On Optimal Audit Mechanisms for Environmental Taxes

Author
Andreas Marcel Oestreich

Presented by
Jui Sen

Background & Research Question

- Environmental protection is a priority & challenge in many countries.
- To reduce emission, economists have frequently recommended unit tax on emission that equalizes marginal damages and marginal benefits of emission.
- EPA is in charge of enforcing this tax. But it cannot observe emission directly and needs costly audit to identify actual emission.
- The number of firms that EPA audits is limited to its budget.
- The budget of audit has been downsized due to economic downturn in past several years. For example, the operating budget of US EPA was reduced by 21% from 2010 to 2016.
- At the same time the increasingly complex production process of some firms makes the audit more costly for those firms.
- Smaller budget of EPA and costly audit amplify the need to identify cost-effective audit mechanism.
- This paper aims to identify optimal audit mechanism that induces firms to choose socially efficient emission while also taking into consideration the EPA's limited audit resources.

Related Literature

- Bayer & Cowel (2009)
  - Suggest audit mechanism in dynamic model i.e. CAM .
  - Consider an oligopolistic industry subject to a profit tax and introduce an audit mechanism where the probability of audit a particular firm depends on that firm's tax report relative to others.
- Ostreich (2015)
  - Considers two types of CAM, first one solely rely on firm's tax report relative to others and the second one is imperfectly discriminating audit mechanism i.e the allocation of audit resource does not solely rely on firm's tax report.
  - Compares the incentives on both emissions and emission reports under two types of CAMs to the random audit mechanism.
  - Both CAM lead to more truthful reporting compare to RAM
  - Depending on their exact specification CAM can induce higher or lower emission among regulated firms.
- None of the above paper tailored to achieve socially efficient outcome.
- This paper step by step derive optimal audit mechanism that makes firms behave according to the socially efficient outcome.
- Optimal audit mechanism does not conform to the simplifying assumption of the audit mechanisms suggested by above paper.

Assumptions

- An industry with n firms that creates emissions as a by-product of their production process.
- Per-firm emission e_i.
- Benefit of firm i from emission $g(e_i)$, which is continuously differentiable.
- Benefit function is assumed to be strictly concave with a maximum at emission level e^0.
- Emissions are taxed at a rate t, which is exogenously given.
- We think of t to induce efficient per-firm emissions e^t, if firms comply with it and choose emission according to $g^{'}(e^t) = t$
- Let K be the number of firm the agency can audit, where $K \leq n$.
- The objective of the EPA is to make all firms comply with the environmental tax, that is all firms choose emission, e^t
- EPA assigns an individual audit probability, p_i to each of the firms.
- Probability of audit depends on the relative difference between relative difference between a firm's emission report and a reference value for reported emission relative to other firms.
- When a firm increase its emission report then the firm's assigned audit probability decreases, and the audit probabilities of all other firms increase.

Assumptions (continue..**)**

- The ‘intensiveness of audit competition’ will vary according to EPA’s audit budget, emission tax and possible penalties.
- EPA is not concerned with tax revenue but only with efficiency in emission.
- EPA is not informed about firm’s actual emission and it faces a binding constraint on the number of firms it can afford to audit.
- There is perfect information between the firms.
- Firms pay taxes on reported emission r_i.
- After an audit, the agency can observe the actual emission and potentially levy a linear penalty θ per unit of under-reported emissions where $\theta \geq t$.

Model

Multi-stage Game:

Stage:1 EPA announces an audit mechanism that will map emission report into audit probabilities upon receiving the report.

Stage:2 Firms choose emission which are observable to other firms.

Stage:3 Firms choose emission reports.

Stage:4 Some of the firms are audited according to announced audit mechanism. Fines for potentially under-reported emissions are levied.

Two-firms case:

- n=2 & K=1
- Firm i's ultimate problem is to choose emission e_i and reporting r_i to maximize expected profit:

$$\max_{r_i \leq e_i} \pi_i = g(e_i) - t\, r_i - p_i \theta(e_i - r_i) \text{ for } i = 1,2 \qquad (1)$$

Random Audit Mechanism

- $n = 2$ & $K = 1$
- $p_i = \frac{1}{2}\ \forall\ r_1, r_2$, for $i = 1,2$.
- RAM can fully implement taxes on emission if the expected marginal cost of under-reporting, $\frac{\theta}{2}$, is larger than the tax rate, t.
- Case: Constrained Auditing budget and capped fine
  - Relation t & θ does not lead to socially efficient emission
  - Only one firm is audited
  - Assumption: $\frac{\theta}{2} < t < \theta$.
- Methodology:
  - At stage three firm chooses emission report:
    - From (1): $-t + \frac{\theta}{2}$ is decreasing in r_i. Hence, firms choose $r_i = 0$
  - At stage two firm chooses emission level:(adjust emission according to marginal expected fine.)
    - From (1) : $g'\left(e^{\frac{\theta}{2}}\right) = \frac{\theta}{2}$
  - Since $g(.)$ is strictly concave and $\frac{\theta}{2} < t$, we have $e^t < e^{\frac{\theta}{2}}$.

Propostion:1

If $\frac{\theta}{2} < t$, the RAM fails to enforce socially efficient emissions. Instead, the RAM induces zero reporting, i.e., $r_i = 0$ for $i = 1,2$ and and emissions that are higher in comparison to socially efficient emissions. The emissions per firm under the RAM are denoted by $e^{\frac{\theta}{2}}$, which is implicitly defined by:

$$g'\left(e^{\frac{\theta}{2}}\right) = \frac{\theta}{2} \text{ for } i = 1,2. \qquad (2)$$

