

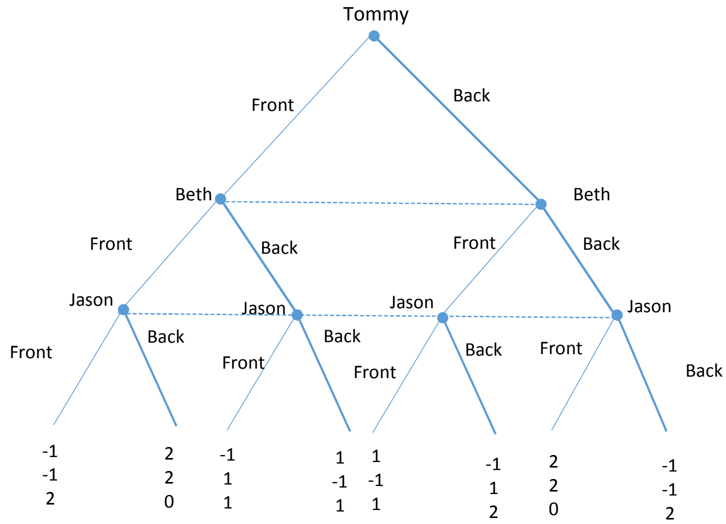
we consider the positive number; and since $t = b$ then $t \simeq 0.56$. Finally, using condition 3 we obtain

$$\begin{aligned} 5j - \mathbf{0.56} &= 2 \\ 5j &= 2.56 \\ j &\simeq 0.51 \end{aligned}$$

Hence, the $msNE = \{(0.56F, 0.44B), (0.56F, 0.44B), (0.51F, 0.49B)\}$.

1. (d) Represent this game in its extensive form (game tree), where Tommy acts first,

Beth acts second and Jason acts third.



2. Consider the game tree in Figure 1 representing the sequential -move version of the Chicken game, where player 1 acts first, and observing its move player 2 responds.

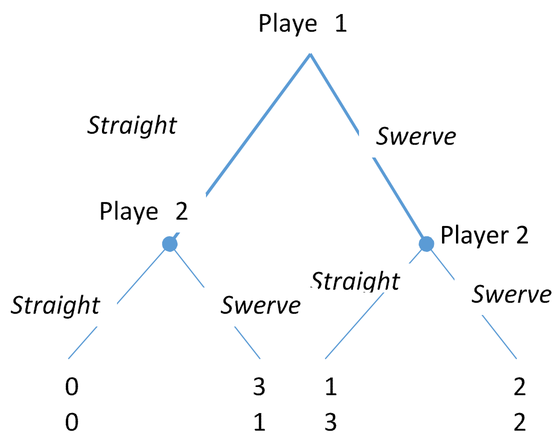
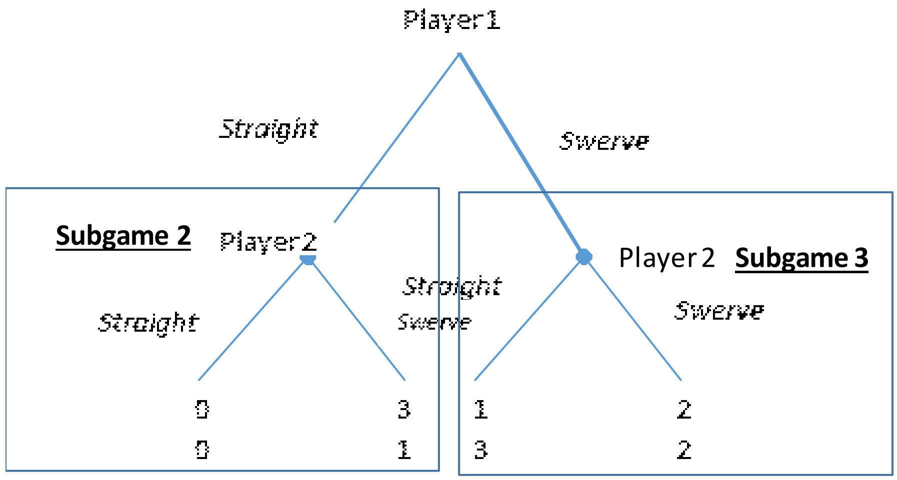


Figure 1

(a) Find the best responses of the second mover, for each of the first mover's actions.



We first circle the subgames in this extensive-form game, where Subgame 1 is the whole game, and Subgame 2 and 3 are initiated after Player chooses Straight and Swerve, respectively.
Subgame 2: After Player 1 chooses Straight, Player 2 responds with Swerve since its payoff from doing so (1, that is, avoiding a heads-on collision but being humiliated by Player 1) is higher than from Straight (0, implying a heads-on collision). Intuitively, Player 2 observes that Player 1 has already chosen Straight, should choose Swerve, which is better than choosing Straight.
Subgame 3: After Player 1 chooses Swerve, Player 2 responds with Straight since its payoff from doing so (3) is higher than from Swerve (2). Hence, Player 2 observes that Player 1 has already chosen Swerve, should choose Straight, which is better than choosing Swerve.
We can see that Player 2 responds by choosing the opposite action as Player 1.

