Recitation #5 (09/27/2019)

1. Consider the following profit function that has been obtained from a technology that uses a single input, z:

$$\pi(p, w) = p^2 w^\alpha$$

where p is the output price, w is the input price and α is a parameter value.

   (a) Check if the profit function satisfies homogeneity of degree one jointly in both p and w. In particular, determine for which values of α this property is satisfied.

   (b) Assuming the value of α for which the profit function satisfies homogeneity of degree one, check if the profit function $\pi(p, w)$ satisfies the following properties: (1) non-decreasing in output price p, (2) non-increasing in input prices w, and (3) convex in prices p and w.

   (c) Calculate the supply function of the firm, $q(p, w)$, and its demand for inputs, $z(p, w)$.

2. Suppose that a firm owns two plants, each producing the same good. Every plant j's average cost is given by

$$AC_j(q_j) = \alpha + \beta_j q_j \quad \text{for} \quad q_j \geq 0, \text{ where } j = \{1, 2\}$$

where coefficient β_j may differ from plant to plant, i.e., if $\beta_1 > \beta_2$ plant 2 is more efficient than plant 1 since its average costs increase less rapidly in output. Assume that you are asked to determine the cost-minimizing distribution of aggregate output $q = q_1 + q_2$, among the two plants (i.e., for a given aggregate output q, how much q_1 to produce in plant 1 and how much q_2 to produce in plant 2.) For simplicity, consider that aggregate output q satisfies $q < \frac{\alpha}{\max_j |\beta_j|}$. (You will be using this condition in part b.)

   (a) If $\beta_j > 0$ for every plant j, how should output be located among the two plants?

   (b) If $\beta_j < 0$ for every plant j, how should output be located among the two plants?

   (c) If $\beta_j > 0$ for some plants and $\beta_i < 0$ for others?