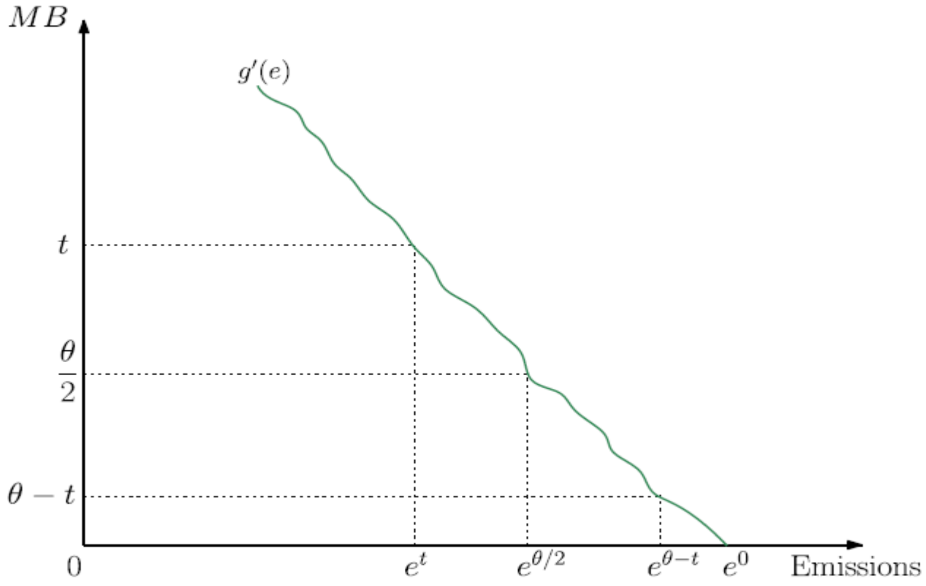

Fig. 1. Illustration of the enforcement problem with emissions per firm on the horizontal axis and marginal benefits (MB) on the vertical axis. The socially efficient emissions level for each firm is e^t while $e^{\theta/2}$ is the higher and socially inefficient per-firm emissions level which results when the common RAM is used.

General Audit Mechanism

- Since RAM is not capable of implementing efficient emissions with capped fines and low auditing budget, more intelligent audit mechanisms are required for this situation.

Definition:1 The audit mechanism is represented by function $p_i: (r_1, r_2) \rightarrow [0,1]$ for $i = 1,2,$ which maps the vector of emission reports into probabilities for each firm of being audited.

- The audit mechanism is supposed to be budget-balancing and symmetric, which we define as the following:

Definition:2 A budget-balancing audit mechanism is defined by:

$$p_1(r_1, r_2) + p_2(r_1, r_2) = 1 \;\; \forall\, r_1, r_2 \geq 0 \qquad (3)$$

- The budget of the agency allows for one audit out of the two firms. Hence (3) implies the probability that firm 1 audited is equal to the probability that firm 2 is not audited and vice-versa.

Definition: 3 A symmetric audit mechanism is defined by:

$$p_1(r_1, r_2) = p_2(r_1, r_2) \; \forall\, r_1, r_2 \geq 0 \qquad (4)$$

- Symmetry of the audit mechanism implies that the audit probability is identical for each firm.
- One implication of symmetry is that if firms' reports coincide then the audit probability is identical for both firms and both firms are audited with probability 1/2.

General Audit Mechanism (cont.)

Lemma: 1 Any differentiable symmetric audit mechanism that exhausts the budget of the EPA for all r_1, r_2 satisfies at $r_1 = r_2$:

$$\left.\frac{\partial^2 p_i}{\partial r_i \partial r_j}\right|_{r_i = r_j} = 0 \text{ for } j \neq i = 1{,}2. \qquad (5)$$

✓ Lemma 1 says that any mechanism allowing a firm to modify its audit probability through its emissions report cannot make the magnitude of this change contingent on the other firm's report when reports coincide.

Proof:

- From (3) $\frac{\partial^2 p_1}{\partial r_1 \partial r_2} + \frac{\partial^2 p_2}{\partial r_1 \partial r_2} = 0 \ \forall \ r_1, r_2 \geq 0$ (26)
- From (4) $\left.\frac{\partial^2 p_1}{\partial r_1 \partial r_2}\right|_{r_1 = r_2} = \left.\frac{\partial^2 p_2}{\partial r_1 \partial r_2}\right|_{r_1 = r_2}$ (29)
- Equation (26) and (29) can both hold if and only if (5) is true.
- If the cross-partial derivative in Lemma 1 was not zero, an audit mechanism would either not be budget-balancing or would not be symmetric.

Methodology:

- The solution concept applied is the SPNE.

General Audit Mechanism (cont.)

Stage:3 Reporting Equilibrium

$$\max_{r_i \leq e_i} \pi_1(p_1(.), e_1, r_1, r_2) = g(e_1) - t\, r_1 - p_1(r_1, r_2)\theta(e_1 - r_1) \quad (6)$$

- Firm chooses emission report to minimize total cost of emission.
- For any $r_i \leq e_i$, it is better for a firm to declare its actual emission if $p_i\theta \geq t$ or $p_i \geq \frac{t}{\theta}$
- Without loss of generality, we can restrict auditing probabilities to a range between the lowest value,

  $\underline{p} = (1 - \frac{t}{\theta})$ and the highest value, $\overline{p} = \frac{t}{\theta}$.

- F.O.C: $$\underbrace{p_1(r_1{}^*, r_2)\theta}_{\substack{Direct\\MB}} - \underbrace{\frac{\partial p_1(r_1{}^*, r_2)}{\partial r_1}\,\theta(e_1 - r_1{}^*)}_{\substack{Indirect\\MB}} = \underbrace{t}_{MC} \text{, at } r_1 = r_1{}^* \in [0, e_1] \quad (7)$$
- S.O.C: $$2\frac{\partial p_1(r_1, r_2)}{\partial r_1} - \frac{\partial^2 p_1}{\partial r_1{}^2}(e_1 - r_1) < 0, \forall\ r_1 \in\ [0, e_1]. \quad (8)$$
- The F.O.C (7) implies that the reporting equilibrium can only be in the interior i.e.: $0 < r_1{}^* < e_1$ if $\frac{\partial p_1}{\partial r_1} < 0$ at $r_1 = r_1{}^*$
- Reporting one more unit of emissions lowers the cost of emissions directly, because the amount of under-reported emissions decreases which lowers the expected fine by $p_1\theta$.

General Audit Mechanism (cont.)

Stage:3 Reporting Equilibrium (cont.)

