Pollution Abatement with Disruptive R&D Investment

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Abstract

This paper examines a model of investment in abatement where firms are allowed to produce output while investing in R&D. The latter, however, increases production costs, thus disrupting first-period output. We identify three equilibrium profiles where firms choose to either: (1) invest in R&D alone (thus rationalizing a common modeling assumption in the literature); (2) produce output alone; or (3) engage in both activities. We evaluate how the emergence of each result is affected by the market structure in which firms compete and by the severity of spillover effects. We then measure welfare levels in each equilibrium profile. Overall, we show that firms endogenously choose to focus in R&D only when the market is concentrated and spillover effects are small. In other type of industries, our findings indicate that firms may focus on output production or engage in both activities under relatively large conditions.

Keywords: Abatement, Emission fees, R&D disruptive effects, Equilibrium profiles, Spillovers.
JEL classification: H23, L12, O32, Q58.

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1 Introduction

The literature analyzing green R&D decisions is extensive, and studies a multitude of topics, from the analysis of suboptimal R&D investment, how is it affected by the number of firms in the industry, spillovers, or incomplete information; among others. A common assumption across these studies is that firms do not choose output and R&D investment simultaneously. That is, in a first period, firms choose how much to invest in green or abatement technologies, in a second period, the regulator responds to these investment decisions by setting environmental policy (e.g., emission fees per unit of pollutant) and then firms compete in the product market; see, for instance, Milliman and Prince (1989), Biglaiser and Horowitz (1994), Katsoulacos and Xepapadeas (1995), Denicolo (1999), Chiou and Hu (2001), Montero (2002), Poyago-Theotoky (2007), Ouchida and Goto (2014), and, more recently, Strandholm et al. (2018) and Haruna and Goel (2019).

The literature generally assumes that, while firms invest in green R&D in the first period, they do not produce output in that period. This modeling assumption can be rationalized using two explanations. First, it can reflect that firms produce first-period output without being affected by their simultaneous R&D investment; and thus output decisions are overlooked since they do not affect equilibrium results in subsequent periods. A second interpretation considers that, while firms invest in green R&D in the first period, they do not have the ability to produce positive units of output; implicitly assuming that their R&D and production decisions are incompatible. In summary, while the first interpretation assumes that R&D and output decisions are perfectly compatible during the first period, the second considers they are completely incompatible.

We argue that the above assumption may be reasonable in specific R&D projects outsourced to other companies (first interpretation) or when it requires firms' full attention (second interpretation). This assumption, however, is not a good description of other green R&D projects. Generally, firms do not consider R&D investment and output decisions as independent (as if one did not impact the other during that period) nor regard them as incompatible (shutting down their operations to focus on green R&D projects during that period). Instead, R&D investments can disrupt production activities —e.g., they require the participation of workers at all levels, reducing their concentration on production activities, and part of the input and equipment is diverted to R&D efforts— entailing a higher cost of production than when no resources are dedicated to R&D. Capital, for instance, may become more difficult to access, as empirically shown by Czarnitzki et al. (2011); or firm productivity may decrease while investing in R&D, as found by Czarnitzki and Thorwarth (2012). Similarly, Rouvinen (2002) finds that R&D investments reduce current productivity in 4 out of the 5 models; with the other model showing a not statistically significant impact.

As a consequence, firms simultaneously invest in R&D while producing positive units of output, and their decisions to focus on only one of these activities depends on the disruption effects that R&D investments have in concurrent production, as well as the relative cost of each activity.[1] In

[1]Examples abound of companies simultaneously investing in green R&D projects in their own facilities while producing output. The Korean manufacturer LG, for instance, reports an investment of $9 billion from 2011-2020,

other words, the literature assumes that firms consider the strategic implications of R&D investment in their future output and profits, but overlooks the potential strategic effects on firms' current output decisions.

Our model considers that, in the first period, every firm has the ability to choose its R&D investment and its output level, allowing for R&D to disrupt production activities. In the second period, the regulator sets the environmental policy, i.e., an emission fee. Observing their investment decisions in the first period and the emission fee, firms choose their output levels.

We seek to identify under which conditions the common modeling assumption (i.e., firms focusing on R&D investment) emerges as an equilibrium prediction. We also investigate in which contexts firms choose to only produce output; not investing any resources in green R&D.[2] Finally, we examine settings where firms choose to do both, as commonly observed in polluting firms in several industries.

For presentation purposes, we first analyze a monopoly, showing that the above three equilibrium profiles can be sustained in equilibrium: one where the firm chooses to invest in R&D alone, another in which it produces output but does not invest in R&D, and another in which the firm carries out both activities simultaneously. Considering a monopoly helps us isolate firm's incentives to invest in green R&D. Intuitively, when a monopolist invests in R&D during the first period, it induces the regulator to respond by setting a less stringent emission fee. The monopolist then faces lower taxes and, importantly, fully captures the tax-saving effect of its R&D investment, both with and without spillover effects. Under oligopoly, however, we demonstrate that every firm has a smaller incentive to invest in R&D since it cannot fully appropriate this tax-saving benefit. As a consequence, more competitive market structures give rise to equilibria where firms choose to focus on production alone without investing in R&D. In contrast, less competitive industries are more likely to sustain equilibria where firms invest in R&D without producing positive units of output; as the existing literature assumed.

We also show that the presence of output disruptions induces firms to reduce their investment, which ultimately leads the regulator to respond with more stringent emission fees. In other words, if regulators ignore output disruptions when designing emission fees, they may be setting too lax environmental policies, inducing a socially excessive pollution. We then evaluate these results in different market structures, finding that, in highly competitive industries we should expect firms either engaging in both activities (output and R&D) when disruptive effects are small; or only producing output when disruptive effects are large. In this setting, firms are then unlikely to focus on R&D alone. In contrast, when few firms compete, all activity profiles can be sustained, including that in which firms focus on R&D alone. A similarly rich pattern arises when R&D generates small

mainly in eco-products that use fewer materials and are more energy efficient; Audi invested Euro 9.5 billion in green R&D in 2011-2015 hiring more than 1,200 experts in electric and lightweight vehicles in that period; and IBM invested $1 billion per year since 2007, mainly directed at efficiency and energy management systems. Similarly, BP reports investing $8 billion in "environmental expenditure" in 2015, and $4 billion in both 2014 and 2013.

[2] This can help explain a recent decision by Global Foundries, a microchip manufacturer, to stop its R&D projects to expand their production scale of existing and older microchip generations; see Albany Business Review, Aug 27, 2018.

(or nil) spillover effects, such as in chemical and pharmaceutical industries which patent most innovations.[3] Our findings then suggest that the common assumption in the green R&D literature, where firms only choose R&D in the first period, can emerge in equilibrium, but only when firms interact in highly concentrated industries with small spillover effects. Otherwise, this behavior is unlikely to be sustained in equilibrium, instead, inducing firms to engage in both activities under large conditions.

Finally, our results indicate that welfare is the highest when firms voluntarily choose to engage in both activities, production and R&D investments, which arises when R&D generates relatively small disruptive effects on production costs. This equilibrium behavior can be promoted by policies reducing R&D disruption, such as helping firms conduct their R&D projects in government-funded research facilities and labs. In contrast, we show that generous R&D subsidies, which reduce firms' investment costs without affecting its disruptive effects, likely induce large shifts in equilibrium behavior leading to welfare losses.

Related literature. Several articles analyze firms' incentives to invest in abatement, seeking to benefit from tax savings in subsequent periods. The literature evaluates these incentives in different market structures, and in distinct policy details.[4] For instance, Montero (2002) examines how environmental R&D in duopoly markets is affected by different environmental policies (emission standards, performance standards, grandfathered tradeable permits, and auctioned permits). In his model, the regulator sets the policy instrument in the first period, and firms respond with their R&D, followed by their output and emissions in the next periods. However, the paper assumes that firms do not have the ability to produce output while investing in R&D, which our paper relaxes.[5]

Other articles investigate the effect of "research joint ventures," where firms coordinate their R&D decisions by maximizing their joint profits; see Chiou and Hu (2001), Poyago-Theotoky (2007), and Ouchida and Goto (2014), among others. In the latter two, for instance, firms invest in R&D

[3] For studies considering green R&D investment with spillover effects, see Katsoulacos and Xepapadeas (1996), Montero (2002), Poyago-Theotoky (2007) and Strandholm et al. (2018). For empirical studies measuring the extent of spillover effects, see Grilleches (1992), Cameron (1998), and Weiser (2005), which report an average private rate of return to R&D around 20-30%, and an estimated spillover of 40-60%.

[4] The literature has extensively analyzed firms' incentives to invest in cost-reducing R&D assuming non-polluting industries; starting in Arrow (1962) which identifies that perfectly competitive markets provide stronger incentives to invest in R&D. The literature was then extended along different dimensions, such as Yi (1999), which assumes firms compete a la Cournot and demonstrates that firms have less incentives to invest in R&D as more firms enter the industry, as opposed to Arrow's (1962) result; Delbono and Denicolo (1990), which compares equilibrium and socially optimal R&D levels under Cournot and Bertrand competition; and Bester and Petrakis (1993), which adds horizontal differentiation to Delbono and Denicolo's (1990) model, identifying that socially excessive investment occurs when goods are sufficiently close substitutes. Other articles include Brander and Spencer (1983), which considers a two-period R&D model where firms compete a la Cournot in the second period, extended by Okuno-Fujiwara and Suzumura (1990) to allow for Bertrand competition and by Qiu (1997) to allow for differentiated products. The literature has also analyzed cooperative R&D decisions, and their welfare effects; see D'Aspremont and Jacquemin (1998), and the extensions in Kamien et al (1992) and Amir (2003).

[5] While most of this literature considers that the number of firms in a polluting industry is exogenous, Katsoulacos and Xepapadeas (1995) allow for endogenous entry. In their setting, the regulator sets the emission fee in the first stage, firms decide to enter in the second stage, and then choose output and abatement efforts; still assuming that investment in abatement does not impact the firm's ability to produce more units of output. They find that, in equilibrium, the emission fee may exceed environmental damages when the market structure is endogenous.

during the first period, the regulator sets the emission fee in the second period, and firms respond with their output decisions in the last period. Ouchida and Goto (2014) work with the same setting as in the Poyago-Theotoky (2007) model, but analyze emissions under subsidies, showing that they can also be welfare improving. Strandholm et al. (2018) consider a similar time structure to investigate the free-riding effects of R&D investment by allowing the regulator to charge a different emission fee to each firm, and Haruna and Goel (2019) apply this setting to a mixed oligopoly (one public and one private firm) to identify optimal pollution tax or subsidy.[6]

Ouchida and Goto (2016) study research joint ventures and R&D cartelization. In the first period, the government decides how much cooperation to allow the firms in the R&D stage (either allow the joint venture where both firms realize aggregate R&D, allow firms to maximize joint profits at the R&D stage, or force firms to choose their R&D non-cooperatively). In the second period, firms choose their R&D cooperation as allowed by the government, and the final three periods correspond to the Poyago-Theotoky (2007) structure (R&D investment, emission fee, and output). McDonald and Poyago-Theotoky (2017) present a model where the regulator pre-commits to an emission fee (effectively switching the first two periods in our model) and allow for two forms of R&D spillover: on the abatement effort (the outcome of the investment) or on the R&D expenditure, showing that the latter leads to lower abatement efforts, higher emissions, and more stringent fees.

