Recitation 11 (November 13th, 2020)

1. Consider an economy with two individuals $i = \{1, 2\}$ with the following quasi-linear utility function

$$u_i(s^i, q^i) = v^i(s^i) + \alpha w^i$$

where s^i denotes the speed at which individual i drives his car, w^i is his wealth, and $\alpha > 0$. The utility that individual i obtains from driving fast is $v^i(s^i)$, which is increasing but concave in speed, whereby $\frac{\partial v^i(s^i)}{\partial s^i} > 0$ and $\frac{\partial^2 v^i(s^i)}{(\partial s^i)^2} < 0$. Driving fast, however, increases the probability of suffering a car accident, represented by $\gamma(s^i, s^j)$. This probability is increasing both in the speed at which individual i drives, s^i, and the speed at which other individuals drive, s^j, where $j \neq i$. Hence, the speed of other individuals imposes a negative externality on driver i, since it increases his risk of suffering a car accident. If individual i suffers an accident, he bears a cost of $c^i > 0$, which intuitively embodies the cost of fixing his car, health-care expenses, etc.

   (a) *Unregulated equilibrium.* Set up individual i's expected utility maximization problem. Take first-order conditions with respect to s^i, and denote the (implicit) solution to this first-order condition as $\widehat{s}^i$.

   (b) *Social optimum.* Set up the social planner's expected welfare maximization problem. Take first-order conditions with respect to s^1 and s^2. Denote the (implicit) solution to this first-order condition as $\overline{s}^i$.

   (c) *Comparison.* Show that drivers have individual incentives to drive too fast, relative to the socially optimal speed, i.e., show that $\widehat{s}^i > \overline{s}^i$.

   (d) *Restoring the social optimum.* Let us now evaluate the effect of speeding tickets (fines) to individuals driving too fast, i.e., to those drivers with a speed $\widehat{s}^i$ satisfying, $\widehat{s}^i > \overline{s}^i$. What is the dollar amount of the fine m^i that induces every individual i to fully internalize the externality he imposes onto others?

   (e) Let us now consider that individuals obtain a utility from driving fast, $v^i(s^i)$, only in the case that no accident occurs. Repeat steps (a)-(c), finding the optimal fine m^i that induces individuals to fully internalize the externality.