Optimal EPA Audit Mechanism With Endogenous Tax

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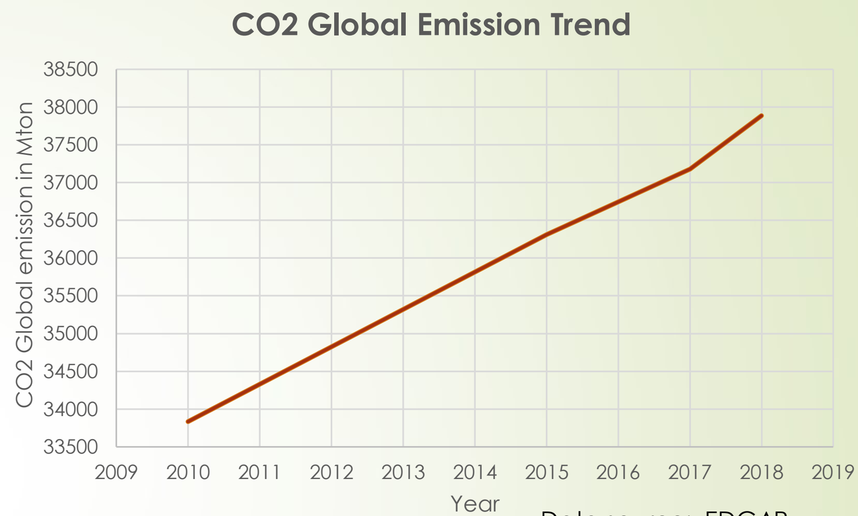
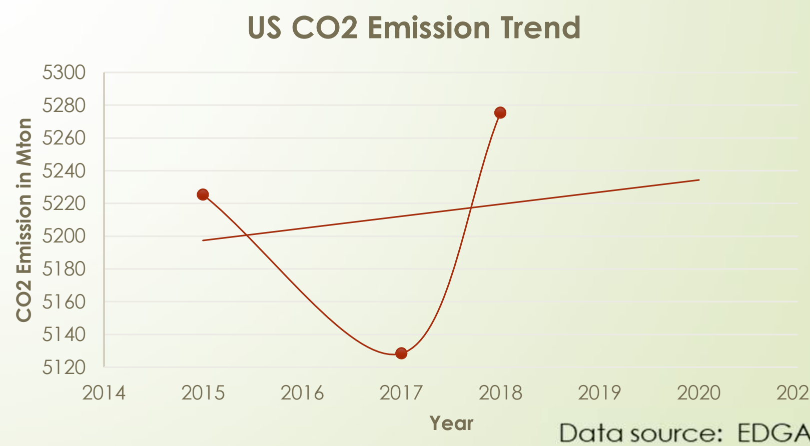
ECONS 582

Motivation

- Global carbon-dioxide emission increased by 11.97 percent from 2010 to 2018
- In USA greenhouse gas emission increased by 3.1 percent from 2017 to 2018



