Recitation #8 (10/23/2020)

1. Let G be the set of compound gambles over a finite set of deterministic payoffs $\{a_1, a_2, ...a_n\} \subset \mathbb{R}_+$. A decision maker's preference relation $\succsim$ over compound gambles can be represented by utility function $v : G \to \mathbb{R}$. Let $g \in G$, and let probability p_i be associated to the corresponding payoff a_i. Finally, consider that the decision maker's utility function $v(\cdot)$ is given by

$$v(g) = (1 + a_1)^{p_1} (1 + a_1)^{p_2} ...(1 + a_n)^{p_n} = \prod_{i=1}^{n} (1 + a_i)^{p_i}$$

   (a) Show that this is *not* a von Neumann-Morgenstern (vNM) utility function.

   (b) Show that the decision maker has the same preference relation as an expected utility maximizer with von-Neumann Morgenstern utility function

$$u(g) = \sum_{i=1}^{n} p_i \ln (1 + a_i) .$$

   (c) Assume now that the decision maker you considered in part (b) has utility function $u(w) = \ln (1 + w)$ over wealth $w \geqslant 0$. Evaluate his risk attitude (concavity in his utility function). Additionally, find the Arrow-Pratt coefficient of absolute risk aversion, $r_A(w, u)$. How does $r_A(w, u)$ change in wealth?

2. Consider the set of deterministic payoffs $\{a_1, a_2, ...a_n\} \subset \mathbb{R}_+$. Studies in regret-based decision making often consider the following utility function: first, define the highest deterministic payoff that could be reached in gamble g by using function

$$h(g) = \max \{a_k : k \in \{1, 2, ..., n\} \text{ and } p_k > 0\} .$$

   Subtracting $h(g)$ from all deterministic outcomes and computing its expected value yields the utility level

$$v(g) = \sum_{i=1}^{n} p_i (a_i - h(g)) = \sum_{i=1}^{n} p_i a_i - h(g)$$

   Intuitively, after event i realizes (which provides a payoff a_i to this individual), the "regretful" decision maker compares such monetary payoff with respect to the highest possible payoff he could have obtained from playing this lottery, $h(g)$. Utility functions of this type hence reflect "regret," as individuals experience a disutility from not receiving the highest possible monetary payoff in the lottery.

(a) Compute the expected value of the following two gambles:

$$g^1 = \left(0, 1, 2; \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right) \quad \text{and} \quad g^2 = \left(1, 4, 5; \frac{1}{2}, \frac{1}{3}, \frac{1}{6}\right)$$

(b) Show that all deterministic outcomes (outcomes with probability 100%) yield the same utility level. That is, $v(a_1) = v(a_2) = ... = v(a_n)$.

(c) Show that the preference relation does not satisfy monotonicity if outcomes are deterministic.