# Homework # 2 EconS501 [Due on Sepetember 14th, 2020]

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1. Consider a $K$-commodity setting in which a consumer's Walrasian demand function is

$$x_j(p, w) = \frac{w}{\sum_{k=1}^{K} p_k} \quad \text{for } j = 1, ...K$$

   (a) Show that this demand function is homogeneous of degree zero in $(p, w)$.

   (b) Show that it satisfies Walra's Law.

   (c) Show that the weak axiom is satisfied.

   (d) Identify the Slutsky substitution matrix for this demand function. Show that it is symmetric and negative semidefinite.

2. Consider a consumer with utility function $u(x_1, x_2, x_3)$ for three goods, where the cross-price elasticities are null. Show that the ratio of substitution effects $\frac{s_{23}}{s_{13}}$ is equal to $\frac{\frac{\partial x_2}{\partial w}}{\frac{\partial x_1}{\partial w}}$.

3. Consider an individual with the following utility function for goods $x$ and $y$, and for the numeraire (good $z$, whose price is normalized to \$1),

$$u(x, y, z) = a(x + y) - \frac{1}{2}\left[b(x^2 + y^2) + 2dxy\right] + z$$

   where $b > |d|$. This assumption implies that the consumer regards goods $x$ and $y$ as differentiated, and we return to this property when solving the exercise. Consumer's budget constraint is $w = z + p_x x + p_y y$, where $p_x, p_y > 0$.

   (a) Solve this consumer's utility maximization problem to find his Walrasian demand for goods $x$ and $y$. Show that Walrasian demands are linear in the price of good $x$ and $y$.

   (b) Interpret your results in terms of parameters $b$ and $d$.

   (c) Evaluate the Walrasian demands in the case that $d = 0$. Interpret.

4. Consider utility function $u(x, y)$, where $x$ and $y$ represent the units of two goods. Assume that $u(\cdot)$ is twice continuously differentiable, strictly increasing and concave in both of its arguments, $x$ and $y$. Assuming that the consumer's wealth is given by $w > 0$, and that he faces a price vector $p = (p_x, p_y) >> 0$, denote his indirect utility function as $v(p, w)$.

(a) Use the indirect utility function $v(p, w)$ to find the consumer willingness to pay for good $y$.

(b) Identify under which condition is this willingness to pay for good $y$ increasing or decreasing in income, $w$. Interpret.