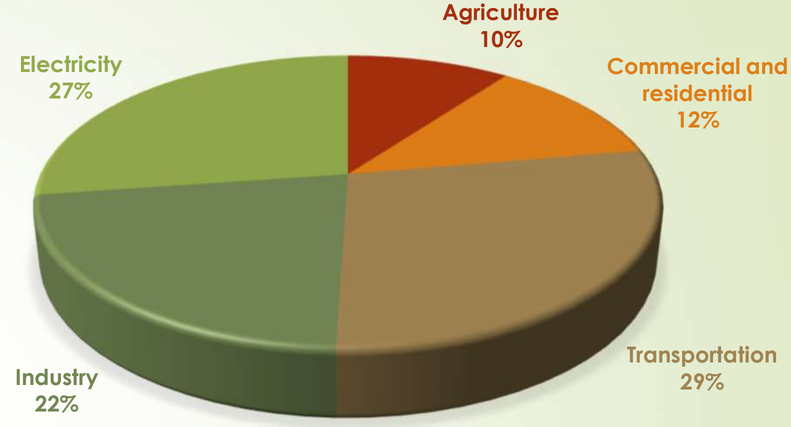
Data source: EDGAR



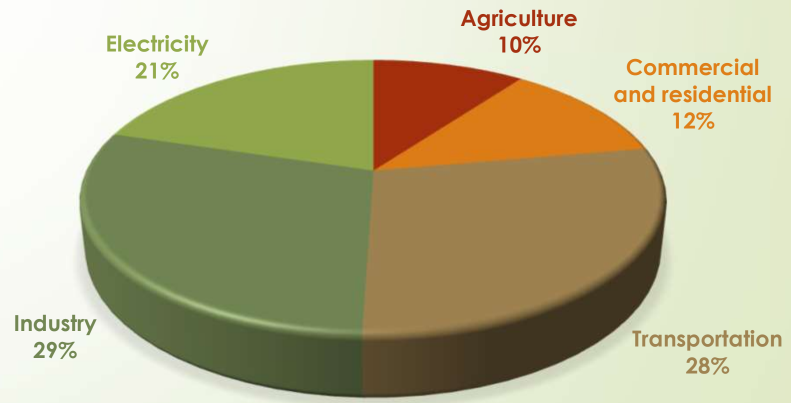
Data source: EDGAR

Motivation

- Considering both direct and indirect emission, industrial sector was the largest contributor of greenhouse gas in 2018, which was 28.9 percent



Us Greenhouse Gas emission sector-wise percentage in 2018 (Direct Emission)



Us Greenhouse Gas emission sector-wise percentage in 2018 (Indirect Emission)

Data Source: EPA's Greenhouse Gas Emission, 2018

Motivation

- US EPA is at the front line of protecting the environment from emission.
- Under GHGRP, approximately 8000 facilities are required to report their emission annually to EPA.
- Due to positive relationship between the level of emission and firms' benefit, how authentic the firms' reporting is a big concern.
- A budget slash has been observed on EPA's operating budget from FY 2010 to 2018 by 14.3 percent.
- Limitation in observing firm's actual emission and bounded budget of EPA intrigue the need to devise an efficient audit mechanism.

Research Question

- What is the optimal environmental audit mechanism when emission tax is endogenous？

Existing Literature and Contribution

- **Macho-Stadler & Perez-Castrillo (2005)**
  - Due to budget and resource constraint, it is optimal to primarily audit **easiest-to-monitor firms and to those firms that value pollutions the less.**
- **Evans, Gilpatric & Liu (2009)**
  - Regulator faces a **trade-off between inducing truthful self-reporting and deterring emissions.**
- **Oestreich (2015)**
  - CAM creates a reporting contest between the firms, which in turn leads to **more truthful reporting compare to RAM.**
- **Oestreich (2017)**
  - Due to fine and budget limitation RAM fails to implement socially efficient emission and author derives **an optimal audit mechanism that capable of implementing socially efficient emission level.**

Existing Literature and Contribution

Macho-Stadler & Perez-Castrillo (2005)	**Evans, Gilpatric & Liu (2009)**	**Oestreich (2015)**	**Oestreich (2017)**	**This Paper**
• Tax is exogenous. • Do not ensure socially efficient emission.	• Tax is endogenous • Do not ensure socially efficient emission.	• Tax is exogenous • Do not ensure socially efficient emission	• Tax is exogenous. • Ensures socially efficient emission. • Does not take into account social cost of untruthful reporting	• Tax is **endogenous.** • Ensures **socially efficient emission.** • Takes into account **social cost of untruthful reporting.**

Assumptions and Model

- Assumptions
  - An industry consisting of two homogenous firms.
  - Firm-i chooses emission level, $e_i \in [0, E]$, where E is the emission level of firms when the pollution is free of cost.
  - One unit of final good produces exactly one unit of emission.
  - Firm-i's inverse demand function is given by $(a - e_i)$
  - EPA cannot perfectly monitor the actual level of emission of a firm without a costly audit.
  - The number of firms to be audited, $k \leq n$.
  - probability of audit, p_i of a firm depends on the level of emission self-reported by the firm i, r_i for $i = 1,2$ and reference value, R (which is exogenous).
  - The audit mechanism is budget-balancing and symmetric.
  - Firms have complete information about each other's emission.