- Reporting emissions lowers the cost of emissions indirectly, because the audit probability decreases, which lowers the expected fine for the remaining under-reported emissions by $\frac{\partial p_1}{\partial r_1}\theta(e_1 - r_1)$, given that $\frac{\partial p_1}{\partial r_1} < 0$ in equilibrium.
- To check how sensitive the emission report of both firms are to change in the emission by firm 1, the author calculates the values for partials $\frac{\partial r_1}{\partial e_1}$ and $\frac{\partial r_2}{\partial e_1}$

Total differentiation for firm 1: $\frac{\partial^2 \pi_1}{\partial r_1^2}\partial r_1 + \frac{\partial^2 \pi_1}{\partial r_1 \partial r_2}\partial r_2 + \frac{\partial^2 \pi_1}{\partial r_1 \partial e_1}\partial e_1 = 0$

Total differentiation for firm 2: $\frac{\partial^2 \pi_2}{\partial r_2 \partial r_1}\partial r_1 + \frac{\partial^2 \pi_2}{\partial r_2^2}\partial r_2 + \frac{\partial^2 \pi_2}{\partial r_2 \partial e_2}\partial e_2 = 0$

- Totally differentiating the system of F.O.C for firm-1 (7) and firm-2 yields:

$$\begin{bmatrix} 2\frac{\partial p_1}{\partial r_1} - \frac{\partial^2 p_1}{\partial r_1^2}(e_1 - r_1) & \frac{\partial p_1}{\partial r_2} - \frac{\partial^2 p_1}{\partial r_1 \partial r_2}(e_1 - r_1) \\ \frac{\partial p_2}{\partial r_1} - \frac{\partial^2 p_2}{\partial r_2 \partial r_1}(e_2 - r_2) & 2\frac{\partial p_2}{\partial r_2} - \frac{\partial^2 p_2}{\partial r_2^2}(e_2 - r_2) \end{bmatrix} \begin{pmatrix} dr_1 \\ dr_2 \end{pmatrix} = \begin{bmatrix} \frac{\partial p_1}{\partial r_1} & 0 \\ 0 & \frac{\partial p_2}{\partial r_2} \end{bmatrix} \begin{pmatrix} de_1 \\ de_2 \end{pmatrix}. \quad (30)$$

General Audit Mechanism (cont.)

Stage:3 Reporting Equilibrium (cont.)

- Applying crammer's rule:

$$\frac{\partial r_1}{\partial e_1} = \frac{\frac{\partial p_1}{\partial r_1}\left(2\frac{\partial p_2}{\partial r_2} - \frac{\partial^2 p_2}{\partial r_2^2}(e_2 - r_2)\right)}{|D|}, \tag{31}$$

$$\frac{\partial r_2}{\partial e_1} = \frac{-\frac{\partial p_1}{\partial r_1}\left(\frac{\partial p_2}{\partial r_1} - \frac{\partial^2 p_2}{\partial r_2 \partial r_1}(e_2 - r_2)\right)}{|D|}, \tag{32}$$

where $|D|$ is:

$$|D| = \left[2\frac{\partial p_1}{\partial r_1} - \frac{\partial^2 p_1}{\partial r_1^2}(e_1 - r_1)\right]\left[2\frac{\partial p_2}{\partial r_2} - \frac{\partial^2 p_2}{\partial r_2^2}(e_2 - r_2)\right] - \left[\frac{\partial p_1}{\partial r_2} - \frac{\partial^2 p_1}{\partial r_1 \partial r_2}(e_1 - r_1)\right]\left[\frac{\partial p_2}{\partial r_1} - \frac{\partial^2 p_2}{\partial r_1 \partial r_2}(e_2 - r_2)\right]. \tag{33}$$

- The key insight for the following analysis is the observation that the design of audit mechanism influences how strongly a firm strategically changes its emission reports when itself or its competitor changes their emissions.

General Audit Mechanism (cont.)

Stage:2 Emission Equilibrium

$$\max_{e_1 \geq 0} \pi_1(e_1, e_2, r_1{}^*(e_1, e_2), r_2{}^*(e_1, e_2)) = g(e_1) - t\, r_1{}^* - p_1(r_1{}^*, r_2{}^*)\theta(e_1 - r_1{}^*) \quad (9)$$

- Firms simultaneously chooses emission while considering how emissions translate into the reporting equilibrium at stage 3.
- F.O.C: $$\frac{d\Pi_1}{de_1} = \underbrace{g'(e_1) - p_1\theta}_{\text{Direct effect}} - \underbrace{\frac{\partial p_1}{\partial r_2}\frac{\partial r_2}{\partial e_1}\theta\left(e_1 - r_1^*\right)}_{\text{Strategic effect}}, \quad \text{for } e_1 \geq 0.$$
- Direct effect: Higher e_1 may have positive profit implication, if the benefit from emission in the production process increase more quickly than the expected cost of e_1 regardless of any strategic effects.
- Strategic effect: $e_1$changes firm 2's reporting behavior by $\frac{\partial r_2}{\partial e_1}$, which affects the audit probability of firm 1, p_1 as well as the expected fine of unreported emissions by $\frac{\partial p_1}{\partial r_2}\theta(e_1 - r_1)$
- Using (3) and (7) the F.O.C can be written as
- $$\underbrace{g'(e_1)}_{MB} = \underbrace{p_1\theta + \frac{\frac{\partial p_2}{\partial r_2}}{\frac{\partial p_1}{\partial r_1}}\frac{\partial r_2}{\partial e_1}(t - p_1\theta)}_{MC}, \text{ for } e_1 \geq 0 \quad (10)$$

General Audit Mechanism (cont.)
Stage:2 Emission Equilibrium (cont.)

- Two conditions for efficient emission: 1. $t = p_1\theta$ or 2. $\frac{\frac{\partial p_2}{\partial r_2}}{\frac{\partial p_1}{\partial r_1}} \frac{\partial r_2}{\partial e_1} = 1$
- Condition 1 is not feasible: Because then using concept of budget balancing, $p_1 + p_2 = \frac{2t}{\theta} = 1$, which contradict $\frac{\theta}{2} < t$.
- Condition 2 is feasible as the three partial of the L.H.S solely depend on the specific design of the audit mechanism.
- Hence, agency can design audit mechanism in a way that influence the choice of emission favorably, but has to figure out which is the optimal way.
- If firms are uninformed about the other firm's emission $\frac{\partial r_2}{\partial e_1} = 0$ and $g^{'}(e_1) = p_1\theta$
- Considering the assumption of Macho-Stadler and Pérez-Castrillo(2006), marginal incentive of the marginal cost of reducing emission, i.e., the function $e_i(p_i\theta)$ is convex, we find following proposition.

General Audit Mechanism (cont.)

Stage:2 Emission Equilibrium (cont.)

- **Propostion:2** Given firms are uninformed about each other's emissions and the function $e_i(p_i\theta)$ is convex [both assumptions are in Macho-Stadler and Pérez-Castrillo,2006], the RAM (or any other audit mechanism that induces $p_1 = p_2 = 1/2$ at $r_1 = r_2$) induces the lowest feasible emissions level in the industry. However, aggregate emissions are larger than efficient emissions in that case.

Proof: Given that efficient emission is not attainable, EPA makes audit decision in order to minimize emission.

$$\min_{p_1,p_2} e_1(p_1\theta) + e_2(p_2\theta) \text{ s.t. } p_1 + p_2 = 1$$

- Choosing any $p_1 \neq p_2$ is not optimal, because any linear combination of a convex function is above that function, i.e., the agency would end up with higher emission.
- If firms are informed about each other’s emission, the agency can induce more intelligent audit mechanism which induce and harness strategic effect between the firms.