1. (b) Find the first mover's equilibrium action and describe the subgame perfect Nash equilibrium of the game.

By backwards induction, Player 1 anticipates that Player 2 will respond with Straight if it chooses Swerve and Swerve if it chooses Straight. Hence, the payoffs pairs after player 1 chooses Straight are $(3, 1)$ since it anticipates that Player 2 will respond with Swerve in Subgame 2. Similarly, payoff pairs after Player 1 chooses Swerve is $(1, 3)$ since it anticipates that Player 2 will respond with Straight in Subgame 3. Hence, $SPNE = \{(Straight, (Swerve, Straight)\}$.

1. (c) Does the equilibrium behavior in the sequential-move game differ from its simultaneous-move version?

Equilibrium behavior in the simultaneous-move version of the Chicken game predicts two Nash equilibria in pure strategies where Player 1 chooses Straight while Player 2 Swerves, or Player 1 selects Swerve while Player 2 chooses Straight. Only the first outcome can be sustained in the sequential-move game where, in equilibrium, Player 1 has chosen Straight, Player 2 is better off responding with Swerve that Straight (that is, avoiding a car crash with Player 1).

3. Consider the game tree in Figure 2, describing a sequential-move game played between two political parties. The Red party acts first choosing an advertising level. The Blue party acts after observing Red party's advertising level (Low, Middle or High) and responds with its own advertising level. Find the SPNE of the game.

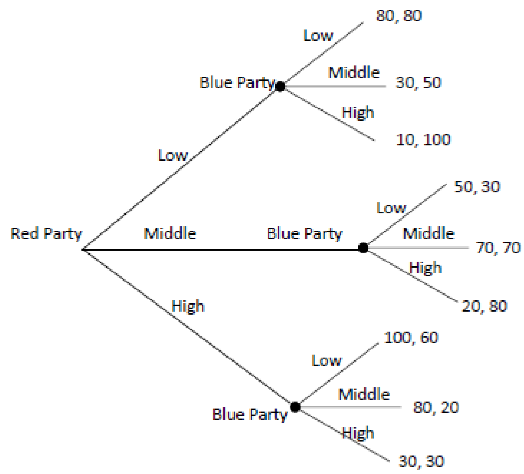


Figure 2

- *Second Mover*. First, we examine the payoffs received by the Blue party in the lower part of the game tree. When analyzing each sub-game independently by comparing the payoffs for the Blue party (the number on the right side of each payoff pair). Starting from the subgame on the left-hand side of the tree, when the Red party chooses Low, we can see that for the Blue party is better to choose High, because $100 > 80 > 50$. In the case of the subgame in the center of the figure, when the Red party chooses Middle, the Blue party have incentives to choose also High, because $80 > 70 > 30$. In the case the Red party chooses High, the Blue party responds with low advertisement expenditure since $60 > 30 > 20$. See blue branches in Figure 3.

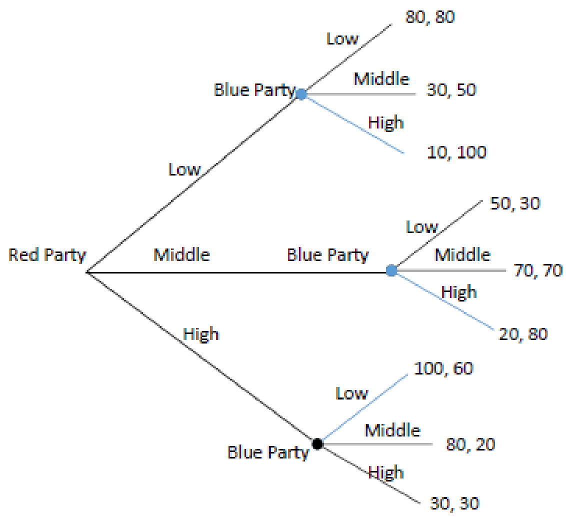


Figure 3

- First Mover. Once the best responses at the subgame for the Blue party are identified, we proceed to identify the best strategy for the Red party by comparing its payoffs in each case. In the case the Red party chooses Low, the response for the Blue party is to choose High, implying that the Red party gets 10. If the Red party chooses Middle, the payoff given the High response of the Blue party is 20. Finally, if Red party's strategy is to choose High, then its payoff is 100 , since the best response by Blue is to choose Low. Hence, since $100 > 20 > 10$, the backwards induction strategy to solve the game indicates that the equilibrium is $SPNE = \{High, (High, High, Low)\}$