Assumptions and Model

- Assumptions
  - Unit tax, t is endogenous and can be obtained by maximizing the social welfare.
  - $0 < \theta < t$ is levied on unreported emission in addition to tax.
  - As the unit tax, t be endogenous and linear penalty, θ also endogenous.
  - Total cost of audit is assumed to be C and cost of one audit is normalized to 1.
- Model
  - Expected Profit:

$$E\pi_i = (a - e_i)e_i - tr_i - p_i(r_i, r_j)(\theta + t)(e_i - r_i) \text{ for } r_i \leq e_i. \quad (1)$$

  - Expected Social Welfare

$$Esw = E\pi_1 + E\pi_2 + \underbrace{t(r_1 + r_2) + p_1(\theta + t)(e_1 - r_1) + p_2\,(\theta + t)(e_2 - r_2)}_{Revenue\ from\ tax\ and\ penalty} - \underbrace{\frac{d}{2}\left(e_1^2 + e_2^2\right)}_{\substack{Environmental\\ Damage}} + \underbrace{\frac{1}{2}e_1^2 + \frac{1}{2}e_2^2}_{CS} - C \quad (2)$$

Assumptions and Model

- Four-stage Game
  - Stage:1 EPA assigns an audit probability to each of the firms with an aim to lower to firm's emission to the socially efficient level.
  - Stage:2 EPA chooses the unit tax that maximizes social welfare and then set penalty for untruthful reporting.
  - Stage:3 Firms choose the level of emission.
  - Stage:4 Firms choose emission reports.

Model analysis and some results

- Stage Four: Reporting equilibrium
  - $\max_{r_i \le e_i} \pi_1(p_1(r_1, r_2), t, e_1, r_1, r_2) = (a - e_1)e_1 - tr_1 - p_1(r_1, r_2)(\theta + t)(e_1 - r_1)$ (3)
  - Firm-1 to be truthful in reporting if and only if $t(e_1 - r_1) \le p_1(\theta + t)(e_1 - r_1)$
  - $\overline{p} = \frac{t}{(\theta+t)}$ & $\underline{p} = 1 - \frac{t}{(\theta+t)} = \frac{\theta}{(\theta+t)}$.

Model analysis and some results

- Stage Four: Reporting equilibrium

- **Lemma 1.** As long as $p_i \geq \frac{t}{(\theta+t)}$, firms become truthful in reporting the level of emission, whereas, if $p_i \leq \frac{\theta}{(\theta+t)}$, firms report no emissions. It gives audit probability range, $(\underline{p}, \overline{p}) = \left[\frac{\theta}{(\theta+t)}, \frac{t}{(\theta+t)}\right]$.

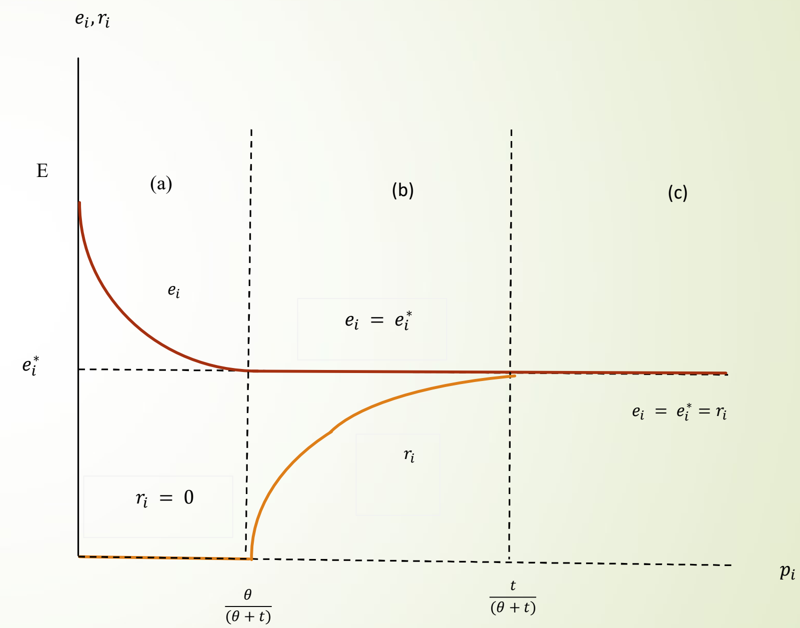


Fig:1 Firm's decision about e_i(blue curve) and r_i (orange curve) with probability of auditing.

