Homework #5 (10/05/2020)

1. Consider a society where every individual i has the following (discontinuous) Walrasian demand function

$$x_i(p) = \begin{cases} \frac{w}{4p} \text{ if } p > k, \\ \frac{w}{2p} \text{ if } p < k, \text{ and} \\ \frac{w}{4p} \text{ or } \frac{w}{2p} \text{ if } p = k \end{cases}$$

where k is a positive constant, while $w > 0$ denotes income.

   (a) Assume that there are only two individuals in this society, 1 and 2. If they both have the same income $w > 0$, find their average demand, i.e., $\frac{x_1+x_2}{2}$. Then show that the average demand $\frac{x_1+x_2}{2}$ takes three possible values at $p = k$.

   (b) Now consider a society with an infinite number of individuals, all with the above Walrasian demand $x_i(p)$ and the same income $w > 0$. Show that now the average demand at $p = k$ now takes all the values between $\frac{w}{4p}$ and $\frac{w}{2p}$.

2. Suppose that a set of I consumers all have homothetic preferences represented by utility functions that satisfy homogeneity of degree one. Consider now the social welfare function

$$W(u_1, u_2, ..., u_I) = \sum_{i=1}^{I} \alpha_i \ln u_i \quad \text{with } \alpha_i > 0 \text{ and } \sum_{i=1}^{I} \alpha_i = 1$$

Show that the optimal wealth distribution rule (emerging from the social welfare maximization problem) is

$$w_i(p, w) = \alpha_i w$$

That is, the optimal wealth distribution rule assigns a constant proportion of wealth to every individual, irrespective of the price level.

3. Suppose that a firm owns two plants, each producing the same good. Every plant j's average cost is given by

$$AC_j(q_j) = \alpha + \beta_j q_j \text{ for } q_j \geq 0, \text{ where } j = \{1, 2\}$$

where coefficient β_j may differ from plant to plant, i.e., if $\beta_1 > \beta_2$ plant 2 is more efficient than plant 1 since its average costs increase less rapidly in output. Assume that you are asked to determine the cost-minimizing distribution of aggregate output $q = q_1 + q_2$, among the two plants (i.e., for a given aggregate output q, how much q_1 to produce in plant 1 and how much q_2 to produce in plant 2.) For simplicity, consider that aggregate output q satisfies $q < \frac{\alpha}{\max_j |\beta_j|}$. (You will be using this condition in part b.)

(a) If $\beta_j > 0$ for every plant j, how should output be located among the two plants?

(b) If $\beta_j < 0$ for every plant j, how should output be located among the two plants?

(c) If $\beta_j > 0$ for some plants and $\beta_i < 0$ for others?