Biglaiser and Horowitz (1995) extend some of the above results by allowing firms to choose whether to patent or license the innovation. In particular, firms invest in R&D in the first period, face emission fees in the second period, patent or license technology in the last period, and choose their output levels in the fourth period; examining firms' investment decisions, and their incentives to adopt their own discoveries or license them to competing firms. Denicolo (1999) considers a similar setting, however, only one firm can invest in green R&D, subsequently choosing whether to use its innovation or license it to other companies. The paper considers two cases, one where the governments sets emission fees before firms invest in R&D, and another in which it responds to their R&D investment.

Our paper then contributes to this literature by considering that R&D investment can be disruptive. We are able to identify under which conditions firms choose to invest in green R&D alone without producing a positive output (as commonly assumed in the literature), when the firm prefers to focus on output production alone without investing in R&D, and when it carries out both activities simultaneously.

In terms of policy implications, our results suggest that, if R&D is disruptive, R&D subsidies may not be welfare improving. Policies that, instead, focus on reducing the disruption are preferable, such as government-industry partnership R&D labs; see, for instance, the National Renewable Energy Laboratory's Integrated Biorefinery Research Facility. Our findings also recommend lowering the fixed cost of entry into industries where R&D activities negatively impacts production costs, since reducing concentration —despite its increased pollution— may yield an overall welfare

[6] Chiou and Hu (2001) assume an exogenous emission fee, as opposed to most articles in this literature which identify the socially optimal emission fee endogenously. They study firms colluding in their R&D expenditures (research joint ventures) or, instead, in the output market, since this practice also reduces their emissions.

gain.

In the next section, we present the model. Section 3 identifies equilibrium behavior under monopoly, which helps us isolate firms' incentives to invest in R&D since, in this market structure, the firm fully appropriates the tax-saving effects of its investment decision. Section 4 extends our analysis to an oligopoly with N firms, finding how our results under monopoly are affected. Section 5 discusses the policy implications of our results and concludes.

2 Model

We consider the following time structure:

1. In period 1, every firm i simultaneously and independently chooses both its R&D investment, z_i, and its output level, x_i.[7]

2. In the second period:

   (a) The regulator chooses a uniform emission fee, t.

   (b) Every firm i responds choosing its output level, q_i.

This time structure represents industries where a future environmental regulation is enacted and firms make R&D investments in preparation for this policy. Firms could, instead, commit in the first period to a production plan across both periods (x_i, q_i). Such production plan, however, would not be credible since every firm i could alter its second-period production q_i at the beginning of this period. Hence, we focus on our above time structure, which avoids time-inconsistency problems. For simplicity, we assume that firms produce the same good in both periods and no time discounting.

First period profits. In particular, the firm's profit function in the first period, while conducting R&D is,

$$\pi_i^{1st} = (a - x_i - x_{-i})x_i - cx_i(1 + \lambda z_i) - \frac{1}{2}\gamma z_i^2$$

where c represents marginal cost of production, satisfying $a > c > 0$, x_i is firm i's first-period output, $x_{-i} = \sum_{j \neq i} x_j$ denotes aggregate first-period output of firm i's rivals, and z_i is firm i's investment in green R&D with cost $\frac{1}{2}\gamma z_i^2$ where $\gamma > 0$. In addition, λ denotes how much R&D increases the firm's current production cost. When $\lambda = 0$, the firm's output decision is unaffected by its R&D investment today, while when $\lambda > 0$ its first-period production is negatively impacted by the firm's R&D efforts during that period. That is, R&D investment increases first-period total and marginal first-period production cost.

[7] For simplicity, we assume that firms produce a non-durable good, so first-period production cannot be stored and sold during the second period.

Second period profits. In the second period, profits become

$$\pi_i^{2nd} = (a - q_i - q_{-i})q_i - cq_i - t(q_i - z_i)$$

where q_i denotes firm i's second-period output, $q_{-i} = \sum_{j \neq i} q_j$ represents its rivals' aggregate output in this period, and the last term indicates that firm i pays an emission fee t per unit of net emissions, $q_i - z_i$. Since R&D investment occurs in the first period, it does not generate disruption effects in second period costs. For simplicity, we abstract from the cost-reducing effect of R&D investments (i.e., second-period marginal cost c does not decrease in firm i's investment, z_i). Appendix 1 shows, however, that our results are qualitatively unaffected when cost-reducing effects are considered.

The next section examines equilibrium behavior, finding output and R&D decisions during the first period, as well as social welfare, and how they are affected when first-period production is disrupted by R&D investment, $\lambda > 0$. For presentation purposes, we first analyze the case of a monopoly, and then extend our results to an oligopoly.

3 Equilibrium analysis - Monopoly

Period 2(b). Solving for the subgame perfect equilibrium of the game, we start from the last period. The monopolist solves[8]

$$\max_{q \geq 0} \; (a - q)q - cq - t(q - z)$$

where $q - z$ denotes net emissions. Output function is then $q(t) = \frac{a-(c+t)}{2}$, entailing profits of $\pi^{2nd}(t) = \frac{(a-c-t)^2}{4} + tz$.

Period 2(a). Anticipating this output function, the regulator chooses the emission fee that solves

$$\max_{t \geq 0} \; SW = CS(q(t)) + PS(q(t)) - Env(q(t))$$

where $CS(q(t)) + PS(q(t)) = (a-c)q(t) - \frac{1}{2}q(t)^2$ denotes the sum of consumer and producer surplus[9], while $Env(q(t)) = \frac{1}{2}d\left[q(t) - z\right]^2$ represents the environmental damage from emissions, $q(t) - z$. Environmental damage is then increasing and convex in emissions, and $d > 1$.[10] Differentiating with respect to emission fee t and solving, yields

$$t(z) = \frac{(a-c)(d-1) - 2dz}{d+1}$$

which is positive as long as $d > d_1 \equiv \frac{a-c}{a-c-2z}$. Cutoff d_1 increases in z, reflecting that the range of d

[8] Since this section analyzes a monopolist's behavior, we drop the firm subscript i everywhere to facilitate our presentation.

[9] This assumes that total tax collection is revenue neutral.

[10] This assumption guarantees a positive emission fee when the firm forgoes investment in R&D.

for which the firm pays an emission fee shrinks as the monopolist's abatement effort, z, increases. For generality, our setting allows for taxes, $t(z) > 0$, or subsidies, $t(z) < 0$. Intuitively, when the environmental damage from pollution is sufficiently severe, $d > d_1$, the environmental externality dominates the market failure arising from monopoly, and thus the regulator sets an emission fee. Otherwise, the underproduction in monopoly creates larger welfare losses than pollution, leading the regulator to subsidize production.

Period 1. In this period, the monopolist anticipates its second-period profits $\pi^{2nd}(t) = \frac{(a-c-t)^2}{4} + tz$, and evaluates them at emission fee $t(z)$, so its second-period profits are

$$\pi^{2nd}(t(z)) \equiv \Pi^{2nd} = \frac{(a-c-z)\,[a-c+d(2+d)z]}{(d+1)^2}.$$

Therefore, the monopolist simultaneously chooses its R&D investment z and first-period output level x to solve.

$$\max_{x,z\geq 0} \underbrace{(a-x)x - cx(1+\lambda z) - \frac{1}{2}\gamma z^2}_{\text{First-period profits}} + \underbrace{\Pi^{2nd}}_{\text{Second-period profits}}.$$

where we assume no discounting. Taking first-order conditions with respect to the choice variables and rearranging we obtain the following equilibrium values for x and z. All proofs are relegated to the appendix.

Proposition 1. *In the first period, the monopolist's equilibrium output and R&D investment are*

$$x^* = \frac{(a-c)\,[d(d+2)(c\lambda-\gamma-2)-c\lambda-\gamma]}{c^2(d+1)^2\lambda^2-2\gamma-2(\gamma+2)d(d+2)} \quad \textit{and}$$
$$z^* = \frac{(a-c)\,[d(d+2)(c\lambda-2)+c\lambda+2]}{c^2(d+1)^2\lambda^2-2\gamma-2(\gamma+2)d(d+2)},$$

respectively. The monopolists chooses to:

1. *invest in R&D, but not produce first-period output, if* $\gamma_1 \geq \gamma > \gamma_2$*;*
2. *produce first-period output, but not invest in R&D, if* $\gamma > \gamma_1$ *and* $\lambda > \lambda_1$*;*
3. *invest in R&D and produce first-period output otherwise;*

where $\gamma_1 \equiv \frac{c^2\lambda^2}{2} + \frac{2}{(d+1)^2} - 2$, $\gamma_2 \equiv \frac{d(d+2)(c\lambda-2)-c\lambda}{(d+1)^2}$, *and* $\lambda_1 \equiv \frac{2d(d+2)-2}{c(d+1)^2}$*. Furthermore, cutoff* γ_1 *satisfies* $\gamma_1 > \gamma_2$ *if and only if* $\lambda > \lambda_1$*.*

As illustrated by Figure 1, the above proposition shows that the firm chooses to engage in both activities (green R&D and positive first-period output) in two cases: (1) when R&D is very inexpensive despite being disruptive (Region Ib); and (2) when R&D exhibits small disruptive effects, regardless of its cost (Region I).[11]

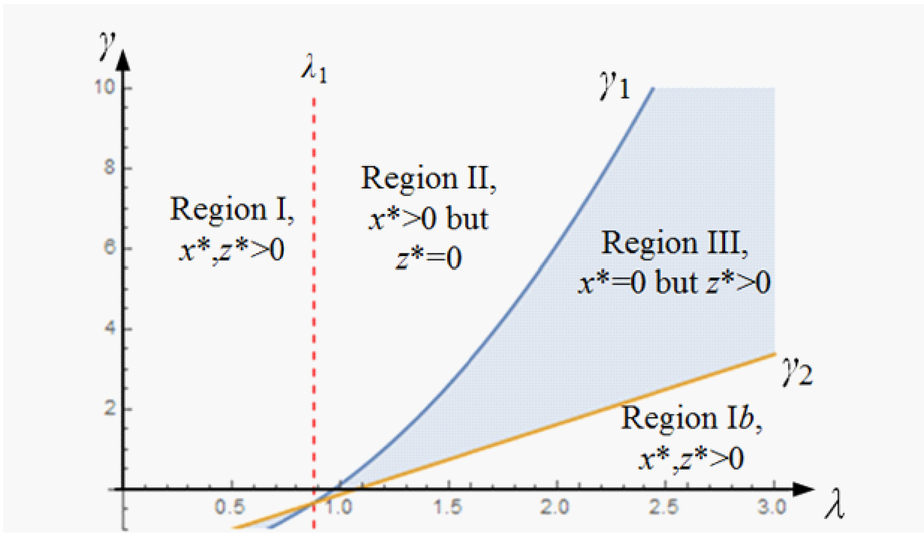

Figure 1. Production and R&D profiles under monopoly.

Relative to Region I, R&D in Region II becomes more disruptive (higher λ), inducing firms to abstain from investing in R&D to ameliorate its production costs. Finally, in the shaded Region III, both λ and γ take intermediate values, leading the firm to invest in R&D, abstaining from producing first-period output. This case helps rationalize a common assumption in the literature analyzing green R&D investment, which assumes that firms do not produce output in the same period they invest in R&D. Our result indicates that this assumption is valid only when R&D is relatively inexpensive and its disruptive effects on the production process during that same period are severe.[12]

Comparative statics. Figure 2a illustrates how our results are affected when d increases (from $d = 3$ to $d = 6$). Relative to Figure 1, Figure 2a is almost identical. Intuitively, when pollution becomes more damaging, the monopolist anticipates a more stringent emission fee in the subsequent period, providing a stronger incentives to increase its R&D investment (to reduce its future emission fees) but also to produce a larger output during the first period (when emission are not taxed yet) than on the second period (when the firm faces emission fees). Overall, the regions

[11] For illustration purposes, Figure 1 assumes $c = 2$ and $d = 3$. In Figure 2a we consider a higher value for parameter d, i.e., $d = 5$. Other parameter combinations produce similar results and can be provided by the authors upon request.

