Global Warming: Prices versus quantities from a strategic point of view

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Outline of the Presentation

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What Is This Research about?-Main Idea

- To determine how strategic choices of instruments affect the interactions in a stock externality game (Global Warming) between cartalized fossil fuel supplier and consumers.

Background Information/Motivation

- The noticeable switch of instruments between fossil fuel suppliers and consumer government.
- For instance, OPEC used price strategies up to 1985 but since 1986 has announced production quotas and allow the market demand to determine the price
- On the other hand, oil products are heavily taxed in most parts of the world, for instance Europe. Despite the fact that oil products are heavily taxed, IEA family prefers quantity instrument for global warming mitigation e.g. Emission Trading System and waxman markey cap and trade bill.

Problem Statement, Research Question and Objective

- However, since the alternative way of regulating the suppliers of fossil fuels is infeasible because they are governments of foreign countries. Then,
- **Broad Research Question:** How do we determine the choice of instruments that could help in mitigating Global warming?
- **Specific Research Question:**What are the outcomes of different combinations of instruments?
- **Objective:**The paper investigates the strategic choices of quantity or price instruments in the dynamic context of global warming when the fossil fuel suppliers and consumers interact

Literature Review: Issues

Non strategic context:

- Weitzman(1974): Mixed price and quantity can give best result;Hoel and Karp(2001): Taxes dominates quotas; Pizer (1999): Under uncertainty, taxes dominants emissions rate control; Pizer (2002): Taxes are efficient than permits for global warming mitigation. Newell and Pizer (2003); price element in quantity-based policies has a great value

Strategic Context:

- Karp (1988): Welfare under Nash quotas is higher than under Nash tariffs; Wirl (1993): Energy taxes may raise the import price of energy; Karp and Zhang (2016): Tax has more leverage than quotas; Strand (2010): Tax policy dominates a cap policy; Montero (2011): Under government commitment, quantity plus subsidies dominate price in fostering R&D.

- Previous studies considered asymmetric differential game and derived equilibria in Markov strategies when a benevolent government of consumers and a cartel of fossil fuel suppliers play in prices (Wirl, 1994; Tahvonen, 1996; Rubio and Esriche, 2001; Liski and Tahvonen, 2004; Wirl, 2007; Dullieuxetal, 2011).

Contribtion to Knowlegde

- This study extends the framework of previous studies by constraining global warming ex ante to include quantities-permits issued by the government and cartelized suppliers of fossil fuel in order to investigate the strategic choices of the instruments.
- The study also considers a situation where one party is able to commit in the short run and thus announces its action prior to the opponent's action. This assumption provides an alternative method should in case the simultaneous move game does not allow for an interior outcome.
- **Question:** How does it differ from Wirl, 1993; 1994; 2007?

Assumptions of Model

- Consumers are passive and the other two act strategically (i.e fossil fuel suppliers and government
- Model ignores depreciation rate
- Model ignores resource constraints
- No extraction costs
- More countries join a global warming compact (Optimistic) and increased cartelization (Pessimistic)

Definition of variables

- q: amount of fossil fuels consumed
- p: price received by the producers after tax or price for a permit has been deducted
- P: final price paid by consumers
- r: discount rate($r > 0$)
- τ: Tax
- π: Price for a permit
- $V(X)$: Producer value function
- $W(X)$: Government value function
- X: Stock externality
- c: external cost

Model and Analysis: Model Setup

Consumers:

$$U(q) = \left(q - \frac{q^2}{2}\right) - pq \tag{1}$$

Where $U(q)$ is the consumer surplus, which consist of the normalized linear-quadratic utility minus the expenses. Hence, $U'(q) = 1 - q = P$ (Market Clearing)

Stock Externality:

$$\dot{X} = q, X(0) = 0 \tag{2}$$

$$C(X) = \frac{c}{2}X^2 \tag{3}$$

Where $\dot{X}$ is the accumulation of CO_2 in the atmosphere and $C(X)$ is the net contribution of emission to global warming

Producer:

$$V(X) = max \int_0^\infty e^{-rt} p(t)q(t)dt \tag{4}$$

Where $P - p = \tau$ or π

Government:

$$W(X) = max \int_0^\infty e^{-rt} \left[q(t) - \frac{q^2(t)}{2} - p(t)q(t) - C(X(t)) \right] dt \tag{5}$$

