

Homework #6 (Due on October 12th, 2020)

1. Consider the following profit function that has been obtained from a technology that uses a single input, z :

$$\pi(p, w) = p^2 w^\alpha$$

where p is the output price, w is the input price and α is a parameter value.

- (a) Check if the profit function satisfies homogeneity of degree one jointly in both p and w . In particular, determine for which values of α this property is satisfied.
 - (b) Assuming the value of α for which the profit function satisfies homogeneity of degree one, check if the profit function $\pi(p, w)$ satisfies the following properties: (1) non-decreasing in output price p , (2) non-increasing in input prices w , and (3) convex in prices p and w .
 - (c) Calculate the supply function of the firm, $q(p, w)$, and its demand for inputs, $z(p, w)$.
2. The profit function, $\pi(p)$, is defined as

$$\pi(p) = \max \{p \cdot y \mid y \in Y\}$$

or alternatively, $\pi(p) \geq p \cdot y$ for every feasible production plan $y \in Y$.

- (a) Show that the profit function $\pi(p)$ is convex in prices.
- (b) Prove that Hotelling's lemma holds for this profit function.
- (c) Show that, if the output function $y(p)$ is differentiable at \bar{p} , then $D_p y(\bar{p})$ is a symmetric and positive semidefinite matrix.