EconS 501 Midterm #1 - October 12th, 2020

Show all your work clearly and make sure you justify all your answers.
NAME_________________________________

1. For each of the following utility functions, show if they are homothetic.

   (a) Cobb-Douglas utility function $u(x, y) = Ax^{\alpha}y^{\beta}$, where $A, \alpha, \beta > 0$.

   (b) Goods are regarded as substitutes $u(x, y) = ax + by$, where $a, b > 0$.

   (c) Goods are regarded as complements $u(x, y) = \min\{ax, by\}$, where $a, b > 0$.

   (d) Stone-Geary utility function $u(x, y) = A\left(x - \overline{x}\right)^{\alpha}\left(y - \overline{y}\right)^{\beta}$, where $A, \alpha, \beta \in \mathbb{R}$ and $\overline{x}, \overline{y} > 0$.

   (e) Highly non-linear utility function $u(x, y) = Ax^{\alpha}y + Byx^{\beta}$, where $A, B, \alpha, \beta > 0$.

2. Consider a firm with the following production function $q = f(w, k) = \frac{\theta}{1+w^{-\delta}k^{-\varepsilon}}$, where $\theta, \delta, \varepsilon > 0$ and w and k denote water and minerals, respectively. The firm has a new technology that allows it to transform all remaining outputs back to the initial amounts of inputs. In addition, the producer can eliminate additional inputs at a cost equal to $\$X$.

   (a) Define the production set and discuss what properties of the production set are not satisfied.

   (b) Identify the sufficient conditions that guarantee that the $MRTS_{w,k}$ is diminishing.

   (c) Find the product-elasticities of the two inputs and comment the type of returns to scale that the function represents. Use a numerical example considering parameters $\delta = \varepsilon = \frac{3}{4}$ and discuss your results.

3. Assume that $L = 2$ and consider a *local* indirect utility function defined in some neighborhood of price-wealth pair $(\widetilde{p}, \widetilde{w})$ by

$$v(p, w) = \left[\frac{w}{p_2} + \frac{1}{b}(a\frac{p_1}{p_2} + \frac{a}{b} + c)\right] \exp\left(-\frac{bp_1}{p_2}\right)$$

   (a) Verify that the local demand function for good 1 is

$$x_1(p, w) = a\frac{p_1}{p_2} + b\frac{w}{p_2} + c$$

   (b) Verify that the local expenditure function is

$$e(p, u) = p_2 u \exp\left(\frac{bp_1}{p_2}\right) - \frac{1}{b}(ap_1 + \frac{a}{b}p_2 + cp_2)$$

   (c) Verify that the Hicksian demand function for good 1 is

$$h_1(p, u) = ub \exp\left(\frac{bp_1}{p_2}\right) - \frac{a}{b}$$

4. Assume that you have a friend, Leo, who is now retired and lives on a fixed income $w > 0$ which does not adjust to inflation. His expenditure function is

$$e(p_1, p_2, u) = (p_1 + p_2)u,$$

where $p_1, p_2 > 0$ denote initial prices. Suppose that prices of goods 1 and 2 increase to p_1' and p_2', respectively.

(a) You want to give him a monetary gift so that he will not be affected by the above price increase. How much money should you give him? That is, find his compensating variation (CV).

(b) Now find his equivalent variation (EV) from the price change, i.e., the change in income needed at initial prices p_1 and p_2 that would have the same effect on utility as would the change in prices, p_1' and p_2'.

(c) Which is larger in this case, CV or EV?

(d) Find his Walrasian demand for each good.

(e) Find his utility function. What is this type of utility function called?