General Audit Mechanism (cont.)

Stage:1 Designing the optimal audit mechanism

Insights from reporting stage

- Anticipating firms' strategic emissions and reporting behavior at stage 2 and stage 3 respectively, this section derives a candidate for the optimal audit mechanism.
- Since firms are symmetric, the author conjectures that there is symmetric SPNE. (In that case emission reports and audit probabilities also coincides)
- When emission reports coincide $\frac{\partial p_1}{\partial r_1} = \frac{\partial p_2}{\partial r_2}$, then from (11) agency is solely concerned about $\frac{\partial r_2}{\partial e_1} = 1$
- It means optimal situation for the agency in equilibrium may occur when firm 2 responds by increasing its reported emission by exactly the same amount as the amount of emission increased by firm 1.
- Using (3) (4) and (32) and Lemma 1 at symmetry:

General Audit Mechanism (cont.)

Stage:1 Designing the optimal audit mechanism (cont.)

Insights from reporting stage (cont.)

- Using (3) (4) and (32) and Lemma 1 at symmetry:

$$\frac{\partial r_2}{\partial e_1} = \frac{-\frac{\partial p_1}{\partial r_1}\frac{\partial p_2}{\partial r_1}}{\left[2\frac{\partial p_1}{\partial r_1} - \frac{\partial^2 p_1}{\partial r_1^2}(e_1 - r_1)\right]\left[2\frac{\partial p_2}{\partial r_2} - \frac{\partial^2 p_2}{\partial r_2^2}(e_2 - r_2)\right] - \left[\frac{\partial p_1}{\partial r_2}\frac{\partial p_2}{\partial r_1}\right]}$$

$$= \frac{-\frac{\partial p_1}{\partial r_1}\frac{\partial p_2}{\partial r_1}}{\left[2\frac{\partial p_1}{\partial r_1} + \frac{\frac{\partial^2 p_1}{\partial r_1^2}}{\frac{\partial p_1}{\partial r_1}}\left(\frac{t}{\theta} - p_1\right)\right]\left[2\frac{\partial p_2}{\partial r_2} + \frac{\frac{\partial^2 p_2}{\partial r_2^2}}{\frac{\partial p_2}{\partial r_2}}\left(\frac{t}{\theta} - p_2\right)\right] - \left[\frac{\partial p_2}{\partial r_2}\frac{\partial p_1}{\partial r_1}\right]}$$

$$= \frac{-\frac{\partial p_2}{\partial r_1}\Big/\frac{\partial p_2}{\partial r_2}}{\left[2 + \frac{\frac{\partial^2 p_1}{\partial r_1^2}}{\left(\frac{\partial p_1}{\partial r_1}\right)^2}\left(\frac{t}{\theta} - p_1\right)\right]\left[2 + \frac{\frac{\partial^2 p_2}{\partial r_2^2}}{\left(\frac{\partial p_2}{\partial r_2}\right)^2}\left(\frac{t}{\theta} - p_2\right)\right] - 1} \tag{38}$$

- We obtain:

$$\left.\frac{\partial r_2}{\partial e_1}\right|_{r_1 = r_2} = \frac{1}{\left(2 + \left(\frac{t}{\theta} - \frac{1}{2}\right)\frac{\partial^2 p_1/\partial r_1^2}{(\partial p_1/\partial r_1)^2}\right)^2 - 1}. \tag{12}$$

General Audit Mechanism (cont.)

Stage:1 Designing the optimal audit mechanism (cont.)

Insights from emission stage

- Combining equation (7) & (8) we get: (which is necessary for a local maximum)

$$\frac{\partial^2 p_1(r_1^*, r_2)/\partial r_1^2}{\left(\partial p_1(r_1^*, r_2)/\partial r_1\right)^2} > -\frac{2}{t/\theta - p_1(r_1^*, r_2)}, \quad at \ r_1 = r_1^* \in [0, e_1].$$

- Setting R.H.S of (12) equal 1 is equivalent to:

$$\left.\frac{\partial^2 p_1/\partial r_1^2}{(\partial p_1/\partial r_1)^2}\right|_{r_1=r_2} = \begin{cases} -\dfrac{\sqrt{2}+2}{t/\theta - 1/2}, \\ -\dfrac{2-\sqrt{2}}{t/\theta - 1/2}. \end{cases}$$

- Using necessary condition for local maximum we can ignore the smaller value of above equation.
- Then relevant characteristics of the optimal audit mechanism leads to:

$$\left.\frac{\partial^2 p_1/\partial r_1^2}{(\partial p_1/\partial r_1)^2}\right|_{r_1=r_2} = -\frac{2-\sqrt{2}}{t/\theta - 1/2}. \tag{13}$$

General Audit Mechanism (cont.)

Stage:1 Designing the optimal audit mechanism

Insights from emission stage (cont.)

- Considering $c \equiv \frac{2-\sqrt{2}}{\frac{t}{\theta}-\frac{1}{2}}$, solving expression for (13) yields (where k & R are constant of integration)

$$p_1(r_1, r_2) = \kappa + \frac{1}{c}\ln(R - r_1), \text{ at } r_1 = r_2 = r^*, \quad (15)$$

Reference value for reported emission

- Reference value, R is an emission level chosen by EPA, which depends on the parameters of the model.
- The gap between the reference value and reported emission influences the assigned audit probability to the reporting firms. **(**The larger the gap, larger the audit probability)
- The author uses $R = e^{\theta-t}$ in the following analysis, defined by $g'\left(e^{\theta-t}\right) = \theta - t$

Implication of symmetry and budget balancing

- To make audit function in (15) symmetric and budget balancing, it is required that, $k = \frac{1}{2} - \frac{1}{c}\ln(R - r_2)$
- Thus, $p_1(r_1, r_2) = \frac{1}{2} + \frac{1}{c}\ln\left(\frac{R-r_1}{R-r_2}\right)$, at $r_1 = r_2 = r^*$ (16)
- Audit mechanism (16) is derived and specific functional form that maps reported emission into audit probabilities in such a way that it gives firms an incentive to choose efficient emission.

General Audit Mechanism (cont.)