Model analysis and some results

- Stage Four: Reporting equilibrium

  - $\underbrace{p_1(r_1^*, r_2)(t+\theta)}_{Direct\ MB} - \underbrace{\frac{\partial p_1(r_1^*, r_2)}{\partial r_1}(t+\theta)(e_1 - r_1^*)}_{Indirect\ MB} = \underbrace{t}_{MC}$ (F.O.C) (4)

  - $\underbrace{2\underbrace{\frac{\partial p_1(r_1, r_2)}{\partial r_1}}_{-}(t+\theta)}_{-} - \underbrace{\underbrace{\frac{\partial^2 p_1(r_1, r_2)}{\partial r_1^2}}_{+}(t+\theta)(e_1 - r_1)}_{+} < 0$ (S.O.C) (5)

  - $r_1^*(e_1, e_2, t, \theta) \in [0, e_1]$ if and only if $\frac{\partial p_1(.)}{\partial r_1} < 0$ at $r_1 = r_1^*$

Model analysis and some result

- Stage Three: Emission equilibrium
  - $\max_{e_i \geq 0} \pi_1(e_1, e_2, r_1^*(e_1, e_2, t, \theta), r_2^*(e_1, e_2, t, \theta)) = (a - e_1)e_1 - tr_1^* - p_1(r_1^*, r_2)(\theta + t)(e_1 - r_1^*)$ (6)
  - $\frac{\partial \pi_1}{\partial e_1} = \underbrace{a - 2e_1 - p_1(t+\theta)}_{Direct\ effect} - \underbrace{\frac{\partial p_1}{\partial r_2}\frac{\partial r_2}{\partial e_1}(t+\theta)(e_1 - r_1)}_{Strategic\ effect} = 0$ (7)
  - Considering budget-balanced audit mechanism from (4) and (7) we obtain:
  - $\underbrace{a - 2e_1}_{MB} = \underbrace{p_1(t+\theta) + \frac{\frac{\partial p_2}{\partial r_2}}{\frac{\partial p_1}{\partial r_1}}\frac{\partial r_2}{\partial e_1}(t - p_1(t+\theta))}_{MC}$ (8)
  - $\frac{\frac{\partial p_2}{\partial r_2}}{\frac{\partial p_1}{\partial r_1}}\frac{\partial r_2}{\partial e_1} = 1$ (9)
  - If equation (9) holds true, firm-1's equilibrium level of emission: $e_1^* = \frac{(a-t)}{2}$ (10)

Model analysis and some result

- Stage Two: Tax equilibrium
  - $\max_{t\geq 0} Esw = (a-t) - \frac{1}{4}(a-t)^2 - \frac{d}{4}(a-t)^2 - C$ (11)
  - $t^* = a\frac{(d-1)}{(d+1)}$ (12)
  - The penalty, θ is being set as, $0 < \theta < a\frac{(d-1)}{(d+1)}$

Model analysis and some results

- Stage One: Designing optimal audit mechanism
  - As our audit mechanism is symmetric, we get, $\frac{\partial p_1}{\partial r_1} = \frac{\partial p_2}{\partial r_2}$
  - From equation (9), we have $\frac{\partial r_2}{\partial e_1} = 1$
  - To check, $\frac{\partial r_2}{\partial e_1}$ totally differentiate first order condition of firm-1 (equation (4)) and firm-2, which yields,

$$\begin{bmatrix} 2\frac{\partial p_1}{\partial r_1}(t+\theta) - \frac{\partial^2 p_1}{\partial r_1^2}(t+\theta)(e_1 - r_1) & \frac{\partial p_1}{\partial r_2}(t+\theta) - \frac{\partial^2 p_1}{\partial r_1 \partial r_2}(t+\theta)(e_1 - r_1) \\ \frac{\partial p_2}{\partial r_1}(t+\theta) - \frac{\partial^2 p_2}{\partial r_2 \partial r_1}(t+\theta)(e_2 - r_2) & 2\frac{\partial p_2}{\partial r_2}(t+\theta) - \frac{\partial^2 p_2}{\partial r_2^2}(t+\theta)(e_2 - r_2) \end{bmatrix} \begin{pmatrix} \partial r_1 \\ \partial r_2 \end{pmatrix} = \begin{bmatrix} \frac{\partial p_1}{\partial r_1}(t+\theta) & 0 \\ 0 & \frac{\partial p_2}{\partial r_2}(t+\theta) \end{bmatrix} \begin{pmatrix} \partial e_1 \\ \partial e_2 \end{pmatrix}$$

