Homework 9 - Due date: Friday, December 11th

1. Consider an economy with 2 consumers, Alessandro and Beatrice, $i = \{A, B\}$, one private good x, and one public good G. Let each consumer have an income of M. For simplicity, let the prices of both the public and private good be 1. In addition, the utility functions of consumer A and B are:

$$\begin{aligned} U^A &= \log(x^A) + \log(G), \quad \text{for individual } A, \text{ and} \\ U^B &= \log(x^B) + \log(G), \quad \text{for individual } B \end{aligned}$$

   Assume that the public good G is only provided by the contributions of these two individuals, that is, $G = g^A + g^B$.

   (a) Find Alessandro's best response function. Depict it in a figure with his contribution, g^A, on the vertical axis and Beatrice's contribution, g^B, on the horizontal axis.

   (b) Identify Beatrice's best response function. Depict it in a figure with her contribution, g^B, on the horizontal axis and Alessandro's contribution, g^A, on the vertical axis.

   (c) *Unregulated equilibrium.* Find the equilibrium contributions to the public good by Alessandro and Beatrice, that is, the Nash equilibrium of this public good game.

   (d) *Social optimum.* Find the efficient (socially optimal) contribution to the public good by Alessandro and Beatrice.

   (e) Use a figure to contrast the Pareto efficient level of private provision and the Nash equilibrium level of provision.

2. Assume that Alaska Airlines is a monopolist in the route Pullman-Seattle. The Washington State Legislature would like to design a policy that induces Alaska Airlines to voluntarily produce an efficient output level. If the firm faces an inverse demand function $p(q)$ with $p'(q) < 0$ and marginal costs $c'(q) > 0$ for all q, show that such a policy must be a subsidy, and determine the exact amount of the subsidy.

3. Consider a monopolist selling two different goods, q_1 and q_2, whose demands are interdependent and given by

$$q_i = a - bp_i + gp_j \quad \text{for } i = \{1, 2\} \text{ and } j \neq i$$

   where $b > 0$ (this guarantees that the demand for a particular good decreases in its own price). In addition, note that if $g > 0$, goods are substitutes, while if $g < 0$, they are

complements (and if $g = 0$ the products are independent of each other). Assume also that $|g| < b$, which guarantees that the own-price effect, represented in parameter b, dominates the cross-price effect, embodied in parameter g. Intuitively, the demand for good i is more sensitive to a given increase in its own price than to the same increase in the price of a different good. Finally, consider that $a > c(b - g)$ which guarantees that output is strictly positive in equilibrium. In order to focus on the interdependence between both products' demands, let us assume that the marginal costs of production coincide across both products. That is, total costs are $TC(q_1, q_2) = cq_1 + cq_2$.

(a) Find the profit-maximizing price for good 1, p_1, and for good 2, p_2, for this monopolist. [*Hint:* Find the profit-maximizing prices rather than output levels.]

(b) Do prices increase or decrease in the parameter that reflects the cross-price effects, g?

(c) Compare prices if $g > 0$ and if $g < 0$ with those where $g = 0$. Explain