Stage:1 Designing the optimal audit mechanism

Limit for the audit probability

- Author has restricted auditing probabilities to the set $[\left(1-\frac{t}{\theta}\right), \frac{t}{\theta}]$ without loss of generality.
- If $p_1(r_1, r_2) \geq \frac{t}{\theta}$, firm 1 is induced to report truthfully. Increasing probability beyond $\frac{t}{\theta}$ cannot improve outcome of the agency.

$$p_1(r_1, r_2) = \frac{t}{\theta}, \text{ if } r_1 \leq R - (R - r_2)\exp(2 - \sqrt{2})$$

- The symmetry of the audit mechanism implies:

$$p_1(r_1, r_2) = 1 - \frac{t}{\theta}, \text{ if } r_1 \geq R - (R - r_2)\exp(-\left(2 - \sqrt{2}\right))$$

The optimal audit mechanism

- A conjecture for the optimal audit mechanism for both firms:
- $$p_i\left(r_i, r_j\right) = \begin{cases} \underline{p} & if \;\; r_1 \geq R - (R - r_2)\exp(-\left(2 - \sqrt{2}\right)) \\ \overline{p} & if \;\; r_1 \leq R - (R - r_2)\exp(2 - \sqrt{2}) \\ \frac{1}{2} + \frac{1}{c}\ln\left(\frac{R - r_1}{R - r_2}\right) & otherwise, \end{cases} \qquad (17)$$

General Audit Mechanism (cont.)

Stage:1 Designing the optimal audit mechanism (cont.)

The optimal audit mechanism (cont.)

- The probability of auditing mainly depends on the relative difference between the wo reports and a reference value R for reported emissions.
- Since, the mechanism is based on a ln-function, it has a natural interpretation.
- When firm i decreases the difference between its emission report and the reference value by one percent, then the firm's assigned audit probability decreased by $\frac{1}{c}$ percentage points.
- Under RAM expected penalty, $\frac{\theta}{2} = t$, then $\frac{1}{c} = 0$
- If $\frac{\theta}{2} < t$, then $\frac{1}{c}$ is positive.
- The smaller the relative audit budget (measured by difference between t and $\frac{\theta}{2}$), the larger the 'intensiveness of competition' induced by the optimal audit mechanism.
- 'intensiveness of competition' means how quickly the audit probability per firm changes in the report.

General Audit Mechanism (cont.)

Stage:1 Designing the optimal audit mechanism (cont.)

The optimal audit mechanism (cont.)

- Figure 2 illustrates audit probability of both firms under the proposed optimal audit mechanism.

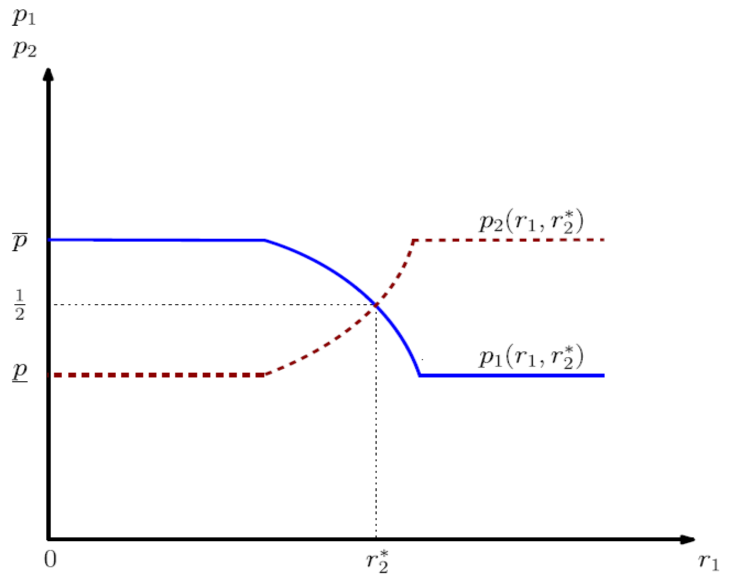

Fig. 2. Sketch of $p_1(r_1, r_2^*)$ and $p_2(r_1, r_2^*)$ under the proposed optimal audit mechanism depending on r_1 with r_2 fixed at $r_2 = r_2^*$.

- $p_1(r_1, r_2{}^*)$, $p_2(r_1, r_2{}^*)$ are measured on the vertical axis dependent on r_1. Report r_2 is fixed at the equilibrium reporting level $r_2{}^*$.
- If the report coincides audit probability of both firms are half.
- Increase in r_1 leads to lower audit probability for firm 1 and higher audit probability for firm 2.

General Audit Mechanism (cont.)
Reporting under the optimal audit mechanism

- Firm 1's stage 3 problem is to choose some $r_1 \in [0, R]$ to maximize:
- $\pi_1 = \begin{cases} -tr_1 - \underline{p}\theta(e_1 - r_1)\ if\ p_1 \leq \underline{p} & Case\ i \\ -tr_1 - p_1\theta(e_1 - r_1)\ if\ p_1 \in \left(\underline{p}, \overline{p}\right) & Case\ ii \\ -tr_1 - \overline{p}\theta(e_1 - r_1)\ if\ \overline{p} \leq\ p_1 & Case\ iii \end{cases}$
- Case:1
  - applies when $e_1 > r_1 \geq R - (R - r_2)\exp(-\left(2 - \sqrt{2}\right))$
  - π_1 is decreasing in r_1, i.e., $\frac{d\pi_1}{dr_1} = 2(\frac{\theta}{2} - t) < 0$
  - Hence, it can never be optimal to report
    - $r_1 > R - (R - r_2)\exp(-\left(2 - \sqrt{2}\right))$ at $r_2 > 0$
    - $r_1 > R(1 - \exp\left(\sqrt{2} - 2\right))$ at $r_2 = 0$
- Case 2:
  - $\frac{1}{2} + \frac{1}{c}\ln\left(\frac{R - r_1}{R - r_2}\right) \in \left(\underline{p}, \overline{p}\right)$
  - r_1^{int} is implicitly defined by, $\frac{d\pi_1}{dr_1} = 0$, i.e.,

General Audit Mechanism (cont.)

