

Recitation #8 - (10/25/2019)

1. Suppose there are two periods, ‘today’ (i.e. period 1) and ‘tomorrow’ (i.e. period 2), and a single consumption good. An individual called Andreu (not necessarily the same person as the first author in Mas-Colell, Whinston and Green’s *Microeconomic Theory*) has preferences over two-period consumption streams that are additively separable. In particular assume his preferences admit a representation of the form:

$$U(x_1, x_2) = u(x_1) + \frac{1}{1+r}u(x_2),$$

where $r > 0$ is the interest rate at which the agent can borrow or lend money, and

$$u(x_t) = -\frac{(x_t - 2)^2}{2}$$

for every time period $t = \{1, 2\}$.

- (a) If his income today is $y_1 = 1$ and he knows that his income tomorrow will also be $y_2 = 1$, and solve for her optimal consumption in each period and calculate the level of discounted lifetime utility he achieves.
 - b. Formulate the agent’s optimization problem and solve for the optimal consumption plan and his level of discounted expected utility. Suppose that Andreu still knows that his income today is $y_1 = 1$. He is, however, uncertain about his future payoff y_2 : It could be high, $y_2^H = \frac{3}{2}$ or low, $y_2^L = \frac{1}{2}$, each happening with equal probability. His problem now is to maximize his discounted expected utility by choosing the initial period consumption x_1 ; his future consumption if his income tomorrow is high, x_2^H ; and his future consumption if income tomorrow is low, x_2^L .
 - (b) Compare your answers to parts (a) and (b).
2. Consider a cumulative distribution function $F(x)$ which first-order stochastically dominates $G(x)$.

- (a) Show that the mean of x under $G(x)$, $\int x dG(x)$, cannot exceed that under $F(x)$, $\int x dF(x)$, i.e.,

$$\int x dF(x) \geq \int x dG(x)$$

Provide now an example where $\int x dF(x) \geq \int x dG(x)$ is satisfied, but $F(x)$ does not first order stochastically dominates $G(x)$.