

EconS 501 Final Exam - December 14th, 2020

Show all your work clearly and make sure you justify all your answers.

NAME _____

1. Consider a setting in which two firms, $N = 2$, have free access to a fishing ground. Every unit of appropriation (e.g., a ton of fish) is sold in the international market which, for simplicity, is assumed to be perfectly competitive. As a result, every firm takes the market price p as given, which is normalized to $p = \$1$. In this case, consider that firms face a different cost function

$$C_1(q_1, q_2) = \frac{q_1(q_1 + q_2)}{S} \quad \text{and} \quad C_2(q_2, q_1) = \frac{\alpha q_2(q_1 + q_2)}{S}$$

where $\alpha \in [0, 1]$ denotes firm 2's cost advantage. When $\alpha = 1$, both firms face the same cost function. However, when $\alpha < 1$, firm 2 benefits from a cost advantage relative to firm 1. S denotes the stock of the resource. In addition, q_i represents fisherman i 's appropriation where $i = \{1, 2\}$.

- (a) Find firm 1 best response function, $q_1(q_2)$. How is it affected by an increase in parameter α ? Interpret.
 - (b) Find firm 2 best response function, $q_2(q_1)$. How is it affected by an increase in parameter α ? Interpret.
 - (c) Compare firm 1 and 2's best response functions.
 - (d) Find the equilibrium appropriation pair (q_1^*, q_2^*) .
 - (e) Which firm appropriates more in equilibrium?
 - (f) Find the equilibrium appropriation pair when firms are symmetric, $\alpha = 1$ and compare it to your result in part (d).
 - (g) How is equilibrium appropriation affected by an increase in parameter α ? Interpret.
2. Consider a firm with production technology $f(x)$, where $x \in [0, 1]$ stands for the input (e.g, gas supply), and $f(x)$ represents the output that is sold in a competitive market at a price p . The firm is subject to an input cost function $g(x)$ that is increasing and convex in input x . This happens, for example, when there is only one company supplying natural gas.

- (a) Define $\varepsilon_g \equiv \frac{\partial x}{\partial g(x)} \frac{g(x)}{x}$ to be the price elasticity of gas supply, measuring the percentage change of gas supply given one percent change in gas price. Setup the firm's profit-maximization problem to maximize $\pi(x) = pf(x) - g(x)x$, and show that

$$pf'(x) = g(x) \left[1 + \frac{1}{\varepsilon_g} \right].$$

- (b) Let $f(x) = x$ and $g(x) = x^\beta$, where $\beta > 1$. Use the expression found in part (a) to identify the optimal gas supply x^* . For simplicity, you may assume that $p = 1$ in the remainder of this exercise.

- (c) *Comparative statics.* How does x^* change with β ? Explain.
- (d) *Numerical example.* Evaluate the firm's optimal gas supply x^* when $\beta = 1$, $\beta = 2$, $\beta = 4$, and $\beta \rightarrow +\infty$. Interpret.

3. Consider an individual with utility function

$$u(x_1, x_2) = \max\{x_1, x_2\}$$

who derives utility from the *maximum* consumption of goods 1 and 2, subject to a budget constraint $p_1x_1 + p_2x_2 \leq w$, where p_1 and p_2 are the prices of goods 1 and 2, respectively, and wealth is $w \geq 0$.

- (a) Find the Walrasian demand. (*Hint:* consider two separate cases: (i) $p_1 < p_2$, and (ii) $p_1 \geq p_2$.)
 - (b) Find the value function and show that the Roy's identity holds.
 - (c) Does the utility function satisfy homotheticity? Interpret your results.
4. Consider that your preference relation over three bundles, x_1 , x_2 , and x_3 , satisfies

$$x_1 \succ x_2$$

$$x_2 \succ x_3$$

$$x_3 \succ x_1$$

- (a) Show that you can be wiped out of your wealth w , where $w > 0$. (*Hint:* Begin with x_3 .)
- (b) Consider an individual with a preference relation that violates rationality because his preferences are incomplete or intransitive. Discuss.

GOOD LUCK!