Reporting under the optimal audit mechanism (cont.)

$$\frac{e_1 - r_1^{int}}{R - r_1^{int}} + \ln\left(\frac{R - r_1{}^{int}}{R - r_2}\right) - 2 + \sqrt{2} = 0 \quad \text{at } r_1 = r_1{}^{int} \qquad (19)$$

- The S.O.C holds concavity with certainty as long as $R > e_1$, which is guaranteed with $R = e^{\theta - t}$
- Interior part of the best response function r_1^{int} is increasing in a convex manner in r_2 . (It can be seen totally differentiating (19) and solving for $\frac{\partial r_1}{\partial r_2}$ and $\frac{\partial^2 r_1}{\partial r_2^2}$ respectively.)

$$\frac{\partial r_1}{\partial r_2} = \frac{(R - r_1)^2}{(R - r_2)(2R - e_1 - r_1)} > 0,$$

$$\frac{\partial^2 r_1}{\partial^2 r_2} = \frac{(R - r_1)^2}{(R - r_2)^2(2R - e_1 - r_1)} > 0.$$

- We can see from (19) firm 1 chooses $r_1 = 0$, if $r_2 < R - \frac{R}{\exp\left(2 - \sqrt{2} - \frac{e_1}{R}\right)}$
- From case (i) $r_1 = R - (R - r_2)\exp(-(2 - \sqrt{2}))$ in case $r_1{}^{int} > R - (R - r_2)\exp(-(2 - \sqrt{2}))$
- Combining the latter with (19): $\overline{r_2} = R + \frac{R - e_1}{(3 - 2\sqrt{2})\exp(\sqrt{2} - 2)} > e^{\theta - t}$, i.e. $\overline{r_2} \notin [0, e^{\theta - t})$
- Hence, resulting BRF never leads to existence of case (i)
- Consequently, in all reporting equilibrium we have $p_1 \in (\underline{p}, \overline{p})$

General Audit Mechanism (cont.)

Reporting under the optimal audit mechanism (cont.)

Case 3:

applies when $if\ \ r_1 \leq R - (R - r_2)\exp(2 - \sqrt{2})$

profit does π_1 not change in r_1, i.e. $\frac{d\pi_1}{dr_1} = 0$

In order to simplify exposition, the author assumes $r_1 = e_1$, without loss of generality.

Aside $e_1 \leq R - (R - r_2)\exp\left(2 - \sqrt{2}\right) \equiv \frac{R - r_2}{R - e_1} < \frac{1}{\exp\left(2 - \sqrt{2}\right)} \approx 0.56$, which necessitates $r_2 > e_1$

Propostion:3 The best response function of firm 1 in term of reporting is given by:

$$r_1(e_1, r_2) = \begin{cases} 0 \quad if r_2 < R - \frac{R}{\exp(2 - \sqrt{2} - \frac{e_1}{R})} \\ e_1 \qquad if\ r_2 > R - \frac{R - e_1}{\exp(2 - \sqrt{2})} \\ r_1^{int}(e_1, r_2)\ otherwise \end{cases} \tag{18}$$

Where the interior reporting best response function $r_1^{int}(e_1, r_2)$ is increasing in e_1 and r_2 as implicitly defined by the first-order condition for a profit-maximizing reporting choice:

$$\frac{e_1 - r_1^{int}}{R - r_1^{int}} + \ln\left(\frac{R - r_1{}^{int}}{R - r_2}\right) - 2 + \sqrt{2} = 0 \quad \text{at } r_1 = r_1{}^{int} \tag{19}$$

Sketch of BRF of firm 1 and firm 2

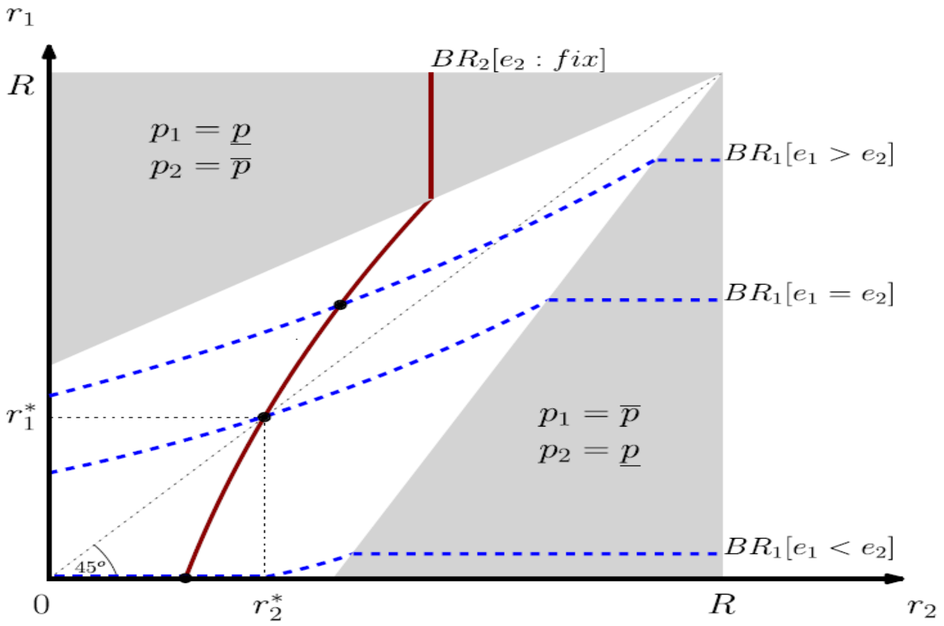

Fig. 3. Sketch of the best reporting response functions for various levels of emissions e_1 with e_2 fixed at $e_2 = e^t$. The curve $BR_2[e_2$: *fix*] is the best response function of firm 2 holding e_2 fixed at $e_2 = e^t$. The curve $BR1[.]$ is the best response function of firm 1 for smaller, equal and larger e_1 in relation to e_2. All three illustrated SPNE are marked with black dots.

- Report of firm 1 on vertical axis and firm 2 on horizontal axis.
- When firms' reports are close together (both are near the 45°-line) then the audit probabilities are in the interior, i.e. have $p_1 \in \left(\underline{p}, \overline{p}\right)$ for $i = 1,2$ which is the situation within the white cone.
- In this case, both firms report some of their emissions, while none of the firms reports truthfully.
- In Fig. 3 the curve $BR_2[e_2, fix]$ is the best response function of firm 2 holding e_2 fixed $e_2 = e^t$.
- The three curves present the $BR_1[.]$ of firm 1 for smaller, equal and larger e_1 in relation to e_2.
- All three illustrated SPNE are marked with three black dots.
- None of the SPNE are outside of the white cone.

- **Propostion:4** The audit mechanism in (17) induces a unique and pure strategy reporting equilibrium at stage 3 of the game for any $e_{i\in}[0, e^{\theta-t})$. In any of these equilibria the audit probability is in the interior, i.e.: $p_i \in \left(\underline{p}, \overline{p}\right)$ for for $i = 1{,}2$.