  - $$\frac{\partial r_2}{\partial e_1} = \frac{-\left[\frac{\partial p_1}{\partial r_1}(t+\theta)\right]\left[\frac{\partial p_2}{\partial r_1}(t+\theta) - \frac{\partial^2 p_2}{\partial r_2 \partial r_1}(t+\theta)(e_2 - r_2)\right]}{(t+\theta)^2\left[\left\{2\frac{\partial p_1}{\partial r_1} - \frac{\partial^2 p_1}{\partial r_1^2}(e_1 - r_1)\right\}\left\{2\frac{\partial p_2}{\partial r_2} - \frac{\partial^2 p_2}{\partial r_2^2}(e_2 - r_2)\right\} - \left\{\left(\frac{\partial p_1}{\partial r_2}(t+\theta) - \frac{\partial^2 p_1}{\partial r_1 \partial r_2}(t+\theta)(e_1 - r_1)\right)\left(\frac{\partial p_2}{\partial r_1}(t+\theta) - \frac{\partial^2 p_2}{\partial r_2 \partial r_1}(t+\theta)(e_2 - r_2)\right)\right\}\right]}$$

Model analysis and some results

- Stage One: Designing optimal audit mechanism

  - $\Rightarrow \left.\frac{\partial r_2}{\partial e_1}\right|_{r_1=r_2} = \frac{-\left[\frac{\partial p_1}{\partial r_1}(t+\theta)\right]\left[\frac{\partial p_2}{\partial r_1}(t+\theta)\right]}{(t+\theta)^2\left[\left\{2\frac{\partial p_1}{\partial r_1}-\frac{\partial^2 p_1}{\partial r_1^2}(e_1-r_1)\right\}\left\{2\frac{\partial p_2}{\partial r_2}-\frac{\partial^2 p_2}{\partial r_2^2}(e_2-r_2)\right\}-\left\{\frac{\partial p_1}{\partial r_2}\frac{\partial p_2}{\partial r_1}\right\}\right]}$ [Considering both budget-balance and symmetry, $\left.\frac{\partial^2 p_i}{\partial r_i \partial r_j}\right|_{r_i=r_j} = 0$]

  - $\Rightarrow 1 = \frac{-\left[\frac{\partial p_1}{\partial r_1}\right]\left[\frac{\partial p_2}{\partial r_1}\right]}{\left[\left\{2\frac{\partial p_1}{\partial r_1}-\frac{\partial^2 p_1}{\partial r_1^2}(e_1-r_1)\right\}\left\{2\frac{\partial p_2}{\partial r_2}-\frac{\partial^2 p_2}{\partial r_2^2}(e_2-r_2)\right\}-\left\{\frac{\partial p_1}{\partial r_2}\frac{\partial p_2}{\partial r_1}\right\}\right]}$

  - $\Rightarrow 1 = \frac{\left[\frac{\partial p_1}{\partial r_1}\right]\left[\frac{\partial p_1}{\partial r_1}\right]}{\left[\left\{2\frac{\partial p_1}{\partial r_1}-\frac{\partial^2 p_1}{\partial r_1^2}(e_1-r_1)\right\}\left\{2\frac{\partial p_2}{\partial r_2}-\frac{\partial^2 p_2}{\partial r_2^2}(e_2-r_2)\right\}-\left\{\frac{\partial p_1}{\partial r_1}\frac{\partial p_2}{\partial r_2}\right\}\right]}$ [as from budget balancing $\frac{\partial p_1}{\partial r_2} = -\frac{\partial p_2}{\partial r_2}$ & $\frac{\partial p_2}{\partial r_1} = -\frac{\partial p_1}{\partial r_1}$]