(4) and (5) are subject to

$$\dot{X} = q, X(0) = 0 \tag{6}$$

$$U'(q) = 1 - q = P \tag{7}$$

Producer:

$$\max_{q(t),X(t)} H = P(t)q(t) - \lambda(t)q(t) \tag{8}$$

Government:

$$\max_{q(t),X(t)} H = q(t) - \frac{q^2(t)}{2} - p(t)q(t) - \frac{c}{2}X^2(t) - \lambda(t)q(t) \tag{9}$$

solving (8) and (9) simultaneously by applying Pontryagain's maximum principle and invoking the fact that at steady $\ddot{X}(t) = 0$ and $\dot{X}(t) = 0$

$$X^* = r/c \tag{10}$$

where X^* is the non-cooperative dynamic game socially efficient outcome with stationary pollution

Model and Analysis: Markov Perfect Equilibria(MPE)

- The MPE analysis is categorized into two strategies:
  - The simultaneous move Nash equilibria.
  - The first mover advantages.
- Going forward, the study analyses different combination of instruments by representing equation (4) and (5) as a dynamic programming using Hamilton-Jacobi-Bellman (HJB) equations.
- All HJBs solutions are subjected to emissions stopping condition (i.e. value matching($V(X_\infty) = 0$ and $W(X_\infty) = -\frac{c}{2r}X_\infty^2$)) and smooth pasting ($V'(X_\infty) = 0$ and $W'(X_\infty) = -\frac{c}{r}X_\infty^2$)
- Due to no depreciation, the stopping level is the identical steady state , which is denoted by X_∞ (i.e. $q = 0 \; \forall \; X > X_\infty$)

Model and Analysis: Markov Perfect Equilibria(MPE)

Prices versus taxes:

Substitute $q = 1 - p - \tau$ into the HJB equations of (4) and (5)

$$rV = \max_{p} (1 - p - \tau)(p + V') \tag{11}$$

$$rW = \max_{\tau} \left\{ \frac{(1-p)^2}{2} - \frac{\tau^2}{2} - \frac{c}{2}X^2 + W'(1 - p - \tau) \right\} \tag{12}$$

Simultaneous maximization of (11) and (12) gives

$$p = \frac{1 - V' - W'}{2}; \; \tau = -W'; \; P = p + \tau = \frac{1 - (V' + W')}{2} \tag{13}$$

substituting (13) into (11) and (12) gives

$$rV = \frac{(1 + (V' + W'))^2}{4}, \tag{14}$$

$$rW = \frac{(1 + (V' + W'))^2}{8} - \frac{c}{2}X^2 \tag{15}$$

Model and Analysis: Markov Perfect Equilibria(MPE)

(14) and (15) can be combined and rewrite as follows:

$$rJ = \frac{3(1+J')^2}{8} - \frac{c}{2}X^2 \tag{16}$$

where $J = V + W$. Applying guess and verify approach ($J = J_1 X + 1/2 J_2 X^2$) gives

$$\widetilde{P} = 1 + \frac{(\sqrt{r^2+3c} - r)}{3}\left(X - \frac{r}{c}\right) \tag{17}$$

$$\widetilde{p} = \frac{r\sqrt{r^2+3c} - (r^2+6c)}{9r}\left(X - \frac{r}{c}\right) \tag{18}$$

$$\widetilde{\tau} = 1 + \frac{2r\sqrt{r^2+3c} - 2r^2 + 6c}{9r}\left(X - \frac{r}{c}\right) \tag{19}$$

Model and Analysis: Markov Perfect Equilibria(MPE)

From (17), (18) and (19) proved the following proposition
Proposition 1: *The consumer price and tax increase up to the choke or backstop price, P and $\tau \to 1$*

$$X_\infty = X^* = \frac{r}{c} = \left(X^* - \frac{r}{c}\right) = 0 \tag{20}$$

Therefore (10) is socially efficient. However, the intertemporal allocation is not efficient since $\widetilde{P}$ is lower for all $X < X_\infty$ and $\widetilde{p}$ declines monotonically to zero, but yet exceed the monopoly price ($p = 1/2$)

$$X < \widetilde{X} = \frac{r^2 + \frac{3}{2}c - \sqrt{r^4 + 3r^2c}}{r^2 + 6c - \sqrt{r^4 + 3r^2c}} X_\infty \tag{21}$$

$(\widetilde{X}/X_\infty)$ with $\lim_{c\to\infty}$*$(\widetilde{X}/X_\infty = 1/4)$*(See: fig 1)
Note:the stopping condition is satisfied.

Prices versus taxes under short run commitments:

- Suppliers reproduce the same result above.
- For government as the first mover, (13) is substituted into the right hand side (12) and then maximizing with respect to τ gives

$$\tau = \frac{1 + v' - 2W'}{3}; p = \frac{1 + W' - 2V'}{3}; P = \frac{2 - (V' + W')}{3} \tag{22}$$

substituting (22) into (11) and (12) yields

$$rV = \frac{(1 + (V' + W'))^2}{9} \tag{23}$$

$$rW = \frac{(1 + (V' + W'))^2}{6} - \frac{c}{2}X^2 \tag{24}$$

The two functional equations are identical to those of taxes versus production quotas under short run commitment. Therefore, the results are shown in the following section.

