Homework (10/28/2020)

1. State the independence axiom. Show that if indifference curves in the Machina triangle are *not* parallel straight lines, then the independence axiom is violated.

2. Consider an individual with preferences over lotteries that satisfy the independence axiom. Answer the following questions.

   (a) Show that the independence axiom implies convexity, i.e., for three different lotteries L, L' and L'', if $L \succ L'$ and $L \succ L''$, then $L \succ \alpha L' + (1-\alpha) L''$.

   (b) Discuss why a decision maker whose preferences violate convexity can be offered a sequence of choices that lead him to a sure loss of money

3. Consider an old professor who is planning to retire next year. He is evaluating risky investments relative to the amount he invests (which we refer to as his reference point, r). Although he never studied prospect theory, his preferences might reveal that he compares investments according to this theory. In particular, for a return x on his investment r, his utility is $100(x-r) - \frac{1}{2}(x-r)^2$ if his return exceeds his investment, $x \geq r$, but becomes $400(x-r) + 2(x-r)^2$ if his return is smaller than his investment, $x < r$. Assume that his expected utility is linear in the probabilities. Intuitively, note that this individual's preferences exhibit loss aversion since the term $(x-r)$ is more heavily weighted when $x < r$ than otherwise.

   (a) Assume that this individual plans to invest \$1,000 and faces two investment options: (1) bonds, that yield \$1,022.54 next year; and (2) stocks, that yield \$900 with probability 0.12 and \$1,100 with probability 0.88. Show that this individual is indifferent between both investment options, and he could thus opt for the riskless bonds.

   (b) Generalize your previous result. That is, assume that the professor has an initial investment $I > 0$, and faces two investment options: (1) bonds yielding outcome I_b with certainty; and (2) stocks, yielding I_{SL} with probability p and I_{SH} with probability $1-p$, where $I_{SH} > I > I_{LH} > 0$. Find under which values of p the equity premium puzzle emerges.

   (c) Consider now an individual who does not use his initial investment as a reference point to evaluate future returns, i.e., $r = 0$. Using the numerical values in part (a), does the equity premium puzzle still arise?