

Homework #6 (Due on October 2nd, 2019)

1. Consider a firm with production function $q = \sqrt{z}$, using one input (e.g., labor) to produce units of output q . The price of every unit of input is $w > 0$, and the price of every unit of output is $p > 0$.

- (a) Set up the firm's cost-minimization problem (CMP), and solve for its conditional factor demand $z(w, q)$ for any output level q . (For now, we write the constraint of the CMP to be $f(z) \geq q$, where the output level q that the firm seeks to reach does not necessarily coincide with that found in part (b), $q(w)$.)
- (b) Evaluate the conditional factor demand $z(w, q)$ at output level $q = q(w)$, to obtain $z(w, q(w))$. Show that it coincides with the unconditional factor demand $z(w, p)$ found in Homework #5 (question #3, part (a)), that is,

$$z(w, q(w)) = z(w, p).$$

- (c) *Shephard's lemma*. Evaluate the CMP's objective function, $w \cdot z$, at the conditional factor demand $z(w, q)$, to obtain the cost function, that is, find $c(w, q) = w \cdot z(w, q)$. Differentiate the cost function with respect to w , and show that your result coincides with the conditional factor demand $z(w, q)$.
 - (d) *Substitution and output effects*. Let us now consider that the firm faces cheaper wages (lower w). Differentiate the unconditional factor demand $z(w, p)$ found in part (a) with respect to w to find the total effect of this price change.
 - (e) Differentiate the conditional factor demand $z(w, q)$ found in part (a) with respect to w to obtain the substitution effect of this price change.
 - (f) Compare your results in parts (d) and (e). Which is the output effect of the change in w ?
2. Consider a firm with Cobb-Douglas production function $q = f(z_1, z_2) = z_1 z_2$, using two inputs (e.g., labor and capital) to produce units of output q . The price of every unit of input 2 is normalized to \$1, whereas the price of every unit of input 1 is $w > 0$. The price of every unit of output is $p > 0$. Repeat the analysis in the previous exercise (all parts including Homework #5 (question #3, part (a)-(b))). Note that for parts (d)-(f) you will need to separately examine own-price effects and cross-price effects since now we deal with two inputs.