  - $\Rightarrow 1 = \frac{1}{\left[\left\{2-\frac{\frac{\partial^2 p_1}{\partial r_1^2}}{\frac{\partial p_1}{\partial r_1}}(e_1-r_1)\right\}^2-1\right]}$ [from symmetry $\frac{\partial p_1}{\partial r_1} = \frac{\partial p_2}{\partial r_2}$]

  - $\Rightarrow 1 = \frac{1}{\left[\left\{2-\frac{\frac{\partial^2 p_1}{\partial r_1^2}}{(\frac{\partial p_1}{\partial r_1})^2}\left(\frac{p_1(t+\theta)-t}{(t+\theta)}\right)\right\}^2-1\right]}$ [from (4) we get, $(e_1 - r_1) = \frac{1}{\frac{\partial p_1}{\partial r_1}}\left(\frac{p_1(t+\theta)-t}{(t+\theta)}\right)$]

Model analysis and some results

- Stage One: Designing optimal audit mechanism
- $\Rightarrow 2-\dfrac{\frac{\partial^2 p_1}{\partial r_1^2}}{\left(\frac{\partial p_1}{\partial r_1}\right)^2}\left(\dfrac{p_1(t+\theta)-t}{(t+\theta)}\right)=\sqrt{2}$
- $\Rightarrow \dfrac{\frac{\partial^2 p_1}{\partial r_1^2}}{\left(\frac{\partial p_1}{\partial r_1}\right)^2}=\dfrac{\sqrt{2}-2}{-\left(\frac{p_1(t+\theta)-t}{(t+\theta)}\right)}=\dfrac{\sqrt{2}-2}{-\left(p_1-\frac{t}{(t+\theta)}\right)}=\dfrac{\sqrt{2}-2}{-\left(p_1-\frac{t}{(t+\theta)}\right)}=\dfrac{\sqrt{2}-2}{\frac{(t-\theta)}{(t+\theta)}}=\dfrac{(\sqrt{2}-2)(t+\theta)}{(t-\theta)}=\dfrac{(\sqrt{2}-2)(t+\theta)}{(t-\theta)}=\dfrac{(\sqrt{2}-2)\left(\frac{a(d-1)}{(d+1)}+\theta(t)\right)}{\left(\frac{a(d-1)}{(d+1)}-\theta(t)\right)}$
- $\Rightarrow \dfrac{\frac{\partial^2 p_1}{\partial r_1^2}}{\left(\frac{\partial p_1}{\partial r_1}\right)^2}=\dfrac{(\sqrt{2}-2)(a(d-1)+\theta(t)(d+1)}{(a(d-1)-\theta(t)(d+1)}$
- $\Rightarrow -\dfrac{1}{\frac{\partial p_1}{\partial r_1}}+\underbrace{\dfrac{(\sqrt{\mathbf{2}}-\mathbf{2})(\boldsymbol{a}(\boldsymbol{d}-\mathbf{1})+\boldsymbol{\theta}(\boldsymbol{t})(\boldsymbol{d}+\mathbf{1})}{(\boldsymbol{a}(\boldsymbol{d}-\mathbf{1})-\boldsymbol{\theta}(\boldsymbol{t})(\boldsymbol{d}+\mathbf{1})}\boldsymbol{R}}_{\textbf{Constant derived from infinite integration}}=\dfrac{(\sqrt{2}-2)(a(d-1)+\theta(t)(d+1)}{(a(d-1)-\theta(t)(d+1)}r_1$
- $\Rightarrow \dfrac{\partial p_1}{\partial r_1}=-\dfrac{1}{\frac{(2-\sqrt{2})(a(d-1)+\theta(t)(d+1)}{(a(d-1)-\theta(t)(d+1)}(R-r_1)}$
- $\Rightarrow p_1(r_1,r_2)=\dfrac{(a(d-1)-\theta(t)(d+1)}{(2-\sqrt{2})(a(d-1)+\theta(t)(d+1)}\ln(R-r_1)+\ v$