Proof:

- From the proof of proposition 3 (case ii) : every $e_i \in (0, e^{\theta-t})$ leads to reporting equilibrium that cause $p_i \in \left(\underline{p}, \overline{p}\right)$ for for $i = 1{,}2$.
- Again, from case (ii), $r_1 = 0$, if $r_2 \leq R - \frac{R}{\exp\left(2-\sqrt{2}-\frac{e_1}{R}\right)}$
- In this case, the best response function of firm 2 is implicitly define by its best response function $\frac{e_2-r_2}{R-r_2} + \ln\left(\frac{R-r_2}{R}\right) - 2 + \sqrt{2} = 0$
- Under the optimal mechanism determinant (33) is positive whenever $p_i \in \left(\underline{p}, \overline{p}\right)$, which is equivalent to:

$$\left[-2 + \underbrace{\frac{e_1 - r_1}{R - r_1}}_{(+)\in(0,1)}\right]\left[-2 + \underbrace{\frac{e_2 - r_2}{R - r_2}}_{(+)\in(0,1)}\right] > 1.$$

- **Propostion:4** The audit mechanism in (17) induces a unique and pure strategy reporting equilibrium at stage 3 of the game for any $e_{i\in}[0, e^{\theta-t})$. In any of these equilibria the audit probability is in the interior, i.e.: $p_i \in \left(\underline{p}, \overline{p}\right)$ for for $i = 1,2$.

Proof: (cont.)

- The set of second-order conditions (8) hold for both firms whenever $p_i \in \left(\underline{p}, \overline{p}\right)$ for $i = 1,2$.

$$-2 + \underbrace{\frac{e_i - r_i}{R - r_i}}_{(+)\in(0,1)} < 0, \quad \mathit{for}\ i = 1, 2.$$

- Both conditions imply global uniqueness of the reporting equilibrium.

- **Proposition:5** The audit mechanism in (17) induces a symmetric reporting equilibrium at

$$e_1 = e_2 = e^t \text{ given by: } r^* = \frac{e^t - R(2-\sqrt{2})}{\sqrt{2}-1} \tag{20}$$

Reporting is zero if $e^t < R(2-\sqrt{2})$, where $R = e^{\theta-t}$.

Proof:

Combining the interior best response function r_1^{int} and r_2^{int} implicitly defined by (19) yields:

$$\frac{e_1 - r_1}{R - r_1} + \frac{e_2 - r_2}{R - r_2} = 4 - 2\sqrt{2}, \quad \text{for } r_1, r_2 \in [0, R). \tag{35}$$

- **Proposition:5** The audit mechanism in (17) induces a symmetric reporting equilibrium at $e_1 = e_2 = e^t$ given by: $r^* = \frac{e^t - R(2-\sqrt{2})}{\sqrt{2}-1}$ (20)

Reporting is zero if $e^t < R(2-\sqrt{2})$, where $R = e^{\theta - t}$.

Proof: (cont.)

- Evaluating (35) at $e_1 = e_2 = e^t$ and $r_1 = r_2 = r^*$ leads to the candidate r^*
- Plug r^* into (8) leads to a negative value, i.e., there is a local maximum at r^*.
- Since, the local maximum is the only stationary point, it has to be global maximum.

Explanation:

- Equilibrium reporting $r^*$is decreasing in R and that equilibrium reporting is never truthful, given that $R = e^{\theta - t} > e^t$.
- Equilibrium reporting is positive if $e^t > R(2-\sqrt{2})$
- Recall, the functioning of the proposed audit mechanism relies on an equilibrium which is non zero.
- As under-reporting of emission is needed to generate strategic effect, under the optimal audit mechanism it is not possible to achieve truthful reporting in equilibrium.
- The equilibrium level of reporting under the optimal audit mechanism is higher in comparison to the level of reporting under RAM when audit resources are low.
- The smaller the reference value, R, the larger is the reporting level in equilibrium.
- However, the smaller the R, the larger is the possibility that the symmetric SPNE does not exist.

➢ **Proposition:6a** Under the optimal audit mechanism, the reports of both firms increase (decrease) when one of the firms increases (decreases) its emissions. The firm that changes its emissions chooses a larger change in its reported emissions than the other firm, i.e.,

$$\frac{\partial r_1^*}{\partial e_1} > \frac{\partial r_2^*}{\partial e_1} > 0, \ \forall \ e_1, e_2 \in [0, e^{\theta - t}],$$

Whenever reports are positive.

Explanation:

- If firm 1 increases its emission by 1 unit, firm 2 increases its emission report by 1 unit.
- Then firm 1 finds its worthwhile to increase its emission report more than firm 2.
- This makes it rather unattractive for imr 1 to increase emission.

Propostion:6b. The audit probability of firm 1, $p_1\left(r_1^*(e_1, e_2), r_2^*(e_1, e_2)\right)$ decreases in $e_1$and increases in e_2.

$$\frac{\partial p_1}{\partial e_2} > 0 > \frac{\partial p_1}{\partial e_1}, \ \forall \ e_1, e_2 \in [0, e^{\theta - t}].$$

Explanation:

Audit probability of a firm decrease in its own emission and increases in its competitor’s emission.

Figure to illustrate the insights of propositions 6a and 6b

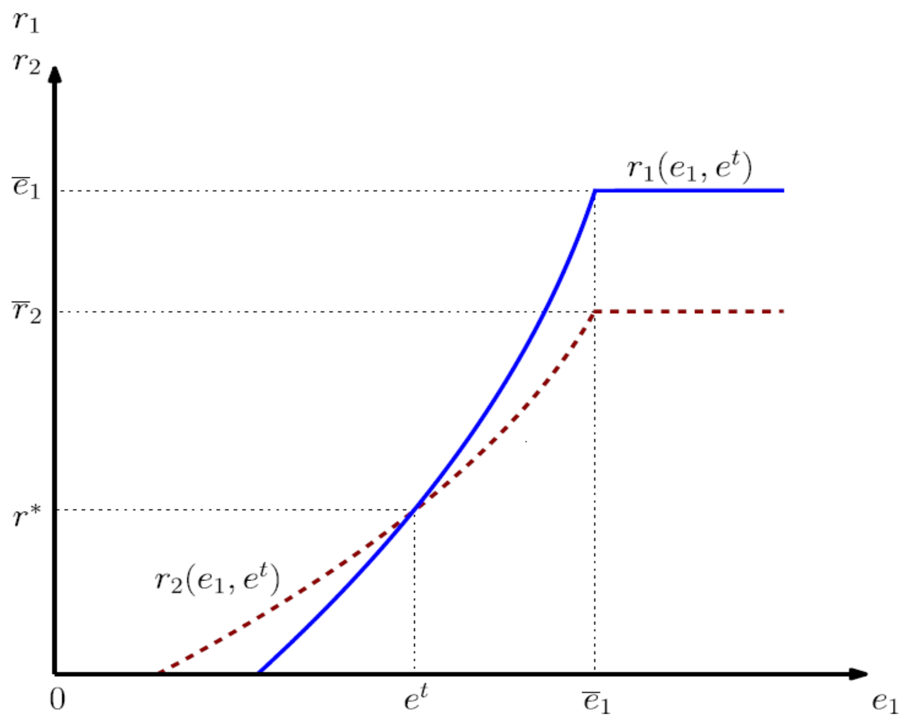

Fig. 4. Sketch of $r_1(e_1, e^t)$ and $r_2(e_1, e^t)$ under the proposed optimal audit mechanism depending on e_1 with e_2 fixed at $e_2 = e^t$.