Model and Analysis: Markov Perfect Equilibria(MPE)

Taxes versus quotas:

Substitute $p = 1 - q - \tau$ into HJB equations of (4) and (5)

$$rV = \max_{q} (1 - q - \tau)q + V'q \tag{25}$$

$$rW = \max_{\tau} \left\{ q\tau + \frac{q^2}{2} - \frac{c}{2}X^2 + W'q \right\} \tag{26}$$

Since tax is linear in the right side of (26), the only Nash equilibrium is inefficient at emission free corner, $(\tau = 1, q = 0)$. Therefore, this problem provides us with a single solution in this section, which is given as follows:

proposition 2:*An interior equilibrium in Markov strategies for a quantity setting supply cartel and a taxing government requires short run commitment of government when setting the intraperiod tax*

Model and Analysis: Markov Perfect Equilibria(MPE)

Proof: Substituting the Nash-cournot reaction obtained from (25)

$$q = \frac{1 - \tau + V'}{2} \tag{27}$$

into the RHS of (25) gives

$$rW = \max_{\tau} \left\{ \frac{W'(1 + V' - \tau)}{2} + \frac{(1 + V' + \tau)}{8} - \frac{\tau^2}{2} + \frac{q^2}{2} - \frac{c}{2}X^2 \right\} \tag{28}$$

maximizing the RHS (28) yields

$$\tau = \frac{1 + V' - 2W'}{3}, \tag{29}$$

which is identical to (22). Substituting this tax (29) into (27) implies the Cartel's output is

$$q = \frac{1 + V' + W'}{3} \tag{30}$$

Model and Analysis: Markov Perfect Equilibria(MPE)

As a consequence,

$$P = \frac{2 - (V' + W')}{3} \tag{31}$$

Substitute (29) and (30) into RHS of (25) and (26) yield (23) and (24).
(23) and (24) can be combined and rewrite as follows:

$$rJ = \frac{5(1 + J')^2}{18} - \frac{c}{2}X^2 \tag{32}$$

where $J = V + W$. Applying guess and verify approach ($J = J_1X + 1/2J_2X^2$) gives

Model and Analysis: Markov Perfect Equilibria(MPE)

$$\widetilde{P} = 1 + \frac{(\sqrt{9r^2 + 20c} - 3r)}{10}\left(X - \frac{r}{c}\right) \tag{33}$$

$$\widetilde{p} = \frac{\sqrt{9r^2 + 20c} - (20c/r + 3r)}{50}\left(X - \frac{r}{c}\right) \tag{34}$$

$$\widetilde{\tau} = 1 + \frac{10c/r + 2(\sqrt{9r^2 + 20c} - 3r)}{25}\left(X - \frac{r}{c}\right) \tag{35}$$

The solution produced the same result as price versus tax when X_∞, but tax is higher and the producer price is lower, such that preemption is eliminated since $\widetilde{p} < 1/2$. The final consumer price is higher, which lower emissions and delays global warming. This result is identical to the case of price versus tax under government short run commitment (See:fig 1).

Model and Analysis: Markov Perfect Equilibria(MPE)

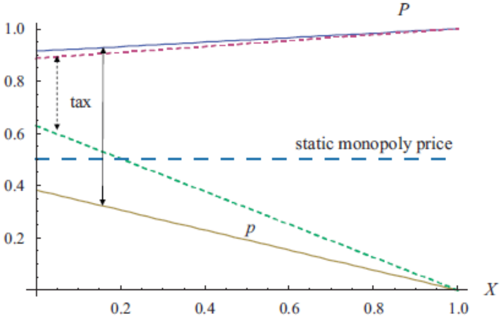

Fig. 1. Comparing different mix of strategies for ($r = 0.05 = c = > X_\infty = 1$): 1. *Tax versus price* (dashing) and Nash equilibrium. 2. *Taxes versus output quotas* with the government having the first move (=outcome of the tax–price game too given this first mover advantage).