Model analysis and some results

- Stage One: Designing optimal audit mechanism
  - As the audit mechanism is both budget-balancing and symmetric,
  - $p_1(r_1, r_2) = \frac{(a(d-1)-\theta(t)(d+1)}{(2-\sqrt{2})(a(d-1)+\theta(t)(d+1)}\ln(R - r_1) + \frac{1}{2} - \frac{(a(d-1)-\theta(t)(d+1)}{(2-\sqrt{2})(a(d-1)+\theta(t)(d+1)}\ln(R - r_2)$
  - $p_1(r_1, r_2) = \frac{1}{2} + \frac{(a(d-1)-\theta(t)(d+1)}{(2-\sqrt{2})(a(d-1)+\theta(t)(d+1)}\ln\left(\frac{R-r_1}{R-r_2}\right)$ at $r_1 = r_2 = r^*$ (13)
  - From lemma 1, $p_1 = \frac{t}{(\theta(t)+t)}$ iff $p_1 \geq \frac{t}{(\theta(t)+t)} \Rightarrow r_1 \leq R - (R - r_2)\exp\left(\frac{a(d-1)(2-\sqrt{2})}{(a(d-1)-\theta(t)(d+1))}\right)$
  - And, $p_1 = \frac{\theta(t)}{(\theta(t)+t)}$ iff $p_1 \leq \frac{\theta(t)}{(\theta(t)+t)} \Rightarrow r_1 \geq R - (R - r_2)\exp\left(\frac{(2-\sqrt{2})\theta(t)(d+1)}{(a(d-1)-\theta(t)(d+1)}\right)$

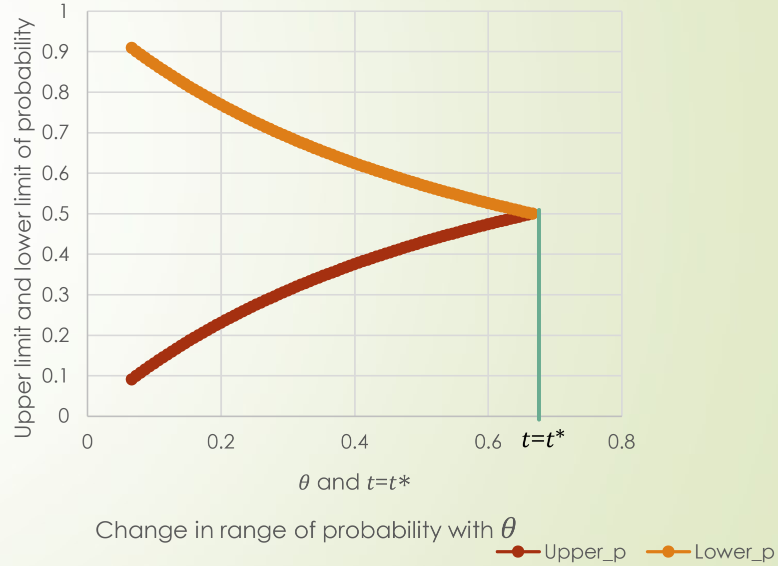
Proposition 1. A speculation for the optimal audit mechanism, which assigns an audit probability to firm-i is given by:

$$p_i(r_i, r_j) = \begin{cases} \frac{\theta(t)(d+1)}{(\theta(t)(d+1)+a(d-1))} & if\ \ r_i > R - (R - r_j)\exp\left(\frac{(2-\sqrt{2})\theta(t)(d+1)}{(a(d-1)-\theta(t)(d+1)}\right) \\ \frac{a(d-1)}{(\theta(t)(d+1)+a(d-1))} & if\ r_i < R - (R - r_j)\exp\left(\frac{a(d-1)(2-\sqrt{2})}{(a(d-1)-\theta(t)(d+1))}\right) \\ \frac{1}{2} + \frac{(a(d-1)-\theta(t)(d+1)}{(2-\sqrt{2})(a(d-1)+\theta(t)(d+1)}\ln\left(\frac{R-r_i}{R-r_j}\right) & otherwise. \end{cases} \quad (14)$$

Model analysis and some results

- Stage One: Designing optimal audit mechanism
  - Analyzing upper limit and lower limit of audit probability depending on change in penalty, θ with tax, $t = t^*$

Proposition 2. If imposed penalty on unreported emission is exactly equal to tax, then RAM ensures socially efficient emission.



Further Research

- Reporting behavior of firms in response to other firms' reporting, own emission and audit probability. (working…)
- Designing audit mechanism using signaling game.

Question?