- Report of firm 1 and firm 2 on vertical axis and emission of firm 1 on the horizontal axis.
- If firm 1 unilaterally deviates upward from $e_1 = e_2 = e^t$ to $e_1 > e^t$ then $r_2(e_1, e^t)$ increases by the exact same amount.

Why the optimal audit mechanism Works?

- In the equilibrium of the game, the two firms choose efficient emission and they are both assigned an audit probability of $\frac{1}{2}$ by the EPA.
- If a firm deviates upwards and chooses higher emissions this firm would benefit directly from its emissions.
- At the reporting stage, the marginal benefit from reporting higher emissions increases for the deviating high-emissions firm, so it subsequently increases its report.
- Proposition 6b says that increasing emissions decreases its own audit probability and increases the audit probability of the low-emissions firm.
- As a strategic reaction, the non-deviating low emissions firm will also increase its reported emissions because, given its increased audit probability, the marginal benefit from reporting higher emissions has increased as well. (exactly same amount)
- As a result, the high-emissions firm finds itself forced to increase its report even more than the low-emissions firm to win the reporting competition, by which we mean that the high-emissions firm ends up with a lower audit probability. (proposition 6a).
- Thus, the high-emissions firm increases its reported emissions over proportionately and faces increased tax payments.

Why the optimal audit mechanism Works? (cont.)

- The outcome of the reporting competition is that the high-emissions firm is assigned an audit probability less than 1/2 and the low-emissions firm is assigned an audit probability greater than ½.
- Higher emissions result in higher benefits and a lower expected fine for under-reported emissions. These two benefits are offset by an overproportionate increase in reporting and hence higher tax payments.
- At the margin, the optimal audit mechanism leads by design to a marginal cost of emissions that is exactly equal to tax t.

➢ **Theorem:1** If the audit mechanism satisfies budget-balancedness and symmetry, then the audit mechanism in (17) induces a symmetric pure strategy emissions equilibrium, where emissions are socially efficient, i.e., $e_1 = e_2 = e^t$, implicitly defined by $g^{'}(e^t) = t$.

The *n*-firm case

- $n > 2$ and $K \geq 1$
- Audit ratio, $k \equiv \frac{K}{n}$
- Assumption 1. The relation between tax t, fine θ and the fraction of firms that can be audited k is given by:

$$k\theta < t < \theta$$

The *n*-firm case (cont.)

- The derived optimal audit mechanism for n firms:

$$p_1(r_1, \ldots, r_n) = \begin{cases} \underline{p} & \text{if } p_1 \le \underline{p}, \\ \overline{p} & \text{if } p_1 \ge \overline{p}, \\ k + \frac{1}{c(n-1)} \ln\left(\frac{(R - r_i)^{n-1}}{\prod_{j \neq i}^{n} (R - r_j)} \right) & \text{otherwise.} \end{cases} \quad (23)$$

- $\overline{p} = t/\theta$ and $\underline{p} = K - (n-1)t/\theta$
- Constant c is:

$$c \equiv \frac{2 - N}{t/\theta - k}, \text{ where: } N = \frac{n - 2 + \sqrt{n^2 + 4n - 4}}{2(n-1)}$$

- N is decreasing in a convex manner in the number of firms n such that $N = \sqrt{2}$ when $n = 2$ and $N \to 1$ when $n \to \infty$.
- The optimal audit mechanism requires reporting equilibrium to be positive.

- **Theorem:1** If the audit mechanism satisfies budget-balancedness and symmetry, then the audit mechanism in (23) induces a symmetric pure strategy emissions equilibrium, where emissions are socially efficient, i.e.: $e_1 = \cdots = e_n = e^t$, implicitly defined by $g'(e^t) = t$.

Findings

- Emission reports of firms are useful for implementing efficient emission even though they are not truthful.
- When firms have no information about each other's emission and EPA has limited resources, the RAM is the most efficient audit mechanism to reduce emissions.
- When firms have complete information about other firms' emission, EPA cannot ensure efficient emission level using RAM with limited resources.
- Deriving an audit mechanism framework that induces a symmetric pure strategy emissions equilibrium, where emissions are socially efficient.

Contribution

- First, it shows that it is possible to enforce socially efficient emissions among regulated firms even if the expected cost of non-compliance using random auditing is lower than the expected cost of compliance.

- Second, the author explicitly derived an audit mechanism for a specific enforcement problem which many EPAs around the globe may face. This is in contrast to some of the previous literature where the audit mechanisms presented were assumed to be exogenous to the enforcement agency (Bayer and Cowell, 2009; Oestreich, 2015) and they did not achieve a socially efficient outcome.

- Third, while it has been argued elsewhere that emission reports are not useful for the EPA when they are not truthful, we find that the reports can be used to implement efficient behavior even though they are not truthful.

- Finally, Theorem 1 strengthens the idea of implementing audit mechanisms that use as a basis the difference between reported emissions and a reference value for reported emissions.

Application of this model

- Applicable for any enforcement agency that makes audit decisions after having received imperfect, costly signal from regulated subject about their compliance effort.

- Other application includes capital requirements for financial institutions, quality control and the monitoring of corporate social responsibility activities.

Limitation & Future Research

- While the optimal audit mechanism induces firms to choose socially efficient emissions, it does not induce truthful reporting when audit resources are low. Considering the social cost of untruthful reporting could be an avenue for future research.

- In this paper, firms are assumed to be symmetric. It is not clear whether efficient emission could be implemented when there is asymmetry between firms.

- The optimal audit mechanism is designed under the assumption that firms have perfect information about each other's emissions. It would be a valuable extension to derive an optimal audit mechanism in a framework of imperfect information among the EPA and the firms.