[12] The crossing point of cutoffs γ_1 and γ_2, λ_1, always occurs in the negative quadrant regardless of the number of firms in the market, yielding the four regions in figure 1. To see this, evaluate γ_1 at the crossing point λ_1, obtaining $\gamma_1(\lambda_1) = \frac{2-6d(d+2)}{(d+1)^4}$, which is negative for all $d > 0.156$; a condition that holds by definition since $d > 1$. Therefore, this crossing point always occurs in the negative quadrant; as depicted in figure 1. In addition, the vertical line depicting cutoff λ_1 in figure 1 lies in the positive quadrant under all admissible parameter values since $\frac{2d(d+2)-2}{c(d+1)^2} > 0$ yields $d > 0.41$, which must hold since $d > 1$ by definition.

of parameter values for which the firm chooses to produce first-period output, invest in R&D, or both are then almost unaffected by a more stringent fee.

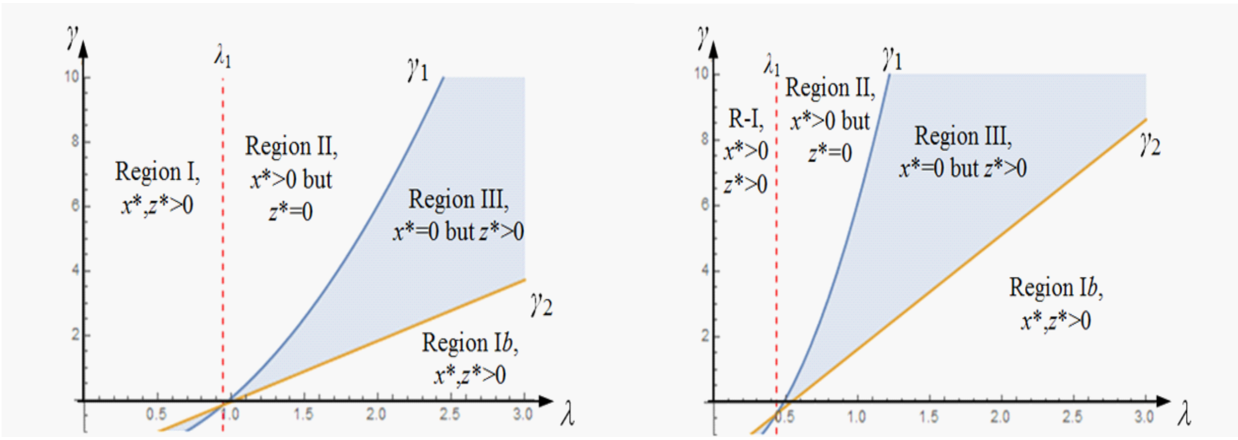

Figure 2a. Higher values of d.

Figure 2b. Higher values of c.

Figure 2b depicts how a more costly output affects our equilibrium results (where c increases from $c = 2$ in Figure 1 to $c = 4$). Relative to Figure 1 (benchmark), the same regions emerge, but the vertical cutoff λ_1 shifts now leftward, shrinking Region I where the firm chooses positive amounts of both x and z, while expanding Regions Ib, II, and III. Intuitively, the firm produces a positive first-period output if disruptiveness and/or the investment cost are sufficiently low.

4 Equilibrium analysis - Oligopoly

When $N \geq 2$ firms compete, our analysis is analogous to the previous section, except for the introduction of disruptiveness parameter λ which now affects every firm's first-period production cost. We assume disruption effects remain within the firm, that is, firm i's R&D investment, z_i, only increases its own first-period cost without affecting its rivals' costs during this period. In the second period, every firm solves

$$\max_{q_i \geq 0} (a - Q)q_i - cq_i - t(q_i - z_i - \beta z_{-i})$$

where Q denotes aggregate output, z_i represents firm i's abatement, $z_{-i} = \sum_{j \neq i} z_j$ is the aggregate abatement from all firm i's rivals, and $\beta \in [0, 1]$ captures the knowledge spillover. Intuitively, $\beta = 0$ indicates that firm i does not benefit from other firms' abatement decisions, while when $\beta = 1$ it benefits from every unit of their abatement.

In this setting, we can obtain equilibrium first-period output and R&D investment in a symmetric fashion to those in Proposition 1 for the monopolist. (For compactness, these equilibrium outcomes are described in the proof of Proposition 2.)[13] From these equilibrium results we find the following proposition, which is analogous to Proposition 1 under monopoly. For presentation purposes, superscript N in the cutoffs denotes oligopoly.

Proposition 2. *Under oligopoly, every firm i chooses:*

1. *invest in R&D, but not produce first-period output, if* $\gamma_1^N \geq \gamma > \gamma_2^N$*;*
2. *produce first-period output, but not invest in R&D, if: (1)* $\gamma < \gamma_2^N$ *and* $\lambda < \lambda_1^N$*; or (2)* $\gamma > \gamma_1^N$ *and* $\lambda > \lambda_1^N$*;*
3. *invest in R&D and produce first-period output otherwise;*

where

$$\gamma_1^N \equiv \frac{c^2\lambda^2}{N+1} - \frac{d\left[\beta(N-1)+1\right]\left[d(\beta+N(\beta(N-2)+N+2)-1\right]+N(N+1)\left[\beta(N-1)+N+1\right]}{N(d+N)^2}, \quad \textit{and}$$

$$\gamma_2^N \equiv \frac{c\lambda\left[d^2N+d\left[N(2\beta+N-1)+2-2\beta\right]-N^2\right]}{N(d+N)^2} - \frac{d\left[\beta(N-1)+1\right]\left[d(\beta+N\left[\beta(N-2)+N+2\right]-1\right]+N(N+1)\left[\beta(N-1)+N+1\right]}{N(d+N)^2}.$$

Furthermore, $\gamma_1^N > \gamma_2^N$ *if and only if* $\lambda > \lambda_1^N$*, where* $\lambda_1^N \equiv \frac{(N+1)\left[d^2N+d[N(2\beta+N-1)+2-2\beta]-N^2\right]}{cN(d+N)^2}$.

Figure 3a depicts the three cutoffs identified in Proposition 2 for the case of $N = 2$ firms. For comparison purposes, we consider the same parameter values as in Figure 1, and assume no spillovers ($\beta = 0$) as a baseline; figure 3b considers a positive spillover $\beta = 1/3$. Graphically, the shaded Region III shifts downward, expanding Region II, shirking Region Ib, and leaving Region I unaffected. These effect are emphasized when the number of firms increases to $N = 4$, or larger, where the shaded Region III is, essentially, pushed to mostly lie in the negative quadrant.[14]

[13] In addition, when R&D is not disruptive ($\lambda = 0$), and $N = 2$, first-period production collapses to $x_i^*|_{\lambda=0} = \frac{a-c}{3}$, and investment in R&D becomes $z_i^*|_{\lambda=0} = \frac{2(a-c)(d(\beta+d+2)-2)}{(\beta+1)d(6(\beta+3)+(\beta+7)d)+2\gamma(d+2)^2}$, thus matching the results in Strandholm et al. (2018).

[14] Figure 3b does not include Region Ib, which lies below cutoff γ_2. This region could, however, be displayed by allowing for larger values of λ in the horizontal axis, but we restricted $\lambda \leq 3$ to make the figure comparable to our previous results in figures 1 and 2. In addition, note that the first case of Region II, $\gamma < \gamma_2^N$ and $\lambda < \lambda_1^N$, is not depicted in figures 3a-3b since this region lies in the negative quadrant, thus not being sustained under admissible parameters values $\lambda, \gamma \geq 0$.

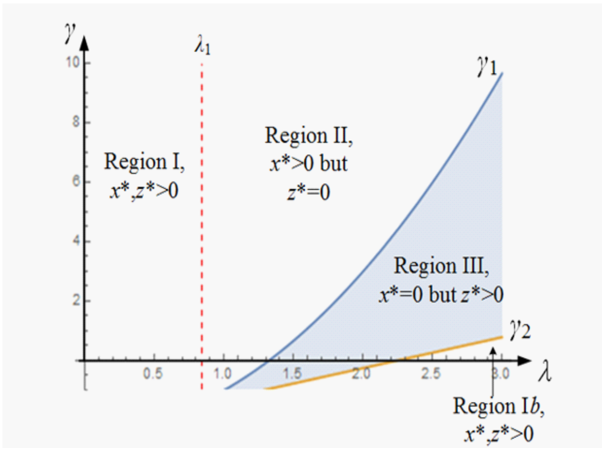

Figure 3a. $N = 2$ and no spillovers.

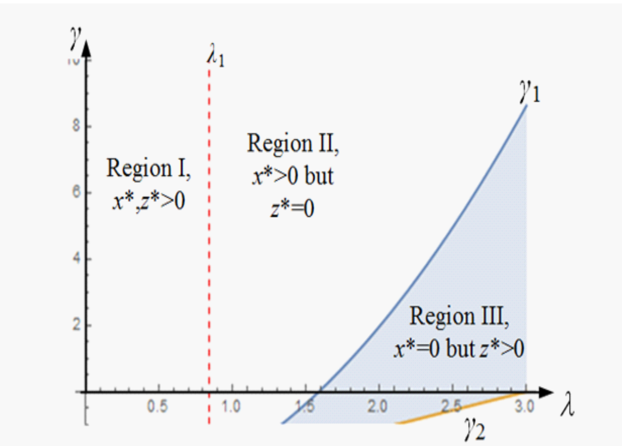

Figure 3b. $N = 2$ and spillovers ($\beta = 1/3$).

Intuitively, the introduction of more firms expands Region II, where firms choose to produce a positive first-period output but abstain from investing in R&D; while shifting shaded Region III downwards, where firms choose to invest in R&D abstaining from first-period production. To understand these effects, note that in this setting every firm i's R&D investment, z_i, reduces aggregate environmental damage $Env(Q(t)) = \frac{1}{2}d\left[Q(t) - Z\right]^2$, where $Q(t)$ denotes aggregate output and Z represents aggregate R&D investment. As a consequence, an increase in z_i reduces the emission fee that the regulator sets on *all* firms. Under monopoly, the firm fully internalized this effect, providing firm i with stronger incentives to invest in green R&D. Under oligopoly, however, firm i has a smaller incentive to invest since it cannot fully appropriate the tax savings effects of its R&D investment. As a result, an increase in the number of firms shrinks the region of parameter values where firms choose to invest in R&D while abstaining from producing first-period output; expanding the region where firms only focus into production ignoring R&D investments. Our analysis shows that, for parameter values where the monopolist produces a positive first-period output and invests in R&D (Region Ib in figure 1), duopolists may choose not to produce first-period output focusing on R&D (Region III in figures 3a-3b). This can be explained by the disruptive effects of R&D and how they generate an unbearable cost for duopolies. While output and overall costs may be lower for a firm under duopoly than monopoly, its revenues are also lower, driving the company to reduce first-period output x_i down to zero, only investing in R&D during this period.

When R&D generates spillover effects, as depicted in figure 3b, Region II expands relative to its area in figure 3a, shifting Regions III and Ib downwards. As a result, firms become less willing to only invest in R&D (as predicted in Region III) or to carry out both activities (as in Region Ib) when a share of their R&D benefits other firms' costs. In contrast, firms choose to focus on first-period production alone under larger parameter conditions.

