EconS 424 - Strategy and Game Theory
Practice Exercises (March 1st and March 3rd, 2021)

1. Consider the following 2x2 game

		Player 2	
		Left	**Right**
Player 1	**Left**	$14, 10$	$0, 3$
	Right	$4, 0$	$8, 8$

   (a) By inspection, what are the pure strategy Nash equilibria?

   (b) Find the additional mixed strategy equilibrium by using the fact that if a player is willing to mix between two or more strategies, she will be indifferent between them in equilibrium.

   (c) Draw the best-response correspondences. Where do they intersect?

2. Consider the following simultaneous-move game played by player 1 (in rows) and player 2 (in columns).

			Player 2	
		x	y	z
	a	$2, 3$	$1, 4$	$3, 2$
Player 1	b	$5, 1$	$2, 3$	$1, 2$
	c	$3, 7$	$4, 6$	$5, 4$
	d	$4, 2$	$1, 3$	$6, 1$

   (a) Which strategy pairs survive the application of iterative deletion of strictly dominated strategies (IDSDS)?

   (b) Using your results from part (a), show that there is no pure strategy Nash equilibrium (psNE) in this game.

   (c) Using your results from part (a), find a mixed strategy Nash equilibrium (msNE) in this game.

3. Assume that the market is composed by two firms with identical costs $C(q) = 10q + q^2$. Firms can either "collude" or "compete." If both collude, they each produce q_m (half the monopoly output Q_m). If one firm colludes and the other competes, the latter produces the output q^{**} that maximizes its profits given that the other firm produces q_m. If both compete, they play Cournot and each produce q_n. Calculate these outputs and the resulting profits if market demand is $Q = 145 - p$. Represent your results as a normal-form game. What is the Nash equilibrium if the game is only played once?

4. In Tacoma, WA there are two suppliers of distilled water, labeled as firm Aqua and firm Blue. Distilled water is considered to be a homogenous good. Let p denote the price per gallon, q_A quantity sold by firm Aqua, and q_B the quantity sold by firm Blue. Both firms are located close to a spring so the only production cost is the cost of bottling. Formally, each firm bears a production cost of $c_A = c_B = \$3$ per one gallon of water. Tacoma's aggregate inverse demand function for distilled water is given by $p = 12 - Q = 12 - q_A - q_B$, where $Q = q_A + q_B$ denotes the aggregate industry supply of distilled water in Tacoma. Solve the following problems:

(a) Solve for firm A's best-response function, $q_A = R_A(q_B)$. Also solve for firm B's best-response function, $q_B = R_B(q_A)$. *Show your derivations.*

(b) Solve for the Cournot equilibrium output levels q_A^c and q_B^c. State which firm sells more water (if any) and why.

(c) Solve for the aggregate industry supply and the equilibrium price of distilled water in Tacoma.

5. Consider two firms competing (a la Cournot), facing inverse demand function $p(Q) = a - Q$, where Q denotes aggregate output, that is,$Q = q_1 + q_2$. Firm 1's marginal cost of production is $c_1 > 0$, while firm 2's marginal cost is $c_2 > 0$, where $c_2 \geqslant c_1$ indicating that firm 2 suffers a cost disadvantage relative to firm 1. Firms face no fixed cost. For compatness, let us present firm 2's marginal cost relative to c_1, so that $c_2 = \alpha c_1$, where $\alpha \geqslant 1$ indicates the cost disadvantage diminishes, while when α is significantly higher than 1 firm 2's disadvantage is severe. This allows us to represent both firms' marginal costs in terms of c_1, without need to use c_2.

   (a) Write down firm's profit maximization problem abd find its best response function.

   (b) Interpret how each firm's best response function is affected by a marginal increase in alpha.

   (c) Find the Nash equilibrium of this Cournot game