Model and Analysis: Markov Perfect Equilibria(MPE)

Permits versus Prices:

$$q = min\left\{\bar{q}, 1 - p\right\} \tag{36}$$

where $\bar{q}$ is the permits issued by the government and p is the cartel's price. Note that not all permit will be used if the price is high.
Maximizing the RHS of the HJB equation of (5)

$$rW = \max_{0 \leq \bar{q} \leq 1-p} \left\{ q - \frac{q^2}{2} - pq - \frac{c}{2}X^2 + W'q \right\} \tag{37}$$

gives

$$\bar{q} = \begin{cases} 1 - p + W', & \text{if } 1 - P + w' \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{37.1}$$

Note that (37) is non-binding since $W' \leq 0$

Model and Analysis: Markov Perfect Equilibria(MPE)

Similarly, replacing q as $1-p$ and maximizing the RHS of the HJB equation of (5)

$$rV = \max_{p \geq 1-\bar{q}} \left\{ p(1-p) + V'(1-p) \right\} \tag{38}$$

gives

$$p = \begin{cases} \frac{1-V'}{2}, & \text{if } \bar{q} \geq \frac{1+V'}{2} \\ 1-\bar{q}, & \text{otherwise} \end{cases} \tag{38.1}$$

Note that the supplier choose $p \geq 1-\bar{q}$ and then Nash reactions of (37.1) and (38.1) is that no interior simultaneous move Nash equilibrium. Therefore,

Proposition 3: *if the government issues permits and the supply cartel sets prices, then only the stopping condition $p = 1$ and $\bar{q} = 0$ is a candidate for a Nash equilibrium in Markov strategies.(see:Fig 2)*

Model and Analysis: Markov Perfect Equilibria(MPE)

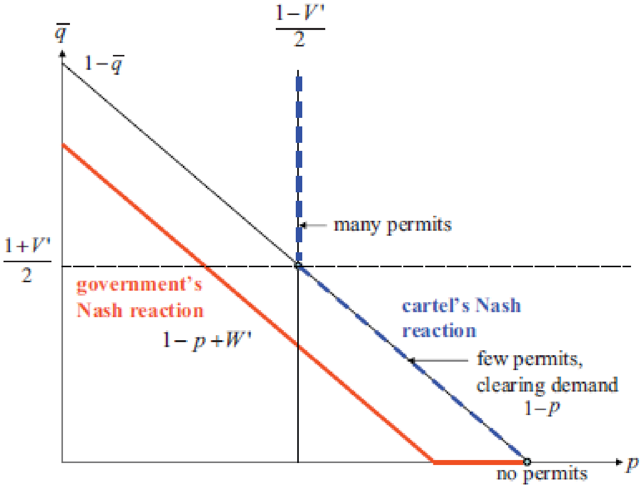

Fig. 2. Intraperiod Nash reaction functions for a permit issuing government and a price setting cartel.

Model and Analysis: Markov Perfect Equilibria(MPE)

Permits versus Prices with short run commitment:
if the cartel moves first, then we have the following
Substituting (37.1) into (38) and maximizing the RHS gives

$$p = \frac{1 + W' - V'}{2} \tag{39}$$

Substitution of this price and the government's Nash reaction (37.1) yields

$$rV = \frac{(1 + (V' + W'))^2}{4}, \tag{40}$$

$$rW = \frac{(1 + (V' + W'))^2}{8} - \frac{c}{2}X^2 \tag{41}$$

(40) and (41) are identical to those of price versus tax.

- Since the cartel moves first, its $\bar{q} < 1 - p$ and that implies positive permits $\pi = 1 - \bar{q} - p$, which means tax revenue remains within the consuming countries.Therefore, permits price is equal to the tax given in proposition 1, $\pi = \tau$.
- This solution above proved **Proposition 4**: *A government issuing permits facing a price settings cartel, which has the first move, reproduces the outcome in proposition 1.*

Model and Analysis: Markov Perfect Equilibria(MPE)

if the government moves first, then we have the following
The cartel will charge at least $p = (1 - V')/2$ and issuing permit above the corresponding market clear $\bar{q} > 1 - ((1 - V')/2) = (1 + V')/2$, then

$$q = min\left\{\bar{q}, \frac{1 + V'}{2}\right\} = \bar{q} \tag{42}$$

if the government's choice of permits is binding.substituting the market clearing price into (37) yields

$$rW = \max_{0 \leq \bar{q} \leq (1+V')/2} \left\{\frac{\bar{q}^2}{2} - \frac{c}{2}X^2 + W'\bar{q}\right\} \tag{43}$$

if the cartel maximizes its rent $(\pi = 0)$, then

$$\bar{q} = \begin{cases} \frac{1+V'}{2}, & \text{if } \frac{1+V'}{2} > -W' \\ 0, & \text{otherwise} \end{cases} \tag{43.1}$$