Comparative statics. Following a similar approach as under monopoly, figures 4a and 4b illustrate how our above duopoly results are affected by changes in the parameters. First, figure 4a assumes that environmental damage per unit of emissions increases from $d = 3$ to $d = 6$, suggesting that, relative to figure 3a, cutoff γ_2 shifts leftward, thus expanding region Ib. Intuitively, as emissions become more damaging, firms intensify their R&D to save on stringent fees. Second, figure 4b considers that production costs increase from $c = 2$ to $c = 4$. Relative to figure 3a, regions I and II shrink while regions III and Ib expand. Intuitively, firms start with relatively high costs, and thus are willing to invest in R&D to benefit from tax savings in the next period (as predicted in regions III and Ib) under a larger set of parameters, i.e., even when R&D generates severe disruptive effects in first-period production (high λ), or when R&D is relatively expensive (high γ).

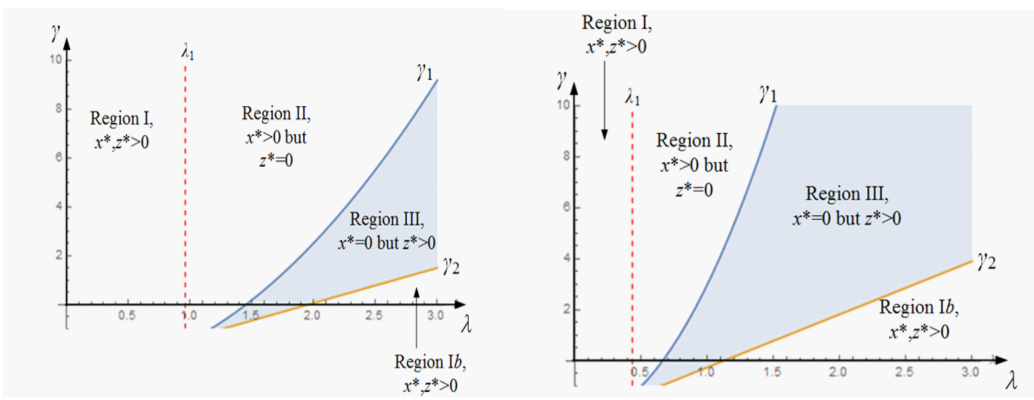

Figure 4a. Higher values of d when $d = 6$. Figure 4b. Higher values of c when $N = 2$.

5 Welfare comparison

In this section, we evaluate social welfare in each of the production/abatement regions identified above. Figure 5 plots social welfare against the disruptiveness parameter λ, considering the same parameter values as in figure 3 and $\gamma = 1/3$. Intuitively, when firms conduct both activities (which occurs in Regions I and Ib, where $x^*, z^* > 0$), an increase in λ makes every unit of R&D more disruptive with first-period production, inducing firms to reduce both their output in this period x^* (as their costs are now larger for a given level of R&D) and their R&D investment z^* (to ameliorate first-period production costs). As a result, social welfare decreases in λ in these two regions; as graphically depicted in the far left- and right-hand areas of figure 5.

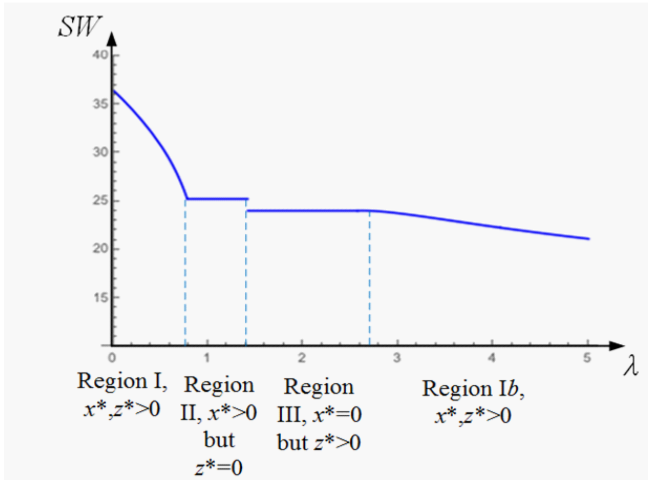

Figure 5. Welfare in regions I-III.

In contrast, when firms do first-period output alone (i.e., $x^* > 0$ but $z^* = 0$ in Region II), an increase in parameter λ does not affect social welfare. In this region, a more disruptive R&D does not increase firms' production costs during the first period since $z^* = 0$. Essentially, firms are not disrupted by R&D since they choose not to invest any resources on this activity in equilibrium. As a consequent, first- and second-period output levels are unaffected by λ, ultimately keeping welfare unchanged. A similar argument applies to Region III, where firms choose to focus on R&D alone, i.e., $x^* = 0$ but $z^* > 0$. In this context, firms choose to shut down their first-period output, $x^* = 0$, thus being unaffected by further increases in its production cost through a more disruptive R&D (higher λ). In other words, firms have even more incentives to shut down their first-period production as λ increases. Since firms do not alter their production or R&D decisions by an increase in λ, overall welfare is constant in λ in Region III.

Figure 6a depicts social welfare under the same parameter values as figure 5, but also includes another set of curves evaluated at a higher environmental damage per unit of output, $d = 6$; indicating that welfare decreases in all regions. This welfare reduction is especially clear in Region II, as firms in this region focus on output production without investing in R&D, yielding an overall increase in emissions which become more harmful in this context. Figure 6b provides a similar comparison, by evaluating welfare at a higher spillover than in figure 5 ($\beta = 1/3$ rather than

$\beta = 0$), suggesting that welfare increases in all regions.[15]

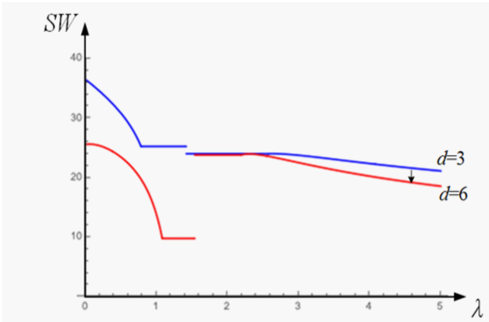

Figure 6a. Higher values of d.

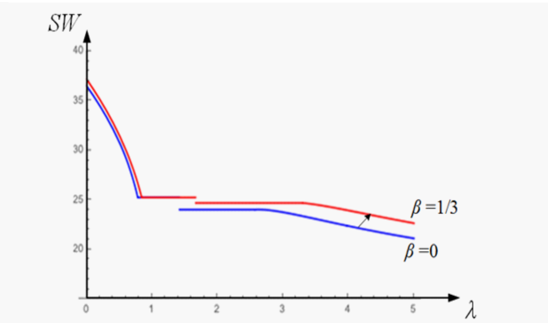

Figure 6b. Higher spillovers.

Figure 7a illustrates how our results are affected when the R&D cost, as captured by parameter γ, increases from $\gamma = 1/3$ in figure 5 to $\gamma = 2/3$. As expected, welfare decreases in all regions, which holds for further increases in γ. Figure 7b evaluates our results when the number of firms increases from $N = 2$ in figure 5 to $N = 4$, showing that social welfare increases significantly. Intuitively, more firms gives rise to two effects: (1) a more intense competition, which increases consumer surplus; and (2) more pollution, which increases environmental damage. In the absence of regulation, the second (negative) effect may dominate the first (positive) effect, yielding an overall reduction in social welfare. When environmental policy is present, however, the regulator sets the emission fee to induce socially optimal output levels, and generate an increase in overall welfare. Finally, note that the increase in N in figure 7b shows that Region III expands significantly, meaning that the equilibrium where firms only invest in green R&D is sustained for a larger range of λ. This result holds because the entry of more firms yields a more stringent fee, inducing every firm to increase its R&D investment in the first period.

[15] Note that the position of cutoffs λ_1, γ_1, and γ_2 changes in the parameter values we consider in each figure, shrinking or expanding the range of λ's supporting each region. For more details on these cutoffs, see Proposition 2.

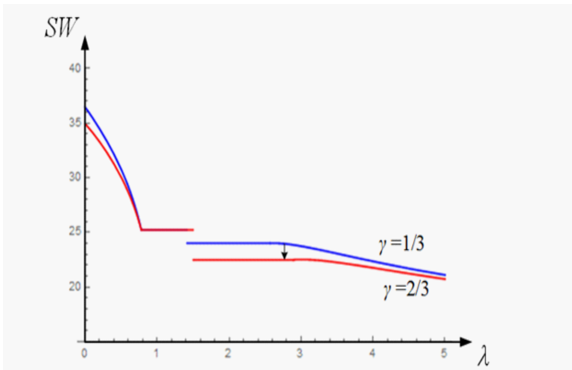

Figure 7a. More expensive R&D.

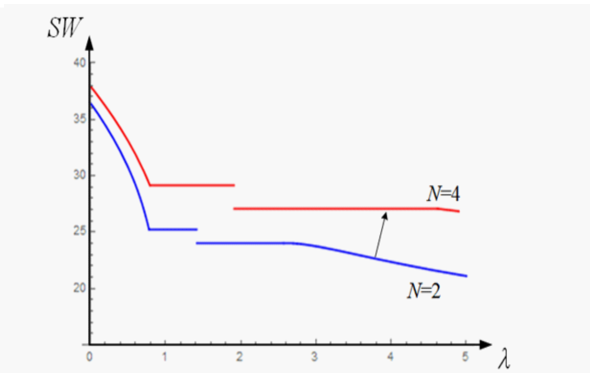

Figure 7b. More firms.

6 Discussion

Emission fees with/without disruptive effects. The emission fee $t(z)$, identified in the second period, is a function of firms' investment in abatement during the first period. When abatement decisions disrupt first-period output and, in turn, the firm responds by reducing its investment in abatement (relative to settings where R&D produces no disruptions in production), the regulator sets a more stringent emission fee. Intuitively, the regulator anticipates larger net emissions in the second period, which call for a higher emission fee. In summary, the presence of output disruptions induces firms to reduce their investment in abatement under large conditions, which ultimately leads to more stringent emission fees in the second period. This finding entails that models overlooking output disruption prescribe a less stringent, suboptimal, environmental policy.

Welfare evaluation with/without disruptive effects. Our results in the previous section also highlight that social welfare is weakly higher when firms endogenously choose to engage in both R&D and first-period output (as predicted in Region I) than in any other equilibrium profile (that is, in any other region). Since this equilibrium behavior emerges when firms face small R&D disruptions (low λ), our findings suggest that subsidy policies that help firms significantly reduce their R&D costs (lower γ) are not necessarily welfare improving; as these subsidies still induce the equilibrium profile in Region I. A similar argument applies if R&D is disruptive, where R&D subsidies induce firms to switch from focusing on output (Region II) to investing in R&D alone (Region III), thus lowering welfare (as illustrated in figures 5 and 6); or to engage in both activities (Region Ib) where welfare is even lower. Our results also suggest that policies reducing the R&D disruption that firms face generate larger welfare gains, as they induce firms to behave as prescribed in Region I where firms engage in both activities. Policies helping firms conduct their R&D activities in government-funded research facilities and labs, or subsidizing the inputs that firms use in both their production process and R&D projects, could help reduce the disruption that latter impose on the former. These policies could promote the emergence of Region I in equilibrium, where firms produce a positive first-period output —thus increasing consumer surplus in this period— and invest in green R&D, which reduces environmental damages in the next period.

