EconS 501 Midterm #2 - November 14th, 2018

Show all your work clearly and make sure you justify all your answers.
NAME__________________________

1. Consider an industry where two firms simultaneously and independently choose how much to recycle. Firm i's marginal cost from recycling is c_i while firm j's marginal cost is c_j, where $c_i \geq c_j > 0$. Firm i receives a benefit from recycling equal to

$$B_i(r_i, r_j) = a - r_i + \beta_i r_j,$$

where r_i denotes firm i's recycling output, r_j indicates firm j's recycling output. In addition, parameter a satisfies $a > c_i, c_j$ and $\beta_i \in [0, 1]$. Intuitively, parameter β_i measures how much firm j's recycling decisions benefits firm i (a positive externality). That is, when $\beta_i = 1$ firm i fully benefits from every unit of firm j's recycling activity, whereas when $\beta_i = 0$ firm j's recycling does not produce any benefit on firm i. Firm j's benefit from recycling is symmetric, that is, $B_j(r_i, r_j) = a - r_j + \beta_j r_i$.

   (a) Set up every firm i's profit-maximization problem, find its best response function $r_i(r_j)$, and discuss whether firms recycling activities are strategic substitutes or complements.

   (b) Identify the equilibrium level of recycling every firm selects, i.e., r_i^* and r_j^*, respectively. How are your equilibrium results affected by costs c_i and c_j? How are they affected by parameters β_i and β_j?

   (c) *Social optimum.* Assume that social welfare only considers the sum of every firms' profits. Identify the socially optimal levels of recycling, i.e., r_i^{SO} and r_j^{SO}.

   (d) *Comparison.* Compare the equilibrium recycling amounts that you found in part (b) and at the social optimum (from part c). Interpret your findings.

   (e) *Numerical example.* Evaluate your results in parts (b) and (c) assuming parameter values $c_i = c_j = c$ and $\beta_i = \beta_j = \frac{1}{2}$. Find social welfare in equilibrium recycling amounts (from part b) and at the social optimum (from part c). Compare these two social welfares, and discuss your results.

2. Consider a setting in which a worker can exert two levels of efforts, e_1 and e_2, where $0 \leq e_i \leq +\infty$ for every $i \in \{1, 2\}$. Intuitively, she may spend some hours a day assembling products, some time presenting them to potential customers, and finally cleaning the store, or keeping records of all costs and receipts. For simplicity, assume that output y_1 (y_2) is only generated from effort e_1 (e_2), and let g_1 (g_2) denote the agent's efficiency at producing output 1 (2, respectively). Each output behaves as follows,

$$y_1 = g_1 e_1 + \varepsilon_1 \text{ and } y_2 = g_2 e_2 + \varepsilon_2$$

Random shocks ε_1 and ε_2, which affect outputs 1 and 2 respectively, follow a bivariate normal distribution $N(\mathbf{0}, \Sigma)$, with expectation 0 for both outputs and variance-covariance matrix of

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}$$

The agent earns a wage of $w = s_1 y_1 + s_2 y_2$, where s_1 and s_2 represent his output shares, with $0 \leq s_i \leq 1$ for $i \in \{1, 2\}$. Cost is increasing and convex in both effort levels, where

$$c(e_1, e_2) = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2$$

such that her payoff function now becomes

$$U = u(w) - c(e_1, e_2)$$

and her utility function is $u(w) = 1 - \exp(-\eta w)$.

(a) Find the certainty equivalent payment of the agent.

(b) How is the agent's certainty equivalent affected when she becomes more risk averse?

(c) How is the agent's certainty equivalent affected when output shocks become more volatile?

3. Consider two consumers with utility functions over two goods, x_1 and x_2, given by

$$\begin{aligned} u_A &= \log(x_1^A) + x_2^A - \frac{1}{2}\log(x_1^B) \quad \text{for consumer } A, \text{ and} \\ u_B &= \log(x_1^B) + x_2^B - \frac{1}{2}\log(x_1^A) \quad \text{for consumer } B. \end{aligned}$$

where the consumption of good 1 by individual $i = \{A, B\}$ creates a negative externality on individual $j \neq i$ (see the third term, which enters negatively on each individual's utility function). For simplicity, consider that both individuals have the same wealth, m, and that the price for both goods is 1.

(a) *Unregulated equilibrium.* Set up consumer A's utility maximization problem, and determine his demand for goods 1 and 2, as x_1^A and x_2^A. Then operate similarly to find consumer B's demand for good 1 and 2, as x_1^B and x_2^B.

(b) *Social optimum.* Calculate the socially optimal amounts of x_1^A, x_2^A, x_1^B and x_2^B, considering that the social planner maximizes a utilitarian social welfare function, namely, $W = U_A + U_B$.

(c) *Restoring efficiency.* Show that the social optimum you found in part (b) can be induced by a tax on good 1 (so the after-tax price becomes $1+t$) with the revenue returned equally to both consumers in a lump-sum transfer.

4. Consider a perfectly competitive industry with N symmetric firms, each with cost function $c(q) = F + cq$, where $F, c > 0$. Assume that the inverse demand is given by $p(Q) = a - bQ$, where $a > c$, $b > 0$, and where Q denotes aggregate output.

(a) *Short-run equilibrium.* If exit and entry is not possible in the industry (assuming N firms remain active), find the individual production level of each firm.

(b) *Long-run equilibrium.* Consider now that firms have enough time to enter the industry (if economic profits can be made) or to exit (if they make losses by staying in the industry). Find the long-run equilibrium number of firms in this perfectly competitive market.