Therefore, the maximum of the RHS of (43) must be at the stopping condition of (43.1). Substituting (43.1) into (43)and (38) yields

$$rV = \left(\frac{1+V'}{2}\right)^2, \tag{44}$$

$$rW = \frac{1+V'}{8}(1+V'+4W') - \frac{c}{2}X^2 \tag{45}$$

The solution proved **Proposition 5**: *There exists no equilibrium in Markov strategies supporting positive quantities. Hence $\bar{q} = 0, p = 1$ results to unique equilibrium as in proposition 3. (43) and (38) satisfied the stopping condition by substituting $\bar{q} = 0, p = 1$ into (43) and (38).*

Permits versus quotas:
supplier plan to sell Q and government deems this too high and issues fewer permits, $\bar{q} < Q$. In this case supplier find it in their interest to supply less $Q < \bar{q}$. Combining the two, gives

$$q = min\{\bar{q}, Q\} \tag{46}$$

the implication on consumer, wellhead and permit prices

$$P = 1 - q, p = 1 - Q, \pi = max\{0, Q - \bar{q}\} \tag{47}$$

substituting (47) into the HJB equations from equation (4) and (5)

$$rW = \max_{\bar{q}} \left\{ q - \frac{q^2}{2} - (1 - Q)q - \frac{c}{2}X^2 + W'q \right\} \tag{48}$$

$$rV = \max_{q} \{(1 - Q)q + V'q\} \tag{49}$$

Model and Analysis: Markov Perfect Equilibria(MPE)

In this case of cooperation, the government can either $\bar{q} = Q$ or $\bar{q} < Q$ if facing supply Q. Therefore,

$$0 \leq \bar{q} = Q + W' \leq Q \tag{50}$$

$W' \leq 0$ is not binding. However, it is sub optimal for producer to produce at $Q > \bar{q}$ then $q = Q \geq \bar{q}$ implies the supplier's Nash reaction,

$$Q = \begin{cases} \frac{1+V'}{2}, & \text{if } \bar{q} \geq \frac{1+V'}{2} \\ 0, & \text{otherwise} \end{cases} \tag{50.1}$$

The unique equilibrium in this case is

$$Q = \bar{q} = 0 \text{ (See Fig 3)} \tag{51}$$

and this solution proved **Proposition 6**: *There exists no Nash equilibrium in Markov strategies with positive emissions if both players choose quantities. Therefore (51) is the unique equilibrium.*

Model and Analysis: Markov Perfect Equilibria(MPE)

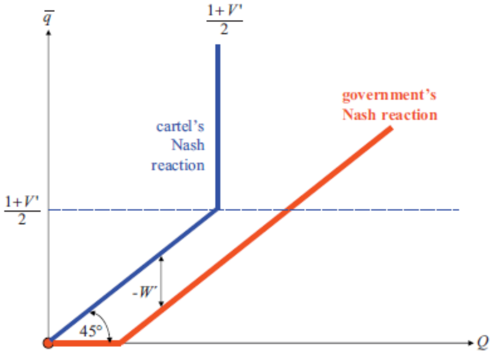

Fig. 3. Intraperiod Nash reactions between a permit issuing government and a quantity setting cartel depending on the shadow prices (V' and W')

Permits versus quotas with short run commitment:

- Allowing government the first move of issuing its permits does not change the result above because substituting the cartels Nash reaction into (48) give the same equation as (43).
- If the cartel has the first move of setting its quota facing a permit issuing government, the result is the same as that of proposition 1 but tax is replaced as price of the permit. This proved **Proposition 7**: *If the cartel has the first move of setting its quota facing a permit issuing government, the same outcomes as preposition 1* .

Conclusion and Summary

Table 1
Comparison of the results.

Government (G)/Monopoly (M) strategies	M-price			M-quota		
	Sim	M 1st	G 1st	Sim	M 1st	G 1st
G-tax	$X \uparrow X^*$	$X \uparrow X^*$	$X \nearrow X^*$	$X = 0$	$X = 0$	$X \nearrow X^*$
G-permit	$X = 0$	$X \uparrow X^*$	$X = 0$	$X = 0$	$X \uparrow X^*$	$X = 0$

M, monopoly; G, consumer government; sim, simultaneous moves; $\uparrow$, convergence; $\nearrow$, slower convergence due to higher consumer price P; $X^* =$ 1st best steady state.

Policy Recommendation

- The current choice of quantities should not persist for global warming mitigation if antagonism between consumer governments and fossil fuel producers intensifies.
- OPEC should switch back to price instrument if consumer governments tax carbon.

Further Research

The following paths for future research seem promising.

- Accounting for political objectives of suppliers and government.
- Another direction for further research is to see if investment in emission abatement technology will change the result.

Thank You