Monopolized vs. Competitive industries. We also found that, when several firms compete, Region Ib cannot be sustained while Region III can be supported for a small set of parameters. In other words, in highly competitive industries, we should expect most firms behaving as predicted by Regions I and II, either conducting both activities (output and R&D) when disruptive effects are small; or only output in both periods when disruptive effects are large, implying that firms would unlikely focus on R&D alone. In concentrated industries, in contrast, all four regions can be sustained under large conditions, including Region III where firms focus on R&D. That is, the common modeling assumption in the literature, which considers that firms only invest in R&D without the ability to produce output simultaneously, emerges as an equilibrium behavior only when firms interact in highly concentrated industries.

Role of spillover effects. Our findings also help us understand settings where spillover effects are significant, as in mature industries or where R&D produces unpatented innovations. In this markets, we identified similar effects as in highly competitive industries, namely, firms choose to focus on first-period production alone under large parameter conditions. In contrast, when spillovers are small (or absent), other production/investment regions can be supported, including that in which firms focus only on investing in R&D, or that in which firms balance both of their activities (i.e., investing in R&D and producing a positive output).

More competition or larger spillovers? Finally, our results suggest that, when a polluting industry is subject to environmental regulation, firm entry may produce a larger increase in social welfare than that emerging from a larger spillover; thus implying that regulators should lower entry costs to attract more firms to the industry rather than purchasing technology licenses from innovations of some companies to distribute them to other firms. This policy implication may be particularly attractive in some industries where regulators can easily lower barriers to entry while increasing spillover effects is relatively difficult.

7 Appendix

7.1 Appendix 1 - Allowing for cost-reducing R&D

In this appendix we test how our main results are affected if R&D investment provides an additional benefit, in the form of reducing firm i's marginal production cost, from c to $c-\alpha z_i$, where parameter $\alpha > 0$ denotes the effectiveness of R&D at reducing second-period marginal costs. When $\alpha = 0$, the production cost is unaffected by R&D (remaining at c), and when $\alpha > 0$ the production cost decreases in firm i's R&D investment z_i. For simplicity, we assume no knowledge spillovers, so firm $j \neq i$ does not benefit from the cost-reducing effect of firm i's investment in R&D, z_i.

For presentation purposes, we first analyze the case of a monopoly, and then extend our analysis to a duopoly, comparing in each case how cost-reducing effects ($\alpha > 0$) affects our results.

7.1.1 Monopoly

Period 2(b). Solving for the subgame perfect equilibrium of the game, we start from the last period. The monopolist solves

$$\max_{q \geq 0} \ (a-q)q - (c-\alpha z)q - t(q-z)$$

where the cost of production in this stage is reduced by R&D at a rate of αz where $\alpha > 0$ is the rate at which R&D decreases production cost. This setting then embodies our main model as a special case when $\alpha = 0$, but otherwise provides more general results. Output function in this setting is $q(t) = \frac{a-(c+t-\alpha z)}{2}$, entailing profits of $\pi^{2nd}(t) = \frac{(a-c-t+\alpha z)^2}{4} + tz$, which are both increasing in the cost-reduction effect of R&D, as captured by the parameter α.

Period 2(a). Anticipating this output function, the regulator chooses the emission fee that solves

$$\max_{t \geq 0} \ SW = CS(q(t)) + PS(q(t)) - Env(q(t))$$

where $CS(q(t)) + PS(q(t)) = (a-c+\alpha z)q(t) - \frac{1}{2}q(t)^2$ now denotes the sum of consumer and producer surplus. Differentiating with respect to emission fee t and solving, yields

$$t(z) = \frac{(a-c)(d-1) + (\alpha-2)dz - \alpha z}{d+1},$$

which increases in α since the derivative $\frac{\partial t(z)}{\partial z} = \frac{(d-1)z}{d+1}$, which is positive given that $d > 1$ by assumption. Intuitively, a larger cost-reducing effect induces the regulator to anticipate more production (and pollution) in the last stage, and thus sets a more stringent fee. In other words, emission fees are more stringent in a setting where R&D can help the monopolist reduce its production and abatement costs than in a context where R&D only reduces emissions.

Period 1. In this period, the monopolist anticipates its second-period profits $\pi^{2nd}(t) = \frac{(a-c-t+\alpha z)^2}{4} + tz$, and evaluates them at emission fee $t(z)$, so its second-period profits are

$$\pi^{2nd}(t(z)) \equiv \Pi^{2nd} = \frac{(a-c+(\alpha-1)z)(a-c+(\alpha+d(d+2))z)}{(d+1)^2}.$$

Therefore, the monopolist simultaneously chooses its R&D investment z and first-period output level x to solve.

$$\max_{x,z \geq 0} \ \underbrace{(a-x)x - cx(1+\lambda z) - \frac{1}{2}\gamma z^2}_{\text{First-period profits}} + \underbrace{\Pi^{2nd}(t(z))}_{\text{Second-period profits}} .$$

Taking first-order conditions with respect to the choice variables and rearranging, we obtain the following equilibrium values for x and z. (For compactness, we include its proof at the end of the appendix.)

Proposition A1. *In the first period, the monopolist's equilibrium output and R&D investment are*

$$x^* = \frac{(a-c)\left[2\alpha(\alpha+c\lambda-1)+d^2(2\alpha+c\lambda-\gamma-2)+2d(2\alpha+c\lambda-\gamma-2)-c\lambda-\gamma\right]}{c^2(d+1)^2\lambda^2+4(\alpha-1)\left[\alpha+d(d+2)\right]-2\gamma(d+1)^2} \quad \text{and}$$
$$z^* = \frac{(a-c)\left[d(d+2)(c\lambda-2)+c\lambda+2-4\alpha\right]}{c^2(d+1)^2\lambda^2+4(\alpha-1)\left[\alpha+d(d+2)\right]-2\gamma(d+1)^2},$$

respectively. The monopolists chooses to:

1. *invest in R&D, but not produce first-period output, if* $\gamma_1(\alpha) \geq \gamma > \gamma_2(\alpha)$*;*
2. *produce first-period output, but not invest in R&D, if* $\gamma > \gamma_1(\alpha)$ *and* $\lambda > \lambda_1(\alpha)$*;*
3. *invest in R&D and produce first-period output otherwise; where*

$$\begin{aligned}
\gamma_1(\alpha) &\equiv \frac{c^2\lambda^2}{2} + \frac{2(\alpha-1)\left[\alpha+d(d+2)\right]}{(d+1)^2} - 2,\\
\gamma_2(\alpha) &\equiv \frac{c\lambda\left[2\alpha+d(d+2)-1\right]+2(\alpha-1)\left[\alpha+d(d+2)\right]}{(d+1)^2}, \text{ and}\\
\lambda_1(\alpha) &\equiv \frac{4\alpha+2d(d+2)-2}{c(d+1)^2}.
\end{aligned}$$

Furthermore, cutoff $\gamma_1(\alpha)$ *satisfies* $\gamma_1(\alpha) > \gamma_2(\alpha)$ *if and only if* $\lambda > \lambda_1(\alpha)$*. For all* $\alpha > 1$, $\gamma_1(\alpha) > \gamma_1(0)$*, and for all* $\alpha > 0$, $\gamma_2(\alpha) > \gamma_2(0)$ *and* $\lambda_1(\alpha) > \lambda_1(0)$*.*

Figure A1a evaluates the above cutoffs at $\alpha = 0.5$. For comparison purposes, the figure also depicts these three cutoffs at $\alpha = 0$, where cost-reduction effects are absent (main model). The figure shows that both cutoff $\gamma_1(\alpha)$ and $\gamma_2(\alpha)$ experience an upward shift when α increases, and that their crossing point, the vertical line of cutoff $\lambda_1(\alpha)$, shifts rightward. Relative to our setting in the main model, Regions I and Ib increase in size while regions II and III shift up and the leftmost boundary shifts right, requiring larger values of R&D and cost disruption for the firm to focus on either only R&D or production in the first stage. Firms now enjoy an additional benefit to R&D, leading them to be more willing to both invest in R&D and produce in the first period for more costly and more disruptive R&D projects. Figure A1b evaluates these cutoffs at stronger cost-reduction effects, $\alpha = 1$, indicating that our above results are emphasized when α increases.

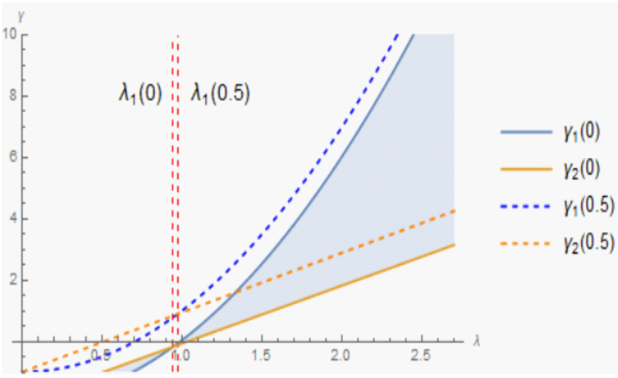

Figure A1a. Effect of $\alpha = 0.5$.

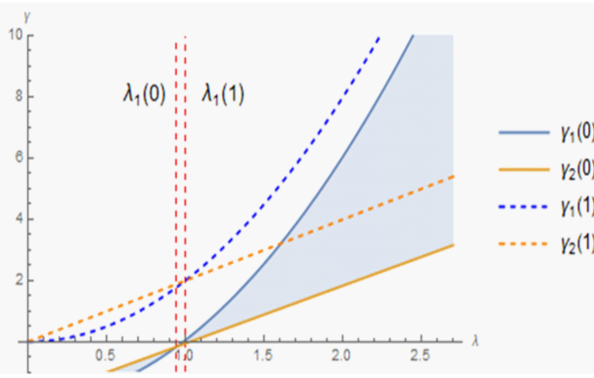

Figure A1b. Effect of $\alpha = 1$.

7.1.2 Duopoly

We now investigate the impact of competition on the above setting. For simplicity, we consider the case of $N = 2$ firms in the following proposition.

Proposition 2. *Every firm i chooses to:*

1. *invest in R&D, but not produce first-period output, if* $\gamma_1^2(\alpha) \geq \gamma > \gamma_2^2(\alpha)$*;*
2. *produce first-period output, but not invest in R&D, if* $\gamma > \gamma_1^2(\alpha)$ *and* $\lambda > \lambda_1^2(\alpha)$*;*
3. *invest in R&D and produce first-period output otherwise; where*

$$\gamma_1^2(\alpha) \equiv \frac{c^2\lambda^2}{3} - \frac{(\beta+1)\left(6\alpha(4+6\beta-3\alpha(\beta+1)) + d^2(\alpha(\beta-13)+3(\beta+7)) + 2dA + 9(\beta+3)\right)}{6(d+2)^2},$$

$$\gamma_2^2(\alpha) \equiv \frac{2c\lambda\left(9\alpha(\beta+1)+3d^2+d(2\alpha+3\beta+6-2\alpha\beta)-6\right) - (\beta+1)9(\beta+3)}{6(d+2)^2} + \frac{-(\beta+1)\left(6\alpha(6\beta+4-3\alpha(\beta+1)) + d^2(\alpha(\beta-13)+3(\beta+7)) + 2dA\right)}{6(d+2)^2},$$

$$\lambda_1^2(\alpha) \equiv \frac{9\alpha(\beta+1)+3d^2+d(2\alpha+3\beta+6-2\alpha\beta)-6}{c(d+2)^2}$$

and $A \equiv \alpha(2(\alpha-3)\beta - 2\alpha - 15$. *Furthermore, cutoff* $\gamma_1^2(\alpha)$ *satisfies* $\gamma_1^2(\alpha) > \gamma_2^2(\alpha)$ *if and only if* $\lambda > \lambda_1^2(\alpha)$.

