Recitation #6 (October 2nd, 2020)

1. Some indirect utility functions $v_i(p, w_i)$, such as those originating from a quasilinear preference relation, can be represented as a convex combination of individual i's wealth, w_i, as follows,

$$v_i(p, w_i) = a_i(p) + b(p)w_i$$

which are often referred as the Gorman form indirect utility function.

   (a) Show that if the indirect utility function can be expressed using the Gorman form, then all consumers exhibit parallel, straight wealth expansion paths at any price vector p. [*Hint:* Use Roy's identity].

   (b) Show also that, if the indirect utility function can be represented using the Gorman form, $v_i(p, w_i) = a_i(p) + b(p)w_i$, with the same $b(p)$ for all individuals, then the associated expenditure function, $e_i(p, u_i)$, can be expressed as

$$e_i(p, u_i) = c(p)u_i + d_i(p)$$

2. Consider a firm with production function $q = \sqrt{z}$, using one input (e.g., labor) to produce one type of output. The price of every unit of input is $w = 8$, and the price of every unit of output is $p > 0$.

   (a) Set up the firm's profit-maximization problem, and solve for its unconditional factor demand $z(8, p)$.

   (b) Evaluate the profit function at the unconditional factor demand $z(8, p)$. Test for convexity of the profit function in output price p.

   (c) Let us now illustrate convexity in output prices by using an alternative approach: (1) evaluate the profit function you found in part (b) at prices $p = 6$, and at $p = 12$. Then, find their convex combination $\alpha\pi(6) + (1 - \alpha)\,\pi(12)$ where $\alpha \in [0, 1]$; (2) evaluate the profit function at the convex combination of the above output prices, that is, $\pi(\alpha 6 + (1 - \alpha)\,12)$. Last, show that the profit function you found in step (1) lies weakly above that found in step (2) for all values of α, that is,

$$\alpha\pi(6) + (1 - \alpha)\,\pi(12) \geq \pi(\alpha 6 + (1 - \alpha)\,12).$$