

Recitation #5 (09/25/2020)

1. An individual consumes only good 1 and 2, and his preferences over these two goods can be represented by Cobb-Douglas utility function

$$u(x_1, x_2) = x_1^\alpha x_2^\beta \quad \text{where } \alpha, \beta > 0.$$

(For generality, we do not impose any assumptions on the sum of the exponents, i.e., $\alpha + \beta$ can satisfy $\alpha + \beta > 1$ or $\alpha + \beta < 1$.) This individual currently works for a firm in a city where initial prices are $p^0 = (p_1, p_2)$, and his wealth is w .

- (a) Find the Walrasian demand for goods 1 and 2 of this individual, $x_1(p, w)$ and $x_2(p, w)$.
- (b) Find his indirect utility function at price vector p , and denote it as $v(p, w)$.
- (c) The firm that this individual works for is considering moving its office to a different city, where good 1 has the same price, but good 2 (e.g., housing) is twice as expensive, i.e., the new price vector is $p' = (p_1, 2p_2)$. Find the value of the indirect utility function in the new location. Let us denote this indirect utility function $v(p', w)$.
- (d) This individual's expenditure function is

$$e(p, u) = (\alpha + \beta) \left(\frac{p_1}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}} \left(\frac{p_2}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} u^{\frac{1}{\alpha+\beta}}$$

Evaluate this expenditure function in the following cases:

1. Under initial prices, p , and maximal utility level $u \equiv v(p, w)$, and denote it by $e(p, u)$.
 2. Under initial prices, p , and maximal utility level $u' \equiv v(p', w)$, and denote it by $e(p, u')$.
 3. Under new prices, p' , and maximal utility level $u \equiv v(p, w)$, and denote it by $e(p', u)$.
 4. Under new prices, p' , and maximal utility level $u' \equiv v(p', w)$, and denote it by $e(p', u')$.
- (e) Find this individual's equivalent variation due to the price change. Explain how your result can be related with this proposal of the worker to his boss: "I would really prefer to stay in this city. In fact, I would accept a salary reduction if I could keep working for the firm in this city."

- (f) How is this individual's consumer surplus affected by the price change? (The change in consumer surplus is often referred to as the "area variation (AV)")
- (g) Which of the previous welfare measures in questions (e) and (f) coincide? Which of them do *not* coincide? Explain.
- (h) Consider how the welfare measures from questions (e) and (f) would be modified if this individual's preferences were represented, instead, by the utility function $v(x_1, x_2) = \alpha \ln x_1 + \beta \ln x_2$.