Figures A2a and A2b evaluate the above cutoffs at the same parameter values as figures 3a and 3b in Section 4, depicting all three cutoffs for the case in which cost-reduction effects are present

($\alpha = 0.5$) and absent ($\alpha = 0$). As in the case of monopoly, when R&D produces cost-reducing effects, cutoffs $\gamma_1^2(\alpha)$ and $\gamma_2^2(\alpha)$ shift upward, while their crossing point, vertical cutoff $\lambda_1^2(\alpha)$, shifts rightward, making firms willing to invest in R&D when it is more disruptive than in the absence of cost-reducing effects.

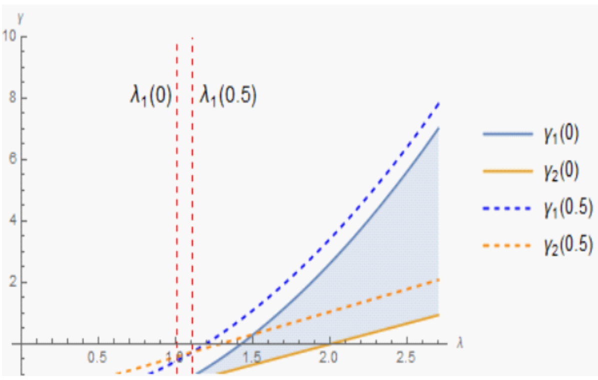

Figure A2a. $N = 2$ and no spillovers.

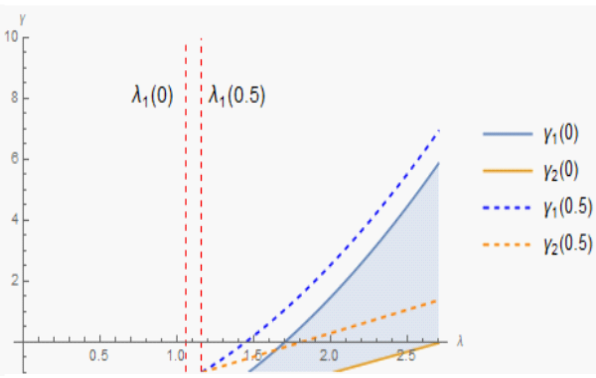

Figure A2b. $N = 2$ and spillovers ($\beta = 1/3$).

7.2 *Proof of Proposition 1*

In the first period, the monopolist simultaneously chooses its R&D investment z and first-period output level x to solve.

$$\max_{x,z\geq 0} \quad \pi = \underbrace{(a-x)x - cx(1+\lambda z) - \frac{1}{2}\gamma z^2}_{\text{First-period profits}} + \underbrace{[(a-q)q - cq - t(q-z)]}_{\text{Second-period profits}}.$$

The first-order conditions are

$$\frac{\partial \pi}{\partial x} = a - c(\lambda z + 1) - 2x = 0$$
$$\frac{\partial \pi}{\partial z} = \frac{a\,[d(d+2)-1] - cd(d+2)(\lambda x + 1) - c\lambda x + c - \gamma(d+1)^2 z - 2d(d+2)z}{(d+1)^2} = 0$$

We can re-arrange these to find

$$x(z) = \frac{1}{2}\,[a - c(\lambda z + 1)]$$
$$z(x) = \frac{(a-c)\,[d(d+2)-1] - c(d+1)^2\lambda x}{\gamma + (\gamma+2)d(d+2)}.$$

Simultaneously solving for x and z in the above two equations, we find that the monopolist's equilibrium output and R&D investment are

$$x^* = \frac{(a-c)\left[d(d+2)(c\lambda-\gamma-2)-c\lambda-\gamma\right]}{c^2(d+1)^2\lambda^2-2\gamma-2(\gamma+2)d(d+2)} \quad \text{and}$$
$$z^* = \frac{(a-c)\left[d(d+2)(c\lambda-2)+c\lambda+2\right]}{c^2(d+1)^2\lambda^2-2\gamma-2(\gamma+2)d(d+2)},$$

respectively.

Next, we examine under which parameter conditions x^* and/or z^* are positive. To do this, we check for which conditions the numerator and denominator of x^* have the same sign. (A similar argument applies to z^*.) First, the numerator of x^* is positive if and only if $\gamma < \gamma_2$, where $\gamma_2 \equiv \frac{d(d+2)(c\lambda-2)-c\lambda}{(d+1)^2}$. Second, the denominator of either x^* or z^* (since both x^* and z^* have the same denominator) is positive if and only if $\gamma < \gamma_1$, where $\gamma_1 \equiv \frac{c^2\lambda^2}{2} + \frac{2}{(d+1)^2} - 2$. Finally, the numerator of z^* is positive if and only if $\lambda > \lambda_1$, where $\lambda_1 \equiv \frac{2d(d+2)-2}{c(d+1)^2}$.

Therefore, for $x^* > 0$, we need that either $\gamma > \max\{\gamma_1, \gamma_2\}$ (which implies that both the numerator and denominator of x^* are negative, making x^* positive) or that $\gamma < \min\{\gamma_1, \gamma_2\}$ (entailing that both the numerator and denominator of x^* are positive). For $z^* > 0$, we need that either: (a) $\gamma < \gamma_1$ and $\lambda > \lambda_1$; or (b) $\gamma > \gamma_1$ and $\lambda < \lambda_1$.

Next, we investigate under which conditions $\gamma_1 > \gamma_2$. This occurs when

$$\frac{c^2\lambda^2}{2} + \frac{2}{(d+1)^2} - 2 > \frac{d(d+2)(c\lambda-2)-c\lambda}{(d+1)^2}.$$

Simplifying, we find that this holds when either $\lambda < 0$ or $\lambda > \lambda_1$. We can ignore the case in which $\lambda < 0$ since $\lambda > 0$ holds by definition. Therefore, $\gamma_1 > \gamma_2$ holds as long as $\lambda > \lambda_1$; otherwise, $\gamma_1 \leq \gamma_2$ is satisfied.

As a result, we only have two situations to consider when evaluating the equilibrium values, that is: (1) $\lambda > \lambda_1$ (and therefore $\gamma_1 > \gamma_2$ is true), and (2) $\lambda < \lambda_1$ (and therefore $\gamma_1 < \gamma_2$ is true). Under each of these assumptions on the value of λ we have four cases:

Case	$\lambda < \lambda_1$ ($\gamma_1 < \gamma_2$)	$\lambda > \lambda_1$ ($\gamma_1 > \gamma_2$)	Case
1	$x^* > 0,\ z^* > 0$	$x^* > 0,\ z^* > 0$	5
2	$x^* > 0,\ z^* = 0$	$x^* > 0,\ z^* = 0$	6
3	$x^* = 0,\ z^* > 0$	$x^* = 0,\ z^* > 0$	7
4	$x^* = 0,\ z^* = 0$	$x^* = 0,\ z^* = 0$	8

Case 1. If $\lambda < \lambda_1$, then $z^* > 0$ if $\gamma > \gamma_1$. For $x^* > 0$ we need that $\gamma > \gamma_2 > \gamma_1$. Both of these conditions can hold simultaneously in Region I.

Case 2. If $\lambda < \lambda_1$, then $z^* = 0$ if $\gamma < \gamma_1$. If $\gamma < \gamma_1 < \gamma_2$, then $x^* > 0$. However, this means that $0 > \gamma_2 > \gamma_1 > \gamma$, which is a violation of our assumption that $\gamma > 0$ and this is not a viable case.

Case 3. If $\lambda < \lambda_1$, then $x^* = 0$ if $\gamma_1 < \gamma < \gamma_2$. If $\lambda < \lambda_1$ and $\gamma > \gamma_1$ then $z^* > 0$. However, this means that $0 > \gamma_2 > \gamma$, which is a violation of our assumption that $\gamma > 0$ and this is not a viable case.

Case 4. If $\lambda < \lambda_1$, then $x^* = 0$ if $\gamma_1 < \gamma < \gamma_2$. For $z^* = 0$ we need that $\gamma < \gamma_1$. We can see that this is a contradiction, so $x^* = 0$ and $z^* = 0$ holding simultaneously is not a viable case.

Case 5. If $\lambda > \lambda_1$, then $z^* > 0$ if $\gamma < \gamma_1$. For $x^* > 0$, we need that $0 < \gamma < \gamma_2 < \gamma_1$. Both of these conditions can hold simultaneously in Region Ib.

Case 6. If $\lambda > \lambda_1$, then $z^* = 0$ if $\gamma > \gamma_1$. For $x^* > 0$, we need that $\gamma > \gamma_1 > \gamma_2 > 0$. Both of these conditions can hold simultaneously in Region II.

Case 7. If $\lambda > \lambda_1$, then $z^* > 0$ if $\gamma < \gamma_1$. For $x^* = 0$, we need that $\gamma_2 < \gamma < \gamma_1$. Both of these conditions can hold simultaneously in Region III.

Case 8. If $\lambda > \lambda_1$, then $z^* = 0$ if $\gamma > \gamma_1$. If $\gamma > \gamma_1 > \gamma_2$ then $x^* > 0$. Hence, both x^* and z^* cannot simultaneously be zero when $\lambda > \lambda_1$.

Therefore, we can summarize the only cases that can be sustained in equilibrium under admissible parameter values. In particular, the monopolist chooses to:

1. invest in R&D, but not produce first-period output, if $\gamma_1 \geq \gamma > \gamma_2$ (which corresponds to Case 7 above, graphically depicted in Region III of Figure 1 in the main body of the paper);

2. produce first-period output, but not invest in R&D, if $\gamma > \gamma_1$ and $\lambda > \lambda_1$ (which corresponds to Case 6, or Region II);

3. invest in R&D and produce first-period output otherwise (which corresponds to Cases 1 and 5, or Regions I and Ib, respectively).

7.3 *Proof of Proposition 2*

Second period. In the second period, every firm i maximizes their profit by choosing second-period output q_i:

$$\max_{q_i} \quad \pi_i = (a - q_i - q_{-i})q_i - cq_i - t(q_i - z_i - \beta z_{-i}),$$

where $q_{-i} = \sum_{j \neq i} q_j$ and $z_{-i} = \sum_{j \neq i} z_j$. The first-order condition is

$$\frac{\partial \pi_i}{\partial q_i} = a - c - 2q_i - q_{-i} - t = 0,$$

which we can re-arrange to find best response function $q_i(q_{-i}) = \frac{a-(c+t)}{2} - \frac{1}{2}q_{-i}$. Invoking symmetry, $q_{-i} = (N-1)q_i$, which helps us obtain output function $q(t) = \frac{a-(c+t)}{N+1}$ for every firm i, which we will use in the second period.

Anticipating this output function $q(t)$, the regulator chooses emission fee t to maximize social welfare

$$\max_t \quad SW = CS(q(t)) + PS(q(t)) - Env(q(t))$$

where

$$\begin{aligned} CS(q(t)) + PS(q(t)) &= \int_0^{Nq(t)} (a-c-x)\, dx - \sum_{i=1}^{N} \frac{\gamma z_i^2}{2} \\ &= \frac{N(a-c)(a-c-t)}{N+1} - \frac{N^2(a-c-t)^2}{2(N+1)^2} - \sum_{i=1}^{N} \frac{\gamma z_i^2}{2}, \end{aligned}$$

and $Env(q(t)) = \frac{1}{2} d \sum_{i=1}^{N} \left(q(t) - z_i - \beta \sum_{j \neq i} z_j \right)^2$. The first-order condition is

$$\begin{aligned} \frac{\partial SW}{\partial t} = \frac{1}{2} \Bigg(& \frac{N^2(a-c-t) - N\left[(N+2)(a-c) + Nt\right]}{(N+1)^2} \\ & -d \sum_{i=1}^{N} \left(\frac{2\beta \sum_{j=1}^{N} z_j - 2(1+\beta) z_i}{N+1} - \frac{2(a-c-t)}{(N+1)^2} \right) \Bigg) = 0 \end{aligned}$$

Simplifying and solving for t, we find

$$t(z_i) = \frac{(a-c)(d-1)}{d+N} - \frac{d(N+1)\left[\beta(N-1)+1\right] \sum_{i=1}^{N} z_i}{N(d+N)}.$$

First period. In the first period, every firm i chooses output x_i and R&D investment z_i to solve

$$\begin{aligned} \max_{x_i, z_i} \quad \pi_i = & \underbrace{(a - x_i - x_{-i})x_i - c x_i (1 + \lambda z_i) - \frac{1}{2}\gamma z_i^2}_{\text{first-period profit}} \\ & + \underbrace{(a - Nq(t(z_i)))]q(t(z_i)) - cq(t(z_i)) - t(z_i)[q(t(z_i)) - z_i - \beta z_{-i}]}_{\text{second-period profit}}, \end{aligned}$$

where $x_{-i} = \sum_{j \neq i}^{N} x_j$. The first-order conditions are

$$\begin{aligned} \frac{\partial \pi_i}{\partial x_i} &= a - c(1 + \lambda z_i) - 2x_i - x_{-i} = 0 \\ \frac{\partial \pi_i}{\partial z_i} &= -\frac{1}{n^2(d+n)^2} \Big[d(N+1)(\beta(N-1)+1) \left((1-\beta) d(N-1) z_i - (1-\beta) d z_{-i} - N(a-c) + N^2 (z_i + \beta z_{-i}) \right) \\ & \quad + \left(d(\beta + (1-\beta)N - 1) + N^2 \right) \left(d(N+1)(\beta(N-1)+1)(z_i + z_{-i}) - (d-1)N(a-c) \right) \\ & \quad + dN(\beta(N-1)+1)(-ad - cN + d(\beta(N-1)+1)(z_i + z_{-i})) \\ & \quad + dN(\beta(N-1)+1)((a-c)N + d(\beta(N-1)+1)(z_i + z_{-i})) \\ & \quad + c\lambda N^2 x_i (d+N)^2 + cdN(d+N)(\beta(N-1)+1) + \gamma N^2 z_i (d+N)^2 \Big] = 0 \end{aligned}$$

In the first period, every firm i's equilibrium output and R&D investment are

$$x^N = \frac{(a-c)}{A}\left[c\lambda\left(d^2N + d(-2\beta + N(2\beta + N - 1) + 2) - N^2\right)\right.$$
$$\left. - \left(d(\beta(N-1)+1)(d(\beta + N(\beta(N-2) + N + 2) - 1) + N(N+1)(\beta(N-1) + N + 1)) + \gamma N(d+N)^2\right)\right],$$
$$z^N = \frac{1}{A}\left[(N+1)\left(d^2N + d(-2\beta + N(2\beta + N - 1) + 2) - N^2\right) - c\lambda N(d+N)^2\right],$$

respectively, where $A \equiv (d+N)^2c^2\lambda^2N - (N+1)\left[d(\beta(N-1)+1)(d(\beta + N(\beta(N-2) + N + 2) - 1)\right.$ $\left.+N(N+1)(\beta(N-1) + N + 1)) + \gamma N(d+N)^2\right]$.

We follow the same process at determining the signs of x^N and z^N. The cutoff γ_1^N is found by solving A for γ, γ_2^N is found by solving the term in the square brackets of x^N for γ, and λ_1^N is found by solving the term in the square brackets of z^N for λ. These cutoffs are

$$\gamma_1^N \equiv \frac{c^2\lambda^2}{N+1} - \frac{d\left[\beta(N-1)+1\right]\left[d(\beta + N(\beta(N-2) + N + 2) - 1\right] + N(N+1)\left[\beta(N-1) + N + 1\right]}{N(d+N)^2},$$
$$\gamma_2^N \equiv \frac{c\lambda\left[d^2N + d\left[N(2\beta + N - 1) + 2 - 2\beta\right] - N^2\right]}{N(d+N)^2}$$
$$- \frac{d\left[\beta(N-1)+1\right]\left[d(\beta + N\left[\beta(N-2) + N + 2\right] - 1\right] + N(N+1)\left[\beta(N-1) + N + 1\right]}{N(d+N)^2}, \text{ and}$$
$$\lambda_1^N \equiv \frac{(N+1)\left[d^2N + d\left[N(2\beta + N - 1) + 2 - 2\beta\right] - N^2\right]}{cN(d+N)^2}$$

Therefore, for $x^N > 0$, we need that either $\gamma > \max\{\gamma_1^N, \gamma_2^N\}$ (which implies that both the numerator and denominator of x^N are negative, making x^N positive) or that $\gamma < \min\{\gamma_1^N, \gamma_2^N\}$ (entailing that both the numerator and denominator of x^N are positive). For $z^N > 0$, we need that either: (a) $\gamma < \gamma_1^N$ and $\lambda > \lambda_1^N$; or (b) $\gamma > \gamma_1^N$ and $\lambda < \lambda_1^N$.

The cutoffs γ_1^N, γ_2^N, and λ_1^N have the same relationship with each other as their $N = 1$ counterparts that we found in Proposition 1. That is, $\gamma_1^N > \gamma_2^N$ holds if $\lambda > \lambda_1^N$, and $\gamma_1^N < \gamma_2^N$ holds if $\lambda < \lambda_1^N$. It is also the case that the crossing point of cutoffs γ_1^N and γ_2^N occurs at the negative quadrant.

When evaluating the equilibrium values, the above analysis leaves us with the same 8 cases as in the proof of Proposition 1. Because the cutoffs in the N-firm case hold the same relationships with each other and the signs of the equilibrium values as in the $N = 1$ case, the analysis is analogous to that of Cases 1-8 in the proof of Proposition 1. This leaves us with only cases 1, 5, 6, and 7 as viable.

Therefore, we can summarize the only cases that can be sustained in equilibrium under admissible parameter values. In particular, every firm i chooses to:

1. invest in R&D, but not produce first-period output, if $\gamma_1^N \geq \gamma > \gamma_2$ (corresponding to Case 7, which is graphically depicted in Region III of Figure 3);

2. produce first-period output, but not invest in R&D, if: (1) $\gamma < \gamma_2^D$ and $\lambda < \lambda_1^D$; or (2) $\gamma > \gamma_1^D$

and $\lambda > \lambda_1^D$ (corresponding to Case 6, or Region II in Figure 3);

3. invest in R&D and produce first-period output otherwise (corresponding to Cases 1 and 5, or Regions I and Ib in Figure 3, respectively).

7.4 Proof of Proposition A1

Third period. In the third period,the monopolist maximizes its profit by choosing third-period output q:

$$\max_q \ \pi = (a-q)q - (c-\alpha z_i)q - t(q-z),$$

and the first-order condition is

$$\frac{\partial \pi}{\partial q} = a - (c-\alpha z) - 2q - t = 0,$$

which we can re-arrange to find an output function

$$q(t) = \frac{(a-(c-\alpha z)) - t}{2}$$

which, as expected, is increasing in the cost-reducing effect of R&D, α.

Second period. In the second period, the regulator chooses emission fee t to maximize social welfare

$$\max_t \quad SW = CS(q(t)) + PS(q(t)) - Env(q(t))$$

where

$$\begin{aligned} CS(q(t)) + PS(q(t)) &= \int_0^{q_i(t)} (a-(c-\alpha z_i)-x)\ dx - \frac{\gamma z^2}{2} \\ &= q(a-c+\alpha z) - \frac{q^2}{2} - \frac{\gamma z^2}{2}, \end{aligned}$$

and $Env(q(t)) = \frac{1}{2} d\,(q(t)-z)^2$. The first-order condition is

$$\frac{\partial SW}{\partial t} = \frac{1}{4}\left[d(a-c-t+\alpha z-2z) - a + c - t - \alpha z\right] = 0$$

Simplifying and solving for t, we find an emission fee

$$t(z_i) = \frac{(a-c)(d-1) + (\alpha-2)dz - \alpha z}{d+1}$$

First period. In the first period, the monopolist chooses output x and R&D investment z to solve

$$\max_{x,z} \pi = \underbrace{(a-x)x - cx(1+\lambda z) - \frac{1}{2}\gamma z^2}_{\text{third period profit}} + \underbrace{(a-q(t(z)))]q(t(z)) - (c-\alpha z)q(t(z)) - t(z)q(t(z))}_{\text{first period profit}},$$

The first-order conditions are

$$\begin{aligned}
&\frac{\partial \pi}{\partial x} = a - c(\lambda z + 1) - 2x = 0 \\
&\frac{\partial \pi}{\partial z} = \frac{1}{(d+1)^2}\left[a(2\alpha + d(d+2) - 1) + c\left(-2\alpha + (d+1)^2\lambda(-x) - d(d+2) + 1\right)\right. \\
&\qquad \left. +2(\alpha - 1)z(\alpha + d(d+2)) + \gamma(d+1)^2(-z)\right] = 0
\end{aligned}$$

In the first period, the monopolist's equilibrium output and R&D investment are

$$\begin{aligned}
x(\alpha) &= \frac{(a-c)\left(2\alpha(\alpha + c\lambda - 1) + d^2(2\alpha + c\lambda - \gamma - 2) + 2d(2\alpha + c\lambda - \gamma - 2) - c\lambda - \gamma\right)}{c^2(d+1)^2\lambda^2 + 4(\alpha - 1)(\alpha + d(d+2)) - 2\gamma(d+1)^2}, \\
z(\alpha) &= \frac{(a-c)(-4\alpha + d(d+2)(c\lambda - 2) + c\lambda + 2)}{c^2(d+1)^2\lambda^2 + 4(\alpha - 1)(\alpha + d(d+2)) - 2\gamma(d+1)^2}.
\end{aligned}$$

We follow the same process as in Proposition 1 to determine the signs of $x(\alpha)$ and $z(\alpha)$. The cutoff $\gamma_1(\alpha)$ is found by solving the denominator of either $z(\alpha)$ or $x(\alpha)$ for γ, cutoff $\gamma_2(\alpha)$ is found by solving the numerator of $x(\alpha)$ for γ, and cutoff $\lambda_1(\alpha)$ is found by solving the numerator of $z(\alpha)$ for λ. These cutoffs are

$$\begin{aligned}
\gamma_1(\alpha) &\equiv \frac{c^2\lambda^2}{2} + \frac{2(\alpha - 1)(\alpha + d(d+2))}{(d+1)^2}, \\
\gamma_2(\alpha) &\equiv \frac{c\lambda(2\alpha + d(d+2) - 1) + 2(\alpha - 1)(\alpha + d(d+2))}{(d+1)^2}, \text{ and} \\
\lambda_1(\alpha) &\equiv \frac{4\alpha + 2d(d+2) - 2}{c(d+1)^2}.
\end{aligned}$$

Therefore, for $x(\alpha) > 0$, we need that either $\gamma > \max\{\gamma_1(\alpha), \gamma_2(\alpha)\}$ (which implies that both the numerator and denominator of $x(\alpha)$ are negative, making $x(\alpha)$ positive) or that $\gamma < \min\{\gamma_1(\alpha), \gamma_2(\alpha)\}$ (entailing that both the numerator and denominator of $x(\alpha)$ are positive). For $z(\alpha) > 0$, we need that either: (a) $\gamma < \gamma_1(\alpha)$ and $\lambda > \lambda_1(\alpha)$; or (b) $\gamma > \gamma_1(\alpha)$ and $\lambda < \lambda_1(\alpha)$.

The cutoffs $\gamma_1(\alpha)$, $\gamma_2(\alpha)$, and $\lambda_2(\alpha)$ have the same relationship with each other as their counterparts that we found in Proposition 1 (evaluated at $N = 1$ and $\alpha = 0$). That is, $\gamma_1(\alpha) > \gamma_2(\alpha)$ if $\lambda > \lambda_1(\alpha)$, and $\gamma_1(\alpha) < \gamma_2(\alpha)$ if $\lambda < \lambda_1(\alpha)$.

When evaluating the equilibrium values, the above analysis leaves us with the same 8 cases as in Proposition 1. Because the cutoffs in this case hold the same relationships with each other and the signs of the equilibrium values as in the $N = 1$ and $\alpha = 0$ case, the analysis used to find the viable cases is analogous. This leaves us with cases 1, 5, 6, and 7 as viable.

7.5 Proof of Proposition A2

Third period. In the third period, every firm i maximizes its profit by choosing third-period output q_i, as follows

$$\max_{q_i} \quad \pi_i = (a - q_i - q_j)q_i - (c - \alpha(z_i + \beta z_j))q_i - t(q_i - z_i - \beta z_j),$$

and the first-order condition is

$$\frac{\partial \pi_i}{\partial q_i} = a - [c - \alpha(z_i + \beta z_j)] - 2q_i - q_j - t = 0,$$

which we can re-arrange to find best response function

$$q_i(q_j) = \frac{a - [c - \alpha(z_i + \beta z_j)] - t}{2} - \frac{1}{2}q_j.$$

In a symmetric equilibrium, $q_i = q_j$, which entails an output function

$$q_i(t) = \frac{a - [c - \alpha(z_i + \beta z_j)] - t}{3}$$

for every firm i, which we use in the second period analysis.

Second period. In the second period, the regulator chooses emission fee t to maximize social welfare

$$\max_{t} \; SW = CS(q(t)) + PS(q(t)) - Env(q(t))$$

where

$$\begin{aligned} CS(q(t)) + PS(q(t)) &= \int_0^{2q_i(t)} (a - (c - \alpha(z_i + \beta z_j)) - x) \; dx - \sum_{i=1}^{2} \frac{\gamma z_i^2}{2} \\ &= \frac{2(a - (c - \alpha(z_i + \beta z_j)))(a - (c - \alpha(z_i + \beta z_j)) - t)}{3} \\ &\quad - \frac{2(a - (c - \alpha(z_i + \beta z_j)) - t)^2}{9} - \sum_{i=1}^{2} \frac{\gamma z_i^2}{2}, \end{aligned}$$

and $Env(q(t)) = \frac{1}{2} d \sum_{i=1}^{2} \left(q(t) - z_i - \beta(z_j)\right)^2$. The first-order condition is

$$\begin{aligned} \frac{\partial SW}{\partial t} = \frac{1}{18}&(d(2(a - c - t + (\alpha - 3)(\beta z_i + z_j)) - 2(a - c - t + (\alpha - 3)(z_i + \beta z_j))) \\ &+ 4(a - c - t + \alpha z_i + \alpha\beta z_j) - 4(2a - 2c + t + \alpha(3\beta + 2)z_i - \alpha\beta z_j)) = 0 \end{aligned}$$

Simplifying and solving for t, we find an emission fee

$$t(z_i) = \frac{2a(d-1) - 2c(d-1) - 3(\beta + 1)d(z_i + z_j)}{2(d+2)}$$

First period. In the first period, every firm i chooses output x_i and R&D investment z_i to solve

$$\max_{x_i,z_i} \pi_i = \underbrace{(a - x_i - x_{-i})x_i - cx_i(1+\lambda z_i) - \frac{1}{2}\gamma z_i^2}_{\text{third period profit}}$$
$$+ \underbrace{(a - q_j(t(z_j)) - q_i(t(z_i))]q_i(t(z_i)) - (c - \alpha(z_i + \beta z_j))q_i(t(z_i)) - t(z_i)[q_i(t(z_i)) - z_i - \beta z_j]}_{\text{first period profit}},$$

The first-order conditions are

$$\frac{\partial \pi_i}{\partial x_i} = a - c(\lambda z_i + 1) - 2x_i - x_j = 0$$
$$\frac{\partial \pi_i}{\partial z_i} = -\frac{1}{6(d+2)^2}\left[-2a\left(9\alpha(\beta+1) + 3d^2 + d(-2\alpha\beta + 2\alpha + 3\beta + 6) - 6\right)\right.$$
$$+ 2c\left(9\alpha(\beta+1) + 3d^2 + d(-2\alpha\beta + 2\alpha + 3\beta + 6) - 6\right) + 6c(d+2)^2\lambda x_i - \alpha^2\beta^2 d^2 z_i + 2\alpha^2\beta d^2 z_i$$
$$- \alpha^2 d^2 z_i + 4\alpha\beta^2 d^2 z_i - 6\alpha\beta d^2 z_i - 10\alpha d^2 z_i - 3\beta^2 d^2 z_i + 12\beta d^2 z_i + 15 d^2 z_i + \alpha^2\beta^2 d^2 z_j - 2\alpha^2\beta d^2 z_j$$
$$+ \alpha^2 d^2 z_j - 3\alpha\beta^2 d^2 z_j - 6\alpha\beta d^2 z_j - 3\alpha d^2 z_j + 6\beta^2 d^2 z_j + 12\beta d^2 z_j + 6d^2 z_j + 10\alpha^2\beta^2 d z_i - 10\alpha^2 d z_i$$
$$- 18\alpha\beta^2 d z_i - 12\alpha\beta d z_i - 18\alpha d z_i + 36\beta d z_i + 6\gamma(d+2)^2 z_i + 36 d z_i - 6\alpha^2\beta^2 d z_j + 6\alpha^2 d z_j + 6\alpha\beta^2 d z_j$$
$$- 30\alpha\beta d z_j - 12\alpha d z_j + 18\beta^2 d z_j + 36\beta d z_j + 18 d z_j - 24\alpha^2\beta^2 z_i - 48\alpha^2\beta z_i - 24\alpha^2 z_i + 72\alpha\beta z_i$$
$$\left. + 24\alpha z_i + 6\alpha^2\beta^2 z_j + 12\alpha^2\beta z_j + 6\alpha^2 z_j + 36\alpha\beta^2 z_j - 12\alpha\beta z_j\ \right] = 0$$

In the first period, every firm i's equilibrium output and R&D investment are

$$x^{(2)}(\alpha) = \frac{1}{3B}(a-c)\left[\left((\beta+1)\left(6\alpha(-3\alpha(\beta+1) + 6\beta + 4) + d^2(\alpha(\beta-13) + 3(\beta+7))\right.\right.\right.$$
$$\left. +2d(\alpha(2(\alpha-3)\beta - 2\alpha - 15) + 9(\beta+3))) + 6\gamma(d+2)^2\right)$$
$$\left. -2c\lambda(a-c)\left(9\alpha(\beta+1) + 3d^2 + d(-2\alpha\beta + 2\alpha + 3\beta + 6) - 6\right)\right],$$
$$z^{(2)}(\alpha) = \frac{1}{B}2(a-c)\left(9\alpha(\beta+1) - d^2(c\lambda - 3) + d(2\alpha(1-\beta) + 3\beta - 4c\lambda + 6) - 4c\lambda - 6\right),$$

respectively, where

$$B \equiv -2c^2(d+2)^2\lambda^2 + 6\gamma(d+2)^2 + (\beta+1)(6\alpha(-3\alpha(\beta+1) + 6\beta + 4)$$
$$+d^2(\alpha(\beta-13) + 3(\beta+7)) + 2d(\alpha(2(\alpha-3)\beta - 2\alpha - 15) + 9(\beta+3))).$$

We follow the same process at determining the signs of $x^{(2)}(\alpha)$ and $z^{(2)}(\alpha)$. Cutoff $\gamma_1^2(\alpha)$ is found by solving the denominator, B, for γ, cutoff $\gamma_2^2(\alpha)$ is found by solving the term in the square brackets of $x^{(2)}(\alpha)$ for γ, and cutoff $\lambda_1^2(\alpha)$ is found by solving the term in the square brackets of

$z^{(2)}(\alpha)$ for λ. These cutoffs are

$$\gamma_1^2(\alpha) \equiv \frac{c^2\lambda^2}{3} - \frac{\beta+1}{6(d+2)^2}\left[-6\alpha(3\alpha(\beta+1)-6\beta-4)+d^2(\alpha(\beta-13)+3(\beta+7))\right.$$
$$\left.+2d(\alpha(2(\alpha-3)\beta-2\alpha-15)+9(\beta+3))\right],$$
$$\gamma_2^2(\alpha) \equiv \frac{1}{6(d+2)^2}\left[2c\lambda\left(9\alpha(\beta+1)+3d^2+d(-2\alpha\beta+2\alpha+3\beta+6)-6\right)\right.$$
$$-(\beta+1)\left[6\alpha(-3\alpha(\beta+1)+6\beta+4)+d^2(\alpha(\beta-13)+3(\beta+7))\right.$$
$$\left.\left.+2d(\alpha(2(\alpha-3)\beta-2\alpha-15)+9(\beta+3))\right]\right]$$
$$\lambda_1^2(\alpha) \equiv \frac{9\alpha(\beta+1)+3d^2+d(-2\alpha\beta+2\alpha+3\beta+6)-6}{c(d+2)^2}$$

Therefore, for $x^{(2)}(\alpha) > 0$, we need that either $\gamma > \max\{\gamma_1^2(\alpha), \gamma_2^2(\alpha)\}$ (which implies that both the numerator and denominator of $x^{(2)}(\alpha)$ are negative, making $x^{(2)}(\alpha)$ positive) or that $\gamma < \min\{\gamma_1^2(\alpha), \gamma_2^2(\alpha)\}$ (entailing that both the numerator and denominator of $x^{(2)}(\alpha)$ are positive). For $z^2(\alpha) > 0$, we need that either: (a) $\gamma < \gamma_1^2(\alpha)$ and $\lambda > \lambda_1^2(\alpha)$; or (b) $\gamma > \gamma_1^2(\alpha)$ and $\lambda < \lambda_1^2(\alpha)$.

The cutoffs $\gamma_1^2(\alpha)$, $\gamma_2^2(\alpha)$, and $\lambda_2(\alpha)$ have the same relationship with each other as their $N = 1$ and $\alpha = 0$ counterparts that we found in Proposition 2. That is, $\gamma_1^2(\alpha) > \gamma_2^2(\alpha)$ if $\lambda > \lambda_1^2(\alpha)$, and $\gamma_1^2(\alpha) < \gamma_2^2(\alpha)$ if $\lambda < \lambda_1^2(\alpha)$.

When evaluating the equilibrium values, the above analysis leaves us with the same 8 cases as in Proposition 2. Because the cutoffs in this case hold the same relationships with each other and the signs of the equilibrium values as in the $N = 2$ and $\alpha = 0$ case, the analysis used to find the viable cases coincides. This leaves us with cases 1, 5, 6, and 7 as viable.

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