

Homework #4 (Due on September 28th, 2020)

1. Consider an Italian household that is seen to purchase quantities of just two goods, spaghetti and cheese. Denote quantities of spaghetti by x and quantities of cheese by y . The household comprises two individuals: Alessandro, whose preference relation can be represented by the utility function $u_A(x, y) = x$ (he is a spaghetti lover) and Beatrice, whose preference relation can be represented by the utility function $u_B(x, y) = y$ (she is a cheese lover).
 - (a) Find the Walrasian demand functions for both Alessandro and Beatrice.
 - (b) Consider that, upon receiving a household wealth level of w , Alessandro and Beatrice agree on evenly dividing the wealth between them (otherwise, a major Italian fight will ensue!). Suppose that you observe the aggregate demands of this household and you interpret it as if it originated from just a single representative consumer. Find the demands of the representative consumer.
 - (c) Assume that the Italian government considers introducing a tax on spaghetti in order to maintain the luxury parties some of their politicians are famous for, and they hire you to evaluate the welfare effects associated to this tax. Recall that the equivalent variation of a change in prices, from p^0 to p^1 is defined as:

$$EV = e(p^0, v(p^1, w^0)) - e(p^0, v(p^0, w^0))$$

If the change in prices is caused by the imposition of a commodity tax on spaghetti, then the deadweight loss (DWL), or excess burden of the tax, is given by:

$$DWL = EV - \sum_{l=1}^L t_l x_l(p^1, w^0)$$

where t_l represents the price change, i.e., $t_l = p_l^1 - p_l^0$, since the new price captures the initial price plus the tax, $p_l^1 = p_l^0 + t_l$. Briefly explain why this measure may be viewed as a deadweight loss for the Italian society.

- (d) Suppose that this Italian household initially faces prices $p^0 = (1, 2)$ and has wealth $w^0 = 300$. In this context, a specific tax of 2 is imposed on spaghetti (i.e., good x) that leads to its price rising to $p_x = 3$ (with the price of cheese, i.e., good y , and the households' wealth both remaining unchanged). Calculate the DWL under the assumption that the household demands originate from one representative consumer.

- (e) Without assuming the existence of a representative consumer, use the individuals' indirect utility functions derived in part (a) to calculate the two individual DWLs. Does its sum coincide with the deadweight loss you found in part (d) where we assumed the existence of a representative consumer?
2. Show that the compensating and the equivalent variation coincide when the utility function is quasilinear with respect to the first good (and we fix $p_1 = 1$). [*Hint*: Use the definitions of the compensating and equivalent variations in terms of the expenditure function (not the Hicksian demand). In addition, recall that if $u(x)$ is quasilinear with respect to good 1, then we can express it as

$$u(x) = x_1 + \phi(x_{-1}),$$

where x_{-1} represents all the remaining goods, $l = 2, 3, \dots, L$.]

3. Consider an individual with utility function $u(q_1, q_2) = q_1^2 + q_2 - 1$, where q_1 (q_2) denotes the units of good 1 (good 2, respectively) that this individual consumes. His income level is denoted by $w \in \mathbb{R}_+$, and prices are both strictly positive, i.e., $\mathbf{p} = (p_1, p_2) \in \mathbb{R}_{++}^2$.
- (a) Determine this individual's Walrasian demand, and his associated indirect utility function.
- (b) Determine this individual's Hicksian demand, $h_1(\mathbf{p}, u)$ and $h_2(\mathbf{p}, u)$, and his associated expenditure function, $e(\mathbf{p}, u)$.

Consider now that this individual's income level is $w = 6$, and the initial vector of market prices is $\mathbf{p}^0 = (4, 3)$. If both prices increase by 50%, determine:

1. (c) The compensating variation of this price increase. Interpret.
- (d) The change in consumer surplus associated to this price increase. Interpret.