Recitation #9 - (10/30/2020)

1. Suppose there are two periods, 'today' (i.e. period 1) and 'tomorrow' (i.e. period 2), and a single consumption good. An individual called Andreu (not necessarily the same person as the first author in Mas-Colell, Whinston and Green's *Microeconomic Theory*) has preferences over two-period consumption streams that are additively separable. In particular assume his preferences admit a representation of the form:

$$U(x_1, x_2) = u(x_1) + \frac{1}{1+r}u(x_2),$$

where $r > 0$ is the interest rate at which the agent can borrow or lend money, and

$$u(x_t) = -\frac{(x_t - 2)^2}{2}$$

for every time period $t = \{1, 2\}$.

   (a) If his income today is $y_1 = 1$ and he knows that his income tomorrow will also be $y_2 = 1$, and solve for her optimal consumption in each period and calculate the level of discounted lifetime utility he achieves.

   b. Formulate the agent's optimization problem and solve for the optimal consumption plan and his level of discounted expected utility. Suppose that Andreu still knows that his income today is $y_1 = 1$. He is, however, uncertain about his future payoff y_2: It could be high, $y_2^H = \frac{3}{2}$ or low, $y_2^L = \frac{1}{2}$, each happening with equal probability. His problem now is to maximize his discounted expected utility by choosing the initial period consumption x_1; his future consumption if his income tomorrow is high, x_2^H; and his future consumption if income tomorrow is low, x_2^L.

   (b) Compare your answers to parts (a) and (b).

2. Consider a cumulative distribution function $F(x)$ which first-order stochastically dominates $G(x)$.

   (a) Show that the mean of x under $G(x)$, $\int x\, dG(x)$, cannot exceed that under $F(x)$, $\int x\, dF(x)$, i.e.,

$$\int xdF(x) \geqslant \int xdG(x)$$

   Provide now an example where $\int xdF(x) \geqslant \int xdG(x)$ is satisfied, but $F(x)$ does not first order